INTERMEDIATE PHYSICS

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SECOND EDITION

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PREFACE

In offering this small book on Elementary Physics to the students, I owe them an explanation for adding one more to the large number of text-books on the subject. While at Delhi, I had published 'Mechanics' and 'Electricity and Magnetism' in separate parts. Since then I have been asked by my pupils to complete the course. It is in fact, with that object in view, that I have ventured to present this elementary treatise to the Intermediate Science students. The book meets with the requirements of the candidates preparing for the Intermediate Examinations of the Punjab University, but it will be found to cover the syllabi of the other Indian Universities as well.

2. An attempt has been made to present the facts in such a way as to enable the student to grasp the fundamental ideas and help him to pass through his examinations. To facilitate the student's work, a brief summary and a good many solved examples have been added at the end of each Chapter. Criticisms and suggestions to improve the book from teachers and students alike will be gratefully received.

3. In the end, it is my pleasant duty to thank the several friends working in the Laboratory for offering valuable suggestions. My thanks are also due to Mr. Rachhpal Singh, B.A., B.T., for seeing the book through the press.

GOVT. COLLEGE, LAHORE.
1st. May, 1931.

B. D. CHHABRA.

PREFACE TO SECOND EDITION

The book has been revised and most of the suggestions received so far, incorporated in the text; which now includes the proposed additions to the syllabus. My sincere thanks are due to the several friends, who very kindly sent me valuable suggestions for its improvement. Changes have only been made where experience has shown that students found unusual difficulties. Further suggestions to make the book more useful to the students will be gratefully acknowledged.

PHYSICS DEPTT.
GOVT. COLLEGE, LAHORE.
1st. January, 1932.

B. D. CHHABRA.

CONTENTS

MECHANICS

CHAPTER I

INTRODUCTION

1. **Introductory** :—Mechanics is that branch of Physics, which deals with motion. Its object is to describe the different kinds of motion and the laws governing them.

To understand completely the various laws governing the motion of bodies, it is essential that the ideas of *Space*, *Time* and *Mass* be clearly understood. Our first step in this direction would be to understand clearly the above mentioned fundamental quantities and the second, to deal with the methods employed to measure them.

(*i*) **Space.**—Suppose you want to have a bath in a tub full of water to the brim. As soon as you get into the tub, some of the water overflows. It is impossible that the water should not overflow, because the water was occupying some space and now you want to occupy the very same space. Every body must have a certain space exclusively to itself; so two bodies cannot occupy the same space at one and the same time. In this universe every star, planet, satellite or even the minutest particle of matter occupies a certain space to the exclusion of other bodies.

(*ii*) **Time.**—Each day, the sun rises, moves higher into the sky, until at mid-day it crosses the meridian, then it sinks lower and finally sets. A man, who is in his senses, thus notices successive events taking place at successive intervals of time.

The interval of time between the sun's highest position on any one day and the same position on the

next day is called the *apparent solar day.* The length of such a day varies throughout the year. But if the lengths of all the days in the year be added and then divided by the number of days in the year, we obtain what is called the *mean solar day.*

(*iii*) **Matter.**—Anything the existence of which can be recognised by the sense of sight, touch, taste, smell or hearing is called *matter.* Bodies are considered to be composed of matter and though we know many of the properties of matter, yet we know very little of its nature. Matter occupies space and possesses weight on the surface of the earth.

Mass.—The mass of a body is the quantity of matter that it contains, a student can have better idea of mass than of any other foregoing terms.

2. Measurement of Fundamental quantities :—The measurement of a quantity such as length is a familiar operation. To perform this, we take a rod and then find out how many times the length of the rod is contained in the length we wish to measure. In order that our measurement should be intelligible to the people, who cannot see the particular rod, it is necessary that the rod used should be of some definite length, which may be accepted as the *standard* and which may be easily procurable. Such a standard may be called a unit of length. A quantity is necessarily measured in terms of a unit of its own kind. In general, we measure the distance between two points in miles or feet, centimetres or inches; the mass of a lump of stone in tons or maunds; the time in days or hours etc. Hence we see, that to measure any quantity, we have to *fix the unit* and *then* to *compare the given quantity* with the *fixed unit.* For example, when it is said that the distance between two fixed points is four feet, it is implied that a certain length called a *foot* has been adopted as a unit and that four of these placed end to end exactly cover the distance.

The choice of a unit is perfectly arbitrary and is settled by convention and convenience. As a result of

the freedom of choice in the selection of units, the systems of all nations are not the same. In this book, we will deal with two distinct systems, the *English* or the *F.P.S.* system and the *French* or the *C.G.S.* system.

The English or Foot-Pound-Second system:—

(*a*) **The unit of length.**—The unit of length is the '*Foot*'. This is one-third of the yard, which is defined by an Act of Parliament* as "The straight line or distance, between the centres of the transverse lines on the two gold plugs in the bronze bar, deposited in the office of the Exchequer, shall be the genuine standard yard at 60° Fahrenheit and if lost, it shall be replaced by its copies."

(*b*) **The Unit of Mass.**—The unit of mass is the '*Pound Avoirdupois*' and is the mass of a lump of Platinum marked "P. S. 1884 1℔." deposited in the Standards Department of the Board of Trade at Westminster.

(*c*) **The Unit of Time.**—The unit of time is the *Mean Solar Second* and is equal to $\frac{1}{24 \times 60 \times 60} = \frac{1}{86400}$th part of the mean solar day.

The Metric or Centimetre-Gramme-Second system:—

(*a*) **The unit of Length.**—The unit of length is the *centimetre.* This is one-hundredth part of the metre, which was defined by the law of the French Republic in 1795 to be the distance between the ends of a rod of platinum made by Borda †, the temperature of the rod being that of melting ice.

(*b*) **The unit of Mass.**—The unit of mass is the *Gramme* and is one-thousandth part of the kilogramme,

* 18 and 19 Victoria, Cap. 72, July 30, 1855.

† At this date, the distance between the pole and the equator along a certain meridian arc of the Earth's surface had recently been measured by Delambre; and it was supposed that Borda's platinum rod at 0°C. represented one ten-millionth of this distance. Further research has, however, shown that this is not so. Thus the standard is the Borda's platinum rod and not the Earth's arc.

which is defined as a certain lump of platinum made by Borda* in 1795 and kept in Paris.

(*c*) **The unit of Time.**—It is the same as in the English system, that is the mean *solar second.*

The units enumerated above are called fundamental units, as all the other units depend upon them. These three units have no connection with one another, but all other units, with which we shall have to deal, are dependent upon those and cannot be arbitrarily chosen, for if an arbitrary unit were selected for the measurement of every physical quantity, without any consideration of the interrelation of the quantities, great confusion would result. The units of other quantities therefore, are made to depend on the fundamental units and are therefore called *DERIVED UNITS.* For example, the units of area and volume depend directly on that of length, they are respectively a square, whose side is one centimetre and a cube, whose edge is one centimetre.

Fundamental Units on the two systems with their submultiples and multiples are given below:—

F.P.S. or English system		C.G.S. or French system	
(*a*) Lengths.			
1 mile	=1760 yards	1 metre	=10 decimetres
1 yard	= 3 feet	1 decimetre	=10 centimetres
1 foot	= 12 inches	1 centimetre	=10 millimetres
(*b*) Masses.			
1 ton	=2240 pounds	1 kilogramme	=1000 grammes
1 pound	= 16 ounces	1 gramme	=1000 mgms.
(*c*) Time.			
1 Solar day	= 24 hours	1 Solar day	=24 hours
1 hour	= 60 minutes	1 hour	=60 minutes
1 minute	= 60 seconds	1 minute	=60 seconds

The relation between the fundamental units in the two systems is shown below with sufficient accuracy.

* It was intended that gramme should be equal to the mass of a cubic centimetre of distilled water at 4°C.

A. LENGTHS.		B. MASSES.	
1 metre	=39·37079 ins. or 1·09362 yds.	1 gramme	=·002205 lb.
1 inch	=2·53995 cms.	1 pound	=453·514 grams.
1 mile	=1·6 k. metre.	C. TIME.	
1 cm.	=·03281 foot	The units are the same.	

3. Properties of mass. Although we know very little about the exact nature of matter, still we know many of its properties. These, we shall have occasion to refer at their proper place in subsequent pages.

The most fundamental property of matter is that it occupies *Volume*, which is defined as the amount of space a body takes up. We all know that the same portion of space cannot be filled up by different portions of matter at the same time; that is, every body occupies a certain portion of space to the exclusion of all other bodies.* The second important property of matter is that *the quantities of matter in two portions of the same homogeneous substance are proportional to the volumes of the two.* Thus if we have two equal cubes of iron, their masses (which are quantities of matter contained in them) are found to be identical and if they be combined into one, the volume as well as the mass of the combined body will be double that of each cube.

Further we see that if the cube of iron considered above be turned into some other shape, *i. e.*, a cylinder, still the quantity of matter will remain the same; hence we come to the conclusion *that the mass of a body is always the same, and is not altered by changing its form.*

4. Sub-divisions of Mechanics.—Mechanics is subdivided into *Kinetics* and *Kinematics.* The latter includes *Dynamics*, *Statics*, *Hydrodynamics* and *Hydrostatics.*

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* An exception to this may appear in the case of absorption of water by a piece of sponge or a lump of sugar. But in this case, the real volume of the sponge or sugar is less than its apparent volume, for their particles are separated by small air spaces and it is into these that the water penetrates.

Kinetics deals with the science of motion *with* reference to the forces causing it.

Kinematics deals with the geometry of motion *without* any reference to the forces causing it.

Dynamics deals with a set of forces acting on a body so as to produce motion.

Statics deals with a set of forces acting on a body in such a way as to keep it in equilibrium.

Dynamics and Statics are generally applied to the Mechanics of solid bodies and that branch of Mechanics which deals with the motion of fluids is called *Hydrodynamics* and the one, which deals with the equilibrium of fluids is known as *Hydrostatics.*

5. Motion. We have used the word motion above so many times, that it seems expedient to define it here. *Motion is the change of position* and that which moves is *matter.* In order to see whether a body is in motion or not, we have to determine its position at different intervals of time and to notice whether it changes or not. *A body which occupies different positions at different intervals of time is said to be in motion.*

Motion and Rest Relative. Motion as defined above is a relative term. A body such as a window, is commonly said to be at rest, when it occupies the same position with respect to other surrounding objects such as houses, trees, telegraphic poles and the like; but we know, the house itself is not actually at rest, for it is moving along with the earth round the Sun. Similarly two men seated in a moving railway carriage are at rest relative to each other and to the carriage; but relative to other objects on the surface of the earth, they are in motion. The planets move round the Sun, the whole Solar system has a motion relative to other stars, which in turn may have their own motion. Thus whenever we want to investigate the motion of a certain object, we have to regard a certain other object as fixed which in practice is not absolutely true. Hence motion and rest are only

relative terms.

Particle. It is a portion of matter, whose volume is considered to be negligibly small.

6. Force. "Any circumstance, the consequence of which is motion in matter is called *Force.*" Thus the idea of force is subsidiary and it may be defined as *any cause which acting on a body, changes or tends to change its state of rest or of uniform motion in a straight line.*

The earliest conception of force is that of muscular exertion and anything capable of producing similar effects. Thus a steam engine exerts force when moving a carriage and gunpowder exerts force on a bullet, when it is passing from the breech to the muzzle of the gun.

In order that an agent may exert force, there must be something to be acted upon, which may offer resistance or impedence. A moving bullet is not exerting any force unless it meets some impedence such as a target. Force is always the mutual action of two bodies, one the agent exerting the force and the second the impedence offering resistance to the agent. Such a pair consisting of a force and resistance taken together is given the name of a *stress.* It is called a *Pressure*, if the two forces are acting towards each other, and a *Tension* if the two forces are acting away from each other.

7. Density. The mass of a given substance depends on its volume, but for bodies of given volume, the mass depends on the substance of which the bodies consist and also on the Physical conditions existing at the time. Thus a big lump of gold has a greater mass than a small piece of it. It is therefore expedient to have a term to denote the mass of a definite volume, say one cubic centimetre of any substance under normal conditions. This term is called *Density*, *which is defined as the mass of unit volume of the given substance* or the ratio of *the mass* of any quantity of it to its *own volume.* From the above definition, it

follows that the mass M grammes of a body is equal to the product of the volume V in cubic centimetres and D the density; for $D = \frac{M}{V}$ or $M = VD$.

7 (*a*). Specific Gravity. Specific gravity of a substance is the ratio of the mass of the given substance to the mass of an equal volume of water at 4°C.* As the mass of 1 c.c. of water at 4°C. is one gramme, therefore the measure of specific gravity or density of a substance is the same. But it is to be remembered that density is necessarily *mass per unit volume*, while specific gravity is a *number*, because it is the ratio of two quantities.

SUMMARY

Mechanics deals with different kinds of motion and the laws governing them.

Space. It is the association of certain sense expressions in a group and its separation from other groups.

Time. It is the recognition of an order of sequence.

Matter. Anything, the existence of which can be recognized by the sense of sight, touch, smell, taste or hearing is called matter.

Mass. It is the quantity of matter in a body.

Volume. It is the amount of space a body occupies.

F. P. S. and C. G. S. systems of units are the two distinct systems of measurement. In the F. P. S system, Foot, Pound and Second are the units of length, mass and time and on the C. G. S. system, Centimetre, Gramme and Second are the respective units.

Motion. Change of position of a body is called motion.

Particle. The smallest portion of matter, which has point existence and no linear dimensions, is called a particle.

Force. That which changes or tends to change a body's state of rest or of uniform motion in a straight line, is called force.

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* Water has its maximum density at 4°C. and therefore that is chosen as the standard for finding the specific gravity of solids and liquids; but for gases, Hydrogen under normal conditions of temperature and pressure is chosen as the standard.

Stress. Force per unit area is called stress.

Pressure. A pair of forces acting towards each other is called a Pressure.

Tension A pair of equal forces acting away from each other is called a tension.

Density. It is mass per unit volume.

Specific gravity. It is the ratio of mass of a given substance to the mass of an equal volume of water at 4°C.

EXAMPLES I

1. Find the number of cubic feet in a litre

1 litre = 1000 c. cms.

= 1000 ('0328)^3 cubic ft for 1 cm. = '0328 ft.

= 03532 cubic ft.

2. The density of a piece of cork is 0·25 grammes per c cm. Find it in lbs. per cubic foot.

1 c. c. = ('0328)^3 cubic ft.

= '00003532 cubic ft

0'25 gms. = '25 × '002205 lb.

Hence a volume of '00003532 cubic foot contains '25 × '002205 lb.

$$\therefore \quad \text{the required density} = \frac{\cdot 25 \times \cdot 002205}{\cdot 00003532} = 15\cdot 608 \text{ lbs.}$$

per cubic foot.

3. Supposing metre to be one ten-millionth part of the distance from the Pole to the Equator. Find the circumference of the Earth in miles.

4. Find the density of a cylinder one foot in height, 6 inches in radius, and 40 lbs. in mass.

5. What is the mass of a sphere of gold 10 cms. diameter? (density of gold = 19'0).

6. The density of mercury is 13'50 grammes per c.c. Find it in lbs. per cubic inch.

7. The inside measurements of a rectangular tank are 10 feet × 5 feet × 5 feet. Calculate its capacity (*a*) in litres (*b*) in cubic feet. Find the mass of water, which it would hold (*i*) in kilogrammes (*ii*) in lbs.

CHAPTER II

KINEMATICS

VELOCITY

8. Motion. It has already been defined *as the change of position of a body.*

Speed. It is the *rate* of change of position of a body, when the *line* and the *direction* along which it moves are *not* taken into account.

Velocity. It is the *rate* of change of position of a body, when the *line* and the *direction* along which it moves *are taken* into account.

To express a velocity completely, besides mentioning the magnitude, it is necessary to denote the direction and the line of motion. Such quantities (requiring specification of line and direction) are called *vector* quantities or *vectors.* Thus speed is not a vector quantity, while velocity is. Hence if a man were to move from O along OA at the rate of 20 feet per second in the direction of OA and his velocity were expressed by $+20$ feet per second; if he were now to move with the same rate in the opposite direction OA', then his velocity in the latter case would be expressed by -20 feet per second.

A′————O———— —A

FIG. 1

Uniform Velocity. The velocity of a body is said to be uniform, when it describes equal distances in the same direction in equal intervals of time, however small those intervals may be. As for example, the motion of a fixed star viewed through a telescope is *uniform Velocity* or the motion of the hand of a clock is *uniform Speed*, inspite of the fact that its direction is constantly changing. In the latter case as the direction continues changing, we use the word

speed and not velocity.

To find the distance S cms. described in time t seconds by a particle moving with a uniform velocity of v cms. per second.

Since the velocity is uniform, the particle passes over v cms. in each second.

Distance traversed in one second $= v$ cms.
Distance traversed in two seconds $= 2v$ cms.
Distance traversed in t seconds $= tv$ cms.

Therefore S (the distance described in t seconds) $= vt$

$$\text{or } v = \frac{S}{t} \qquad \ldots.(1)$$

This result gives us a method of measuring uniform velocity. Uniform velocity is thus obtained by dividing the distance traversed in any interval of time by that time.

The unit of velocity on the *F P.S.* system is the velocity of a body, which describes 1 foot per second and on the *C.G S.* system, it is the velocity of a body, which describes 1 cm. per second.

Variable Velocity. When a body moves over unequal distances in equal intervals of time, its velocity is said to be variable. Thus the velocity of a stone thrown upwards is an example of variable velocity for, as it goes up, its velocity goes on decreasing, until it is reduced to zero; or the velocity of a train which starts from one station and reaches the next is variable; because at first on starting, its velocity increases and then decreases, when it approaches the next station.

Average Velocity. The average velocity of a particle moving over a given distance in a given time is defined as the velocity, with which the particle moving uniformly would describe the same distance in the same time. *It is found by dividing the distance traversed by the time taken to traverse it.*

Hence the variable velocity of a body at any instant is measured by the average velocity of the body for a very small interval of time, including the

given interval.

Thus $v=\frac{ds}{dt}$ where ds denotes a very small distance traversed in a very short interval of time dt.

9 Composition of Velocities. A body may have more than one velocity impressed upon it simultaneously. Consider a man moving along the foot-boards of a railway train in the direction of motion of the train along a platform. In this case the man shall obviously reach the farther end of the platform in a shorter time than when he were stationary and the train alone in motion, for when both are moving in the same direction, the actual velocity, with which the man approaches the farther end of the platform, is equal to the sum of the velocity of the man and that of the train.

On the other hand, if he were to move along the foot-boards in the opposite direction, evidently it would take him a longer time to reach the other end of the platform. In this case the actual velocity, with which the man approaches the farther end of the platform is equal to the difference of the velocity of the train and that of the man.

Thus, if a body possess two velocities simultaneously in the same straight line, the actual velocity of the body will be equal to either the sum or difference of the separate velocities according as the two velocities are in the same or in opposite directions along the line.

Resultant. The *single velocity*, which is equivalent to two are more velocities, impressed on a particle, is called the **Resultant** of those velocities and each of the two or more velocities separately is called **a component** velocity.

But the two velocities may not be in the same straight line. Consider a man moving diagonally across a railway compartment from A to B (fig. 2) in one second. Then if the compartment were at rest,

his velocity would be equal to AB. Suppose the train to be in motion and let AA' represent the velocity of the compartment. Draw BC equal and parallel to AA', join AC. Then AC represents the resultant velocity of the man in magnitude as well as in direction. The man, no doubt, walks across the opposite corner B of the carriage; but meanwhile the compartment has moved on and A is at A' and B at C. So the man finds himself at C.

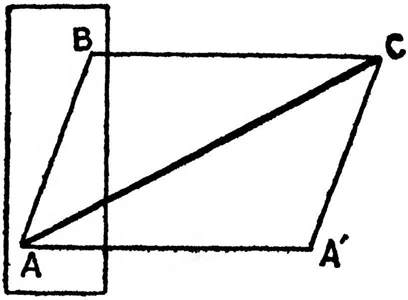

FIG. 2

This principle is known as the **parallelogram law of velocities**, which may be enunciated thus:—*If a particle possess simultaneously two velocities represented in magnitude and direction by two adjacent sides of a parallelogram, these are equivalent to a single resultant velocity represented by the diagonal of the parallelogram, passing through their point of intersection.*

Let OA, OB (fig. 3), represent the two velocities u and v respectively. Complete the parallelogram $OACB$, and draw the diagonal OC. Then OC shall represent the resultant velocity.

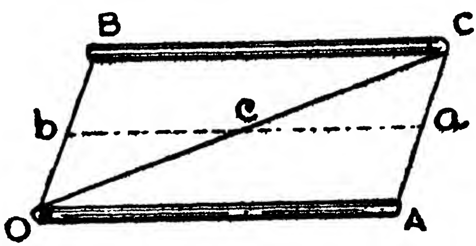

FIG. 3

Imagine OA to be a hollow tube u cms. long and moving parallel to itself with uniform velocity v, and always having the end O on the line OB. Suppose further that at the instant at which the tube begins to move from the position OA, a marble is sent in the tube

itself with a velocity u. Then owing to this latter motion, the marble at the end of one second would be at A, but owing to the motion of the tube itself, the point A at the end of one second, would have come to C. Thus the marble will be at C. Hence the marble has moved in one second from O to C with uniform velocity. Therefore OC represents the resultant velocity.

To show that OC actually represents, the path of the marble, let us imagine that the tube occupies the position bca in $\frac{1}{n}$ th of a second. Then the distance Ob traversed by the tube $= v \times \frac{1}{n} = \frac{v}{n}$ and distance bc traversed by the marble in $\frac{1}{n}$ th of a second is $= \frac{u}{n}$; but by the similarity of Δs Obc and OBC we have

$$\frac{bc}{Ob} = \frac{BC}{OB} = \frac{OA}{OB} = \frac{u}{v} \text{ or } \frac{bc}{\frac{v}{n}} = \frac{u}{v}$$

$$\therefore \quad bc = \frac{u}{n}$$

and the distance traversed by the marble in the tube in $\frac{1}{n}$th of a second is $= \frac{u}{n}$.

Therefore the marble must be at the point c (a point on the diagonal AC) at the end of $\frac{1}{n}$ th of a second, now n may be given any value and thus we show that OC represents the actual path of the marble.

If the two velocities u and v are inclined to each other at an angle θ, then the resultant velocity R can be easily proved to be $\sqrt{u^2+v^2+2uv\cos\theta}$ *

When $\theta=90^\circ$, $R=\sqrt{u^2+v^2}$, for $\cos 90^\circ=0$.

10. Triangle of velocities. *If two velocities im-*

* For proof of this, the student should refer to Trigonometry (Solution of Triangles)

pressed on a particle be represented by two sides of a triangle taken in order, their resultant is represented by the third side taken in the reverse direction. The proof of this follows directly from the parallelogram law of velocities, for if OAC (fig. 4) be the triangle and if OA and AC represent two velocities possessed simultaneously by a particle, then OC represents the resultant velocity, *i. e.* velocities OA and AC can be replaced by a velocity OC, thus proving the above

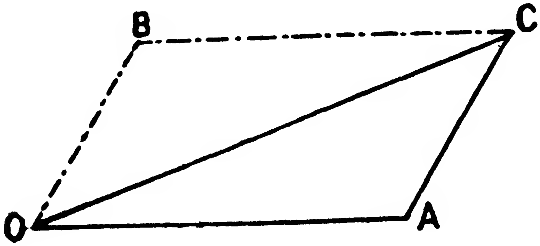

FIG. 4

theorem. If now a velocity represented by CO be impressed, then the resultant effect on the particle will be zero, for when two equal and opposite velocities act on a particle it remains in equilibrium. *Thus if a particle possess velocities represented in direction and magnitude by the three sides of a triangle taken in order, it remains at rest. The converse of this is also true*, i. e. *when three velocities acting on a particle are in equilibrium, then it is always possible to represent them both in magnitude and direction by the three sides of a triangle taken in order.*

11. Polygon of Velocities. *If several velocities impressed upon a particle be represented completely by all but one of the sides of a polygon, taken in order, their resultant is represented by that side taken in the opposite direction.*

Let velocities OA, OA_1, OA_2 and OA_3 act on a particle at O. Draw OA equal and $\parallel$ to OA; and AB equal and $\parallel$ to OA_1. Further draw BC equal and $\parallel$ to OA_2 and CD equal and $\parallel$ to OA_3; then OD will give the resul-

tant of the above velocities.

At first consider two velocities OA and OA_1, (Fig. 5). The resultant of these two velocities is equal to OB (by the triangle of velocities). Now consider three velocities OA, OA_1 and OA_2, the resultant of OB and BC is OC (by the triangle of velocities) but OB is the resultant of OA and OA_1, therefore OC is the resultant of OA, OA_1 and OA_2.

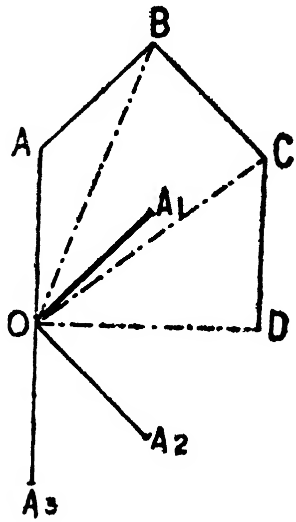

FIG. 5

Thus as a general rule, it is clear that if the resultant of several velocities is required, it is necessary to draw from any point O a line OA to completely represent first velocity and then from A, the extremity of OA to draw a line AB to represent the second velocity and from B to draw a line BC to represent the third velocity and so on. Then if D be the last point thus found, OD is the resultant velocity.

We notice however, that if D were to coincide with the point O, then the resultant would be equal to zero and the body would be at rest. *Thus we can say that if several velocities impressed on a particle be represented by the sides of a closed polygon taken in order, the particle will remain at rest.*

12. Resolution of Velocities. The inverse process to that of "compounding" is called resolution and is an operation which is highly useful in tackling problems of applied mechanics. One velocity can be resolved into a pair of velocities called its **components**, such that they are equivalent to the given velocity. All that is required is that the two components shall

be represented by the sides of a parallelogram, of which the diagonal represents the velocity, whose resolution is required.

Thus if it were required to find the components of a velocity represented by XY in the directions OA and OB, all that is needed is to draw a line XA from X parallel to OA and a line XB from X parallel to OB and then to complete the parallelogram $XBYA$ (fig 6) The velocities represented by XA and XB are components of XY, and they are parallel respectively to OA and OB. This method of finding components is called the graphic method.

From the above, it is evident, that a velocity can

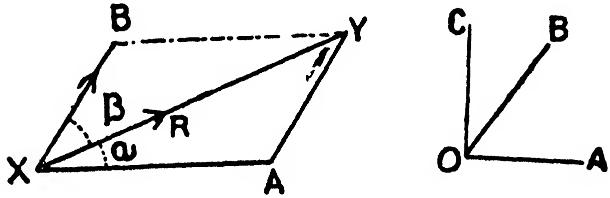

FIG. 6

be resolved into an innumerable number of pairs of components depending upon the direction and angle between the two components; but the case, when the two components are at right angle to each other deserves special consideration. This is of very frequent occurrence and in this case it is easy to find out the **resolutes**, for if one of them makes an angle α (fig. 7) with the resultant then the component in that direction would be $R\cos\alpha$ for $\frac{OA}{OC}=\cos\alpha$ or $OA=OC\cos\alpha=R\cos\alpha$, and similarly the component in the other direction would be $R\cos(90-\alpha)$ or $R\sin\alpha$. Hence, when a velocity is re-

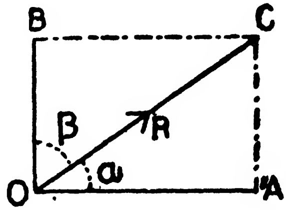

FIG. 7

solved into two other velocities, mutually perpendicular to each other, *the resolute in each direction is found by multiplying the original velocity by the cosine of the angle between it and the direction of original velocity.*

13. Relative velocity. All motion, with which we are concerned is relative, and we know that a body A is in motion relative to another body B, when the *length* or *direction* of the line AB joining A and B varies. Sometimes cases occur, where it is desirable to determine the motion of one body relative to another which itself is in motion. To do this, it is essential to understand clearly that the relative velocity of two bodies is not altered by superposing on both, the same velocity. For example, the relative motion of two men moving in a Railway compartment is the same, whether the compartment is in motion or at rest. The relative velocity of a moving body A with respect to B is found by imparting to both* of them such a velocity as will reduce B to rest; then the resultant velocity of A is its relative velocity with respect to B.

SUMMARY

1. Speed. The rate of change of position of a body without any reference to the direction and line of motion is called speed.

2 Velocity. The rate of change of position of a body when the direction and the line of its motion are taken into account, is called velocity. Velocity is a vector quantity while speed is a scalar one.

3. Uniform Velocity. Velocity is said to be uniform, when a body describes equal distances in equal intervals of time, however small.

4. The distance traversed by a body in t seconds moving with a uniform velocity u is given by $S=ut$.

5. Variable Velocity. Velocity is said to be variable, when a body describes unequal distances in equal intervals of time.

6. Average Velocity $v=\frac{ds}{dt}$

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* By so doing, the relative velocity will remain unaltered.

7. The process of compounding two velocities into one is called composition of velocities, and its reverse process is called the resolution of velocities.

8. Resultant. The single velocity, which is equivalent to two or more velocities, is called the resultant of those.

9. Parallelogram Law of Velocities. If a particle possess simultaneously two velocities, represented completely by the two adjacent sides of a parallelogram, these are equivalent to a single resultant velocity represented by the diagonal of the parallelogram, passing through their point of intersection.

Triangle of Velocities. If a particle possess velocities represented in direction and magnitude by the three sides of a triangle taken in order, it remains at rest.

Relative Velocity of a moving body A with respect to another moving body B is found by adding to both of them, such a velocity as will reduce B to rest, then the resultant velocity of A is its relative velocity with respect to B.

EXAMPLES

1. A body moves at the rate of 60 ft. per sec. What distance would it travel in 5 minutes?

Here $u = 60$ ft. per second

and $t = 5 \times 60$ seconds

But $S = ut$

$\therefore S = 60 \times 300 = 18{,}000$ feet

2. Find the speed of the earth round the Sun, assuming it to describe in 365 days a circle of 92000000 miles radius.

The circumference of the earth's orbit is $2\pi r$

$$= 2 \times \frac{22}{7} \times 92000000 \text{ miles}$$

$$\text{or} = \frac{44}{7} \times 92000000 \times 1760 \times 3 \text{ feet.}$$

365 days are equal to $365 \times 24 \times 3600$ seconds

Hence in $365 \times 24 \times 3600$ seconds the earth moves $\frac{44}{7} \times 3 \times 1760 \times 92000000$ feet. But speed $= \frac{S}{t}$

$$\therefore \text{Speed} = \frac{44 \times 3 \times 1760 \times 92000000}{7 \times 365 \times 24 \times 3600}$$

$$= 97691 \text{ feet per second.}$$

3. A monkey is climbing the mast of a ship at the

rate of 6 feet per second. Find the actual velocity of the monkey, when the ship itself is moving at 15 miles per hour due east.

The two velocities are at right angles to each other and by the parallelogram law the resultant R in this case is given by

$$R^2 = u^2 + v^2$$

but $u = 6$ feet per second

and $v = \dfrac{15 \times 1760 \times 3}{60 \times 60} = 22$ feet per second

$\therefore \; R^2 = 6^2 + 22^2$ or $R = \sqrt{520} = 22{\cdot}80$ feet per second and in a direction making an angle θ [the tangent of which is $\dfrac{6}{22}$] with due east.

4. A man swims across a river half a mile wide at the rate of 3 miles per hour. The river is flowing at the rate of 4 miles per hour. Find (*i*) the resultant velocity of the swimmer (*ii*) the distance down the stream the swimmer reaches the opposite bank.

Here u the velocity of the swimmer $= \dfrac{3 \times 1760 \times 3}{60 \times 60} = \dfrac{22}{5}$ feet per sec.

and v the " " " stream $= \dfrac{4 \times 1760 \times 3}{60 \times 60} = \dfrac{88}{15}$ feet per sec.

and as the two velocities are perpendicular to each other the resultant velocity $R =$

$$\sqrt{\left(\frac{22}{5}\right)^2 + \left(\frac{88}{15}\right)^2} = \frac{22}{3} \text{ feet per second.}$$

The velocity of the swimmer across the stream is $\dfrac{22}{5}$ feet per second and the width of the river $= \frac{1}{2} \times 1760 \times 3$ $= 264$ feet, therefore the time taken by the swimmer to reach the opposite bank $= \dfrac{2640}{\frac{22}{5}} = 600$ seconds.

During this time, *i. e.* 600 seconds, the water would

take the swimmer $\frac{88}{15} \times \frac{600}{1}$ ft.$=3520$ feet down.

Caution. The distance traversed in any direction when the two velocities are at right angles to each other is found by supposing as if the other velocity were absent.

5. One of the resolutes of a velocity of 60 miles per hour is a velocity of 30 miles per hour, find the other component.

Suppose that the given resolute makes an angle α with the given velocity.

Then we have $30=60 \cos \alpha$

$\therefore \cos \alpha=\frac{1}{2}$ or $\alpha=60^\circ$

The other resolute must be $60 \times \sin \alpha=60 \cdot \frac{\sqrt{3}}{2}$

$=30\sqrt{3}$ miles per hour.

6. A body is moving along a straight line OX with a velocity of 4 feet per second. When it reaches the point X, a blow is given to it, causing it to move along XZ at right angles to OX with a velocity of 3 feet per second. Find the magnitude and direction of the velocity impressed upon the body at X.

Draw the figure as shown, so that XB represents the velocity of the body at 4 feet per sec. before the blow and XZ represents the velocity of the body at 3 feet per second after the blow. Join BZ and complete the parallelogram.

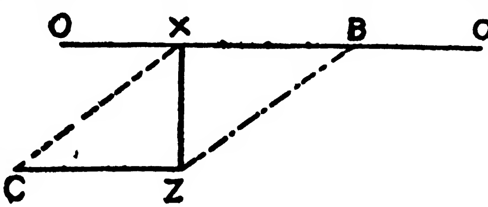

FIG. 8

Then XZ is the resultant of XB and XC.

Therefore XC represents the velocity, which must have been given to the body when at X.

But $XC^2=XZ^2+ZC^2$

$=3^2+4^2$

$\therefore XC=5$ feet per second and in the direction as shown.

7. Rain is falling vertically with a velocity of 3 miles per hour and a man is walking due east at the rate

of 4 miles per hour. At what angle to the vertical must he hold his umbrella to make the rain drops hit the top at right angles?

Impress upon both a velocity equal to 4 miles per hour due west. The velocity of man would thus be equal to zero and the resultant velocity of the rain would be equal to 5 miles per hour and would appear to be in a direction making an angle, the sine of which is $\frac{3}{5}$ with the east.

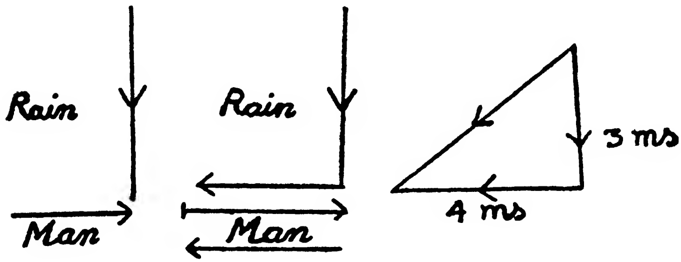

Fig. 9

Thus if the man wants that the drops should fall at rt.$\angle$s to the top of the umbrella, then he should incline it at the above angle, *i.e.* 37° to the east.

8. A ship is sailing at the rate of 10 miles an hour and a sailor climbs the mast 100 feet high in 10 seconds. Find his velocity relative to earth.

9. A velocity represented by one side AB of an equilateral triangle ABC becomes changed into one represented by the side AC, find the change in velocity.

10. A velocity of 10 miles an hour to the east is changed into one of 10 miles an hour to the north. Find the change in velocity.

11. An æroplane flying due north at the rate of 40 miles an hour is carried westward by the wind at the rate of 20 miles per hour. What is its resultant velocity?

12 A stone falls through 40, 70 and 100 feet in three consecutive seconds. What is its average speed?

13. A boat is rowed upstream at 8 miles per hour through the water, which is flowing at the rate of 5 miles per hour. What is the resultant velocity of a man on board the boat, when he walks (*i*) from bow to stern and (*ii*) from stern to bow at 4 miles per hour?

14. A train is travelling at the rate of 25 feet per

second when a rifle bullet fired at right angles to the train passes through the opposite window of a carriage 6 feet wide, with a velocity of 200 feet per second. Draw a graph to show the path of the bullet through the carriage.

15. To a passenger on a ship steaming northwards, with a velocity of 15 miles per hour, the wind appears to blow from the north-west with a velocity of $15\sqrt{2}$ miles per hour. Find the true direction and velocity of the wind.

CHAPTER III

ACCELERATION

14. Acceleration. So far we have fixed our attention on uniform motion, but the velocity of a body may change either in magnitude or in direction or in both. The *rate of change of velocity* is known as *acceleration.*

Acceleration may be Uniform or Variable. It is said to be *uniform*, when velocity increases or decreases by equal amounts in equal intervals of time, *however small.* It is measured by the velocity gained in that direction in a certain time, divided by the time taken to gain it. And it is said to be *variable*, when the velocity does not increase or decrease by equal amounts in equal intervals of time; and it is measured at *any instant* by the change in velocity, which would take place in one second, if during that second, the velocity were to change uniformly.

It should be noted clearly that the numerical measure of an acceleration is the *number of units of velocity added per second* and we have already seen that velocity is measured by the number of units of space traversed per second. Thus to express velocity we speak of so many centimetres per second. When dealing with acceleration, we speak of so *many Centimetres per second per second.*

The latter phrase is rather perplexing for the beginners. The repetition of the words per second seems to them unnecessary ; but the mystery is solved, if instead of the abbreviated form, it is put in its actual form *i.e.* " an acceleration in which in *each second*, a velocity of so many feet per *second* is added."

Unit Acceleration. (*i*) *C.G.S. system.*—A particle

has unit acceleration, when its velocity increases in each second by one centimetre per second.

(*ii*) *F.P.S. system.*—A particle has unit acceleration, when its velocity increases in each second by one foot per second.

If the units of distance and time be changed, the numerical measure of a given acceleration is also changed.

Uniform acceleration. *For the present we will deal with uniform acceleration in the direction of motion.*

To determine the velocity of a body after t seconds moving with uniform acceleration of a feet per second per second and having an initial velocity of u feet per second.

Let the initial velocity be u feet per second
the acceleration be a ,, per sec. per sec.
and, the final velocity be v ,, after t seconds

In one second a velocity of a centimetres per second is added,

therefore at the end of 1 second the velocity is $u+a$
at the end of 2 seconds the velocity is $u+2a$
at the end of 3 seconds the velocity is $u+3a$
and so on

At the end of t seconds the velocity is $u+t.a$, but as the final velocity is v ft. per second,

therefore we have $\mathbf{v=u+at}$....... . ...**(2)**

If however, the velocity decreases with the time, the acceleration is negative and we have $\mathbf{v=u-at}$.

To represent the velocity of a uniformly accelerated body graphically.

Draw a horizontal line OX, (fig 10) to denote time and a vertical line OY to represent velocity. Choose suitable scales to represent time and velocity. Mark off along OY a length OR to represent the initial velocity u. Through R draw a line $RS \parallel$ to OX and mark off P', P, P'' etc. along the line OX to denote $(t-1)$, t, $(t+1)$ *etc.* seconds, and from these points

draw $P'Q'$, PQ, $P''Q''$ *etc.*, normals to RS making $S'Q'=(t-1)a$, $SQ=ta$, $S''Q''=(t+1)a$ and so on. Then these lines would represent the increments of velocity up to the end of $(t-1)$, t, $(t+1)$ *etc.* seconds, and the lines $Q'P'$, QP, $Q''P''$ *etc.* represent the actual velocities at the end of $(t-1)$, t, $(t+1)$ *etc.* seconds. *The line $RQ'QQ''$ represents the velocity curve and as shown in the figure, it is a straight line.*

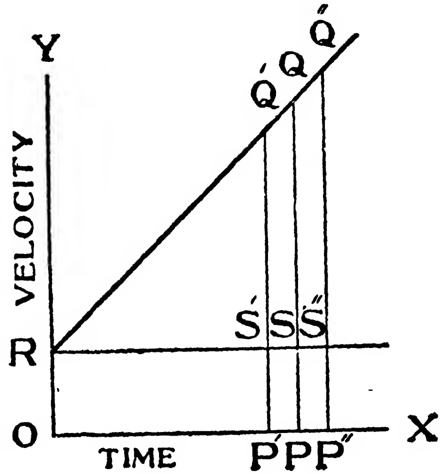

FIG. 10

The acceleration is given by the *tangent* of the angle QRS. Its value is the same for all points on the curve because PQ is a straight line.

To find Graphically the space traversed in *t* seconds by a particle moving with uniform acceleration of *a* feet per second per second, when the particle starts with an initial velocity of *u* feet per second.

$RQ'QQ''$ (fig. 10) represents a velocity curve. Let us suppose that $P'Q'$ and PQ are very near to each other, so that we may neglect the change in velocity, which occurs in the very short interval of time θ represented by $P'P$.

Let the velocity represented by PQ be equal to v. Then the space traversed in time θ would clearly be equal to $v\theta$, for v the velocity may be assumed to be very nearly constant when θ is assumed to be very small. Therefore the space traversed would be equal to the number of units of area in the narrow figure

$QQ'P'P$. Now the whole figure can be divided into a large number of such rectangles, each of which represents the space passed over in the small time θ represented by the distance between consecutive ordinates. Hence the space passed over in the time represented by OP is equal to the area $OPQR$.

From the figure we see that this area is made up of the rectangle $OPSR$ and the triangle RSQ.
This area $= OR \times OP + \frac{1}{2} RS \times SQ$.

$\therefore \quad S = u \times t + \frac{1}{2} t \times at$ (for SQ *represents increase in velocity in time t*)

or $\mathbf{s = ut + \frac{1}{2} at^2}$ ....**(3)**

The space traversed by a uniformly accelerated body can also be found out by multiplying the average velocity by the time. But the average velocity is obtained by halving the sum of the initial and final velocities *i e.* by $\frac{u+v}{2}$. The vertical line drawn from a point midway between O and P (not shown in (Fig. 10) would be of height $u + \frac{1}{2}at$ or $v - \frac{1}{2}at$.

Where u = the initial velocity
v = the final velocity at the end of t seconds
and SQ (Fig. 10), the increase of v over u, is equal to at.
Any of the three expressions, *i.e.*

(*i*) $u + \frac{1}{2}\,at$, (*ii*) $v - \frac{1}{2}\,at$ and (*iii*) $\frac{1}{2}\,(u+v)$

when multiplied by the time t will give the distance travelled in that time.

The average velocity $V = \frac{u+v}{2}$ and $S = \frac{u+v}{2} \times t$ (3*a*)

$$\text{or } S = \left(\frac{u+at+u}{2}\right) t, \text{ for } v = u + at$$

$$\text{or} \quad \mathbf{s = ut + \tfrac{1}{2} at^2}$$

To find the velocity at a given distance from the starting place.

Let u be the initial velocity
v ,, ,, final velocity at the end of t seconds
S ,, ,, distance traversed during the time t
and a be the acceleration.

We have proved that (*i*) $v - u = at$

and $(ii)\ S = \frac{u+v}{2}t$

or $u+v=\frac{2S}{t}$, by simple transposition.

Multiplying the corresponding sides of the above equation, we get

$$v^2-u^2=2as. \quad \ldots\ldots \quad (4)$$

Formulæ connected with uniform acceleration.

We have proved the following formulæ, in which the symbols have the meanings attached to them in the preceding section.

$$v=u+at \ldots \ldots (2)$$
$$s=ut+\tfrac{1}{2}at^2 \ldots (3)$$
$$s=\tfrac{1}{2}(u+v)t. \ldots (3a)$$
$$v^2=u^2+2as. \ldots (4)$$

If the initial velocity u be zero, then the above equations can be written as

$$v=at \ldots\ldots 2$$
$$s=\tfrac{1}{2}at^2 \ldots\ldots 3$$
$$s=\tfrac{1}{2}vt \ldots\ldots 3a$$
$$v^2=2as \ldots 4$$

15. * **Falling Bodies.** When a body is allowed to fall to the earth's surface from a point above, it is found that within small range, the acceleration is fairly the same and uniform for all bodies. This acceleration is generally spoken of as "acceleration due to gravity," and is denoted by the letter g. The value of g in the *C.G.S.* system varies between 983·11 cms. per second per second at the poles to 978 10 cms., at the equator; and is generally taken as 981 cms. per second per second in calculations. In the *F.P.S.* system it is taken as 32 feet per second per second

Thus the formulæ referred to above, in the case of falling bodies can be re-written as

$$v=u+gt \ldots\ldots 2b$$
$$s=ut+\tfrac{1}{2}gt^2 . \quad 3b$$
$$v^2=u^2+2gh \ldots 4b$$

* The cause of this is dealt with in Chapter V.

where h is the height from which the body falls.

In the case of a body projected vertically upwards the formulæ may be written as

$$v = u - gt \ldots \qquad \ldots (2b')$$
$$s = ut - \tfrac{1}{2}gt^2. \quad \ldots (3b')$$
$$v^2 - u^2 = -2gh \quad \ldots (4b')$$

where h is the height to which the body is projected.

16. Composition and Resolution of Accelerations. A particle may have two or more accelerations communicated to it simultaneously, then the resultant effect is found in the same way as in the case of velocities, *i.e.* by the parallelogram law. The proof is the same as given above for velocities; except that we have to write acceleration for velocity.

EXAMPLES

1. A particle has an acceleration of 32·0 feet per sec. per sec. Express it in (*i*) cms. per sec. per sec. and (*ii*) in yards per minute per minute. (1 ft. = 30·48 cms.)

(*i*) In 1 sec. a velocity of 32·0 feet per sec. is added

∴ In 1 sec. a velocity of $32{\cdot}0 \times 30{\cdot}48$ cms. per sec. is added.

∴ the acceleration is $32{\cdot}0 \times 30{\cdot}48$

$= 975{\cdot}36$ cms. per sec. per sec.

(*ii*) In 1 sec. a velocity of 32·0 feet per second is added

∴ „ „ „ „ $\frac{32}{3}$ yards „ „ „

or „ „ „ „ $\frac{32}{3} \times \frac{60}{1}$ „ minute „

∴ 1 minute „ „ $\frac{32 \times 60 \times 60}{3}$ yards „ „

∴ the acceleration is $\frac{32 \times 60 \times 60}{3}$ yds. per min. per min.

This is equal to 38400 yards per minute per minute.

2. Convert an acceleration of 40 in *C.G.S.* system, to one in which the units are a kilometre and an hour.

1 cm. $= \frac{1}{100000}$ of a kilometre,

and 1 sec. $=\frac{1}{3600}$ of an hour.

In 1 sec. a velocity of 40 cms. per sec. is added, or

In 1 sec. a velocity of $\frac{40\times 1}{100000}$ kms. per sec. is added, or

In 1 sec. a velocity of $\frac{40}{100000}\times\frac{3600}{1}$ kms. per hour is added.

Therefore a velocity of $\frac{40\times 3600\times 3600}{100000}$ kms. per hour is added after every hour.

This is expressed as 5184 kms. per hour per hour.

3. A particle moving from rest with uniform acceleration has a velocity of 192 feet per second, after 6 seconds. Find its acceleration.

We have $v=u+at$

or $192=0+a.\,6$

$\therefore \quad a=\frac{192}{6}=32$ feet per sec. per second.

4. A particle starts with a velocity of 3 cms per sec. and an acceleration of 1 cm per sec per sec. Find (*i*) its velocity after 6 seconds, (*ii*) distance traversed in 10 seconds and (*iii*) the distance traversed when the body acquires a velocity of 5 cms. per sec.

(*i*) $v=u+at \quad \therefore \quad v=3+1\times 6$

velocity after 6 secs. would be 9 cms. per sec.

(*ii*) $s=ut+{}^1/_2at^2$

$s=3\times 10+{}^1/_2.1\times 10^2$

or $s=30+50=80$ cms.

(*iii*) $v^2-u^2=2aS$

$5^2-3^2=2.1\,S$

$\therefore \quad S=8$ cms.

5. A particle starts with a velocity u and an acceleration $-a$; find (*i*) the time when it would come to rest, (*ii*) the time when it passes through the starting point again.

(*i*) Let the time be t.

Then we have v the final velocity $=0$

$\therefore \quad 0=u-at$

or $t=\frac{u}{a}$

(*ii*) Let the time be T.

Then we have $S=0$ for the particle is again at the starting point.

Then we have $0=uT-\frac{1}{2}aT^2$

$\therefore$ either $T=0$ . . (*i*)

or $u-\frac{1}{2}aT=0$ (dividing both sides by T)

$$\therefore \quad T=\frac{2u}{a}$$

6. Two bodies A and B are 138 feet apart. A starts from rest and moves towards B with an acceleration of 5 feet per second per second ; while B starts at the same time to meet A and moves with uniform velocity of 8 feet per second. When and where do they meet?

Let t be the time after which they meet

Then A traverses a distance $=\frac{1}{2}\, 5.\, t^2$ feet,

and B ,, ,, ,, $=8.\, t$ feet.

But both of them together cover a distance $=138$ feet; therefore we have $138=8t+\frac{5}{2}\, t^2$ or $t=6$ seconds,

and they meet at a distance of $8\times 6=48$ feet from B.

7. Find the distance traversed over by a falling body in the nth second of its motion.

Let S_1 be the space traversed up to the beginning and

S_2 ,, ,, ,, ,, ,, end of the nth second

then $S_1=\frac{1}{2}g\,(n-1)^2$

and $S_2=\frac{1}{2}gn^2$

$\therefore \quad S_2-S_1=\frac{1}{2}g\{n^2-(n-1)^2\}$

$=\frac{1}{2}g(2n-1)$.

8. A ball is projected upwards with a velocity u; find (*i*) the greatest height to which it will rise, (*ii*) the time of rising to the greatest height and (*iii*) the time of falling to the ground.

(*i*) $v^2-u^2=-2gh$

$$\therefore \quad h=\frac{u^2}{2g}$$

(*ii*) Let the time be t_1, then we have

$v=u+at_1$

$\therefore \quad 0=u-gt_1$

$$\text{or } t_1=\frac{u}{g}$$

(*iii*) Let the time be t_2, then we have

$$S = \tfrac{1}{2} g t_2^2$$

$$\text{or } h = \tfrac{1}{2} g t_2^2 = \frac{u^2}{2g} \quad \text{.. from } (i) \text{ above,}$$

$$\therefore \quad \tfrac{1}{2} g t_2^2 = \frac{u^2}{2g} \text{ or } t_2 = \frac{u}{g}.$$

Hence the time of rising up is equal to the time taken to fall through the same height.

9. A train passes three consecutive quarter-mile posts. It takes 20 seconds between the first and the second, and 25 seconds between the second and the third. Find its retardation, supposed uniform.

10. A body moves from rest and has its velocity uniformly accelerated. If it describe 40 feet in the first two and a half seconds of its motion, what distance does it describe in the next second?

11. What acceleration is needed to get up a speed of thirty miles an hour in a half-mile run? What is the break retardation, that could destroy this motion in 100 yards What retardation would stopping in 8 yards represent?

12. A train moving with an acceleration —3, has a velocity 45 when passing a particular point. How much further would it go?

13. When a balloon has ascended vertically at a uniform rate for 6 seconds. a stone let fall from it reaches the ground in 9 seconds after leaving the balloon. Find the velocity of the balloon and the height from which the stone is let fall.

14. If the measure of an acceleration be 264, when a yard and a minute are the units of length and time, find its measure, when a mile and an hour are the units of length and time.

15. A stone is dropped from a balloon moving vertically upwards with a uniform velocity of 60 feet per second and reaches the ground in 4 seconds. Find the height of the balloon (*i*) when the stone was dropped, and (*ii*) when the stone reaches the ground.

16. You throw a cricket ball and catch it after 5 seconds, how high would it go and with what velocity would it return to you?

17. A falling body describes 100 feet in the last second of its motion. Find how far it must have fallen and

also the time taken.

18. From a cliff 400 feet high a bullet is fired horizontally with a velocity of 44 feet per second Find the time and the distance from the foot of the cliff, when the bullet reaches the ground. Also find how far it will roll along the surface, if on reaching the earth, it is subjected to a retardation of 16 feet per second per second.

19. A stone is thrown vertically upwards and just reaches a point 80 feet above the point of projection. Find the time taken by the stone to reach the highest point.

20. A body moves in a straight line with uniform acceleration of 40 feet per second per second Find the time necessary to increase its velocity by 10 miles per hour.

21. A ball thrown up from the top of a tower with a velocity of 40 feet per second reaches the ground after 5 seconds. Find the height of the tower?

CHAPTER IV

LAWS OF MOTION

17. Of the various Natural Sciences, Mechanics is an experimental science, though the experimental side is obscured by its mathematical form. The Laws of Motion cannot be proved like mathematical theorems for they are generalizations from experience. Their justification however, lies in the accuracy, with which we can predict the behaviour of bodies under the influence of the given forces. Astronomy, which deals with the movements of heavenly bodies is built on the laws of motion and it is a matter of common knowledge, that the movements of heavenly bodies are predicted very accurately years in advance and these predictions correspond exactly with the actual observed facts. Thus we conclude that there could possibly be no error in the principles on which the calculations depend.

Momentum. So far we have studied motion without any reference to the moving body; but it is evident that in every case of motion there must be some matter that moves. Let us observe the effect of quantity of matter (*i. e.* mass) of the moving body on its motion. To do so, suppose a pound weight is allowed to drop from a height of one foot. To stop it a certain amount of muscular effort is required, but it will be stopped without much difficulty. Suppose now that instead of one pound weight, one maund weight was dropped from the same height, the velocity would be the same ; but the effort required to stop it, would be much greater. Thus the *effort necessary to stop a moving body depends in the first instance upon its mass.*

Again suppose that the same one pound weight is

dropped from a height of 100 feet instead of one foot; the effort required to stop it would be much greater than in the first case. Why? Because, the velocity with which it reaches the hand now is ten times as large as the previous velocity. Thus we learn that in the *second instance, the effort required to stop a moving body depends upon its velocity.* Hence, consistent with these facts, it is proper to speak of the quantity of motion in a body as proportional to its *speed* and *mass.* This 'quantity of motion' is termed momentum. It is on account of the great momentum of the moving masses of air and dust particles that trees and telegraphic poles are uprooted. Thus *momentum of a body is the property, which it possesses by virtue of its mass and velocity conjointly and it is always measured by the product of the mass and the velocity*

$$M = mv \quad . \qquad (5)$$

Unit of momentum. The unit of momentum is the momentum of a unit mass moving with unit velocity. No special name has been given to this unit, though many have been suggested.

Change of momentum. If the velocity of a moving body is changing, its momentum must also change since the mass of a body remains constant therefore the *rate of change of momentum is equal to the mass multiplied by the rate of change of velocity, i. e. mass × acceleration.*

Thus if the velocity of a mass m be u and after t seconds its velocity be v,

then change of momentum $= m\,(v-u)$

and the " " in one second $= \dfrac{m\,(v-u)}{t}$

$$\text{but } \frac{v-u}{t} = a \text{ for } v = u + at$$

$\therefore$ change of momentum per unit of time $= m \times a$.

18. Newton's laws of motion.

First law. "Every body preserves or continues in its state of rest or of uniform motion in a straight line, except in so far as it is compelled to change

that state by impressed forces"

Second law. "Rate of change of momentum is proportional to the impressed force and takes place in the direction of the straight line in which the force is impressed."

Third law. "To every action there is always an equal and opposite reaction, that is to say, the actions of two bodies upon each other are always equal and directly opposite."

The first law. The first point to notice about this law is that it includes the definition of force, for it states that without force there can be no change of motion. If the resultant force is zero, then acceleration or (change of motion) is zero. A body must continue in a state of rest or of uniform motion in a straight line, unless some force acts upon it to change it. *It is in keeping with the general axiom that no effect happens without a cause.* It is not likely that a piece of matter should change its own state (whether of rest or of motion) without some external cause. Thus if cash disappears from a safe, we invariably ask: Who has removed it?—the suggestion that the cash may have moved itself being rejected as improbable.

The sole cause of change of motion in matter is *force.* This furnishes us with a definition of matter in terms of force.

Thus matter is that, which requires force to change its position. This property of matter is called **Inertia.** This inability of matter to change its position is Newton's test of matter. This is beautifully explained by the help of the experiment shown in fig 10 (a) A stone hangs by a string a and another string b hangs downwards from the stone. If a jerk be given to the lower string it breaks because due to inertia of the stone the force of the jerk cannot produce any effect above it. If however, the force applied be slowly increased then

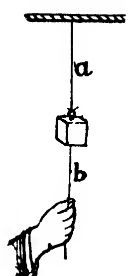

FIG. 10 (a)

the upper string breaks, because the force on it will be equal to the sum of the applied force and the weight of the stone. To answer whether electricity, ether or heat is matter, we have to inquire, 'Does either of these require force to change its state of motion?' (This is the simple test of matter)

The second part that a moving body should continue moving uniformly in a straight line for ever unless acted upon by some impressed force, seems at first to be contrary to experience; but this is due to the impossibility of attaining the necessary conditions.

The nearer we approach the circumstances in which the law is stated to be true, the more nearly do observations agree with the results, which should follow from the law. A ball slides a longer distance on ice than on a rough road. Thus the rate at which the ball comes to rest is not an inherent property of the body or its motion; but is due to something in which the surface plays a part. The law states 'if it were possible to eliminate all the forces (friction, air resistance, etc.) the motion would continue uniformly in a straight line for ever

The second law. The second law of motion suggests how a force is to be measured. We have seen that the force required to start or stop a body depends upon the momentum generated or destroyed. Thus, if a force F were applied to a certain mass m, a certain quantity of momentum would be generated; but if the same force were applied to a greater mass, the quantity of momentum must still remain the same, though the velocity generated in the latter case would be smaller. Thus force may be measured by the momentum generated by it per second and this is independent of all other things except force Thus if the above force F change the momentum of the body of mass m from mu to mv in t seconds, then the change of momentum $=\dfrac{mv-mu}{t}$ per second which by

the II Law should be proportional to F. Thus we may write $F \propto \frac{m(v-u)}{t}$.

or $F = K\frac{m(v-u)}{t}$

For the sake of convenience, we may choose such a unit of force as may give to K a value equal to unity.

The above equation will then be written as $F = \frac{m(v-u)}{t}$. Suppose $m=1$ gramme, $t=1$ sec. and $v-u=$ 1 cm. per sec; then F should be equal to unity. Thus we get the *unit of force, as that force which acting on a unit mass for unit time, produces unit change of momentum*; but $\frac{v-u}{t} = a$ (the acceleration).

Thus Force is equal to mass × acceleration.

i. e. $\mathbf{F = ma}$ **(6)**

And the unit force may be defined as that force, *which produces a unit acceleration in a body of unit mass.* It may also be defined as that force, which acting on a unit mass (initially at rest) for a unit time causes it to move with unit velocity.

In the C. G. S. system of units, the unit of force is called a **dyne** and is that force, which acting on a mass of one gramme produces in it an acceleration of one centimetre per second per second.

In the F. P. S. system of units, the unit force is called a **Poundal** and is that force, which acting on a mass of one pound produces in it an acceleration of one foot per second per second. One poundal equals 13825·38 dynes.

A pound weight is about 32 poundals (see next Chapter) and a gramme weight is about 981 dynes. These standards known as gravitational units are frequently used as practical units of force. The student should make himself thoroughly familiar with translating gravitational units into absolute and *vice versa.*

18 (*a*). **Composition and Resolution of forces.** Force like velocity and acceleration is a vector quantity. The resultant of a number of forces acting simultaneously on a body cannot be found by simple arithmetical addition. Parallelogram method affords a means of finding the resultant of two forces. Thus we may enunciate it as follows: *When two forces acting on a particle are represented both in magnitude as well as in direction by two lines drawn from a point and a parallelogram is completed with them (the two lines) as its adjacent sides, then their resultant is represented in all respects by the diagonal of the parallelogram passing through the same point.*

The above statement known as the parallelogram law of forces is experimentally proved as follows:—A wooden board held firmly in the vertical plane carries two smooth pulleys at the two ends. Three threads each carrying a scale-pan at one end, are tied to a very small ring. Two of these threads pass over the two pulleys. The weights in the three scale-pans are so adjusted as to allow the whole system to come into equilibrium. Let the weights (along with the weight of the respective scale-pans when in equilibrium) be w_1, w_2 and w_3 respectively. Draw lines *oa*, *ob* and *od* parallel to the threads on the paper attached to the board. Choose some suitable scale and mark off *oa* and *ob* to represent w_1 and w_2 respectively. Complete the parallelogram *oacb* and draw the diagonal *oc* and also produce *od* backwards, *oc* and the prolongation of *od* should coincide, if the experiment is performed carefully. Measure the length *oc* and show that on the scale already adopted, it (*oc*) represents w_3. But w_3 is the resultant of w_1 and w_2, because it keeps them in

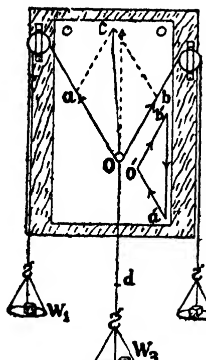

FIG. 10 (*b*)

equilibrium. Therefore we see that the resultant is represented in magnitude as well as in direction by the diagonal of the parallelogram, whose adjacent sides represent the two forces.

The three forces w_1, w_2 and w_3 acting at the point o keep it in equilibrium, therefore in accordance with the triangle method, it should be possible for us to represent the three forces completely by the three sides of a triangle taken in order. From any point o' draw $o'a'$ and $o'b'$ parallel and equal respectively to oa and ob. Join $b'a'$ Show that it is parallel and equal to co. Thus we come to the conclusion that when three forces acting at a point keep it in equilibrium, it is possible for us to represent them by the three sides of a triangle taken in order. This is known as the law of triangle of forces.

19. Physical Independence of forces. The second law besides furnishing us with the measure of force as shown above, gives us a very important principle, called the Physical Independence of forces. *It states that the change of momentum due to a force takes place along the line of action of the force and in the direction in which the force acts.* By the line of action is meant the straight line, in which the force would cause a body at rest to move, when no other forces are acting upon it But the law says nothing as to whether the body is at rest or in motion, whether it is acted upon by any force or not. Hence it follows that (*i*) a force acting on a body would produce the same change of momentum as it would have done if the body were at rest. (*ii*) If more than one force act on a body simultaneously, each would produce the same change of momentum as it would have done, if the other forces were absent. *This principle is known as Physical Independence of forces.*

Illustration.—When a ball is dropped from the mast-head of a ship, it always falls at the foot of the mast, whether the ship is at rest or in motion and

the same time is always spent in falling because the ship when in motion imparts the same velocity to the ball as its own and its downward fall remains unaffected by the horizontal velocity.

Professor Clark Maxwell has re-stated the second law in the following language:—

"*The change of momentum of a body is equal to the impulse, which produces it and is in the same direction.*"

By impulse is meant the product of the force and the time for which it acts. Therefore the effect of a force to produce motion depends upon time also. The hammer blow is a very great force, while it lasts; but as its duration is small, its impulse may not be as great as that of a much smaller force applied continuously for some time.

Thus if force F act for a time t on a body of mass m and change its velocity from u to v, the impulse Ft will be equal to the change of momentum.

i.e. $\mathbf{F \times t = m \times (v - u)}$ **(7)**

The third law of motion or conservation of momentum. Newton's third law states that to every action there is an equal and opposite re-action. What is meant by *action and reaction* is shown by Newton's own illustrations in Principia. "If a man press a stone with his finger," he says, "his finger is also pressed by the stone. If a horse draw a stone by means of a rope, the horse is equally drawn towards the stone by the rope." In the latter example what then is the cause of the horse moving the stone? The answer is very obvious. The horse presses the ground backwards with his hoofs and the ground in turn presses the horse forward. Thus there are two forces acting on the horse (*i*) The pressure of the ground tending to push the horse forward and (*ii*) the tension of the rope which tends to draw the horse backwards. It is the balance of the first over the second that gives a forward movement to the horse.

20. The third law suggests the principle of conservation of momentum, for it states that if two bodies A and B act upon one another, the momentum lost by A along any straight line is equal to the momentum gained by B along the same line. Thus when a moving cricket ball is hit with a bat, both bodies are in motion at the time of collision. By the third law, the force exerted by the bat upon the ball is equal and opposite to the force exerted by the ball upon the bat. Therefore the change of momentum produced by one would be equal and opposite to the momentum produced by the other. Thus we say that *whenever a transference of momentum takes place between two bodies, the momentum gained by one body is lost by the other.* Or the total momentum of any mechanical system, measured in any direction, is constant; and is not affected by any interaction between the parts.

Illustration.—When a gun is fired it kicks, unless held firmly close to the shoulder. In the latter case the gun and the man constitute but one body, the mass of which is much greater than that of the bullet, but the momentum of the bullet and that of the gun and man combined will be equal and opposite.

21. Deduction of first and third laws from the second. Clark Maxwell has shown that Newton's three laws are actually contained in Newton's second law. The first and the third are mere corollaries from the second. The second law states that the change of momentum is proportional to the impressed force; if there is no force there is no change of momentum. But the mass of a body remains the same, therefore either the body must be at rest or should continue moving uniformly in a straight line. This is Newton's first law. The third law follows from the first, for if the mutual forces exerted by the two parts of the *same body* on each other were not equal and opposite, they would not be in equilibrium ; and consequently the two parts might by their action cause the body to move, the possibility of which is denied by the first

law. Hence the mutual actions of the two parts must necessarily be equal and opposite and this is the third law. Thus the first and third laws are mere deductions from the second law, which is sometimes spoken of as *the law of motion.*

22. Uniform Circular Motion. According to Newton's first law a body continues moving with uniform velocity in a *straight line*, when no force acts upon it. It is evident from this that if a body moves with uniform velocity in a *circle*, some force must be acting on it. We will find the magnitude and direction of this force, which makes a body describe a circle.

Experiment.—Tie a stone to the end of a string and whirl it round. While so doing let off the string, the stone will not describe a circle but will be seen to fly away tangentially. This shows that a force acts along the string which makes the body describe a circle

Suppose a body describes the circle AB with centre O and uniform velocity v. Let the radius of the circle be denoted by R. Let us further suppose that in a small interval of time t the body moves from

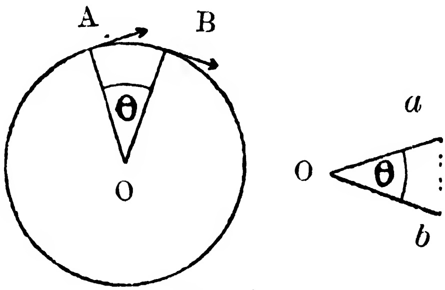

FIG. 11

A to B. From a point o draw oa equal and parallel to the velocity v at A and ob equal and parallel to the velocity v at B. Join ab then the change in velocity of the body in moving from A to B in time t is given by ab (by the Theorem of the Triangle of Velocities) or in

unit time by $\frac{ab}{t}$.

Now the velocity of the body at every instant of its motion is perpendicular to the radius. Hence $\angle aob = \angle AOB$*. Let this be denoted by θ (in circular measure). Then change in velocity per unit time, called acceleration, is given by $\frac{ab}{t}$

$$\text{or} \quad \frac{r\theta}{t} \text{ for } ab = oa \times \theta = r\theta.$$

$$\text{But } \theta \text{ is equal to} \frac{AB}{R} = \frac{vt}{R}$$

$$\text{Substituting this we get acceleration } = \frac{v}{t} \cdot \frac{vt}{R}$$

$$= \frac{\mathbf{v}^2}{\mathbf{R}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Now if θ be very small, *i.e.* if A and B be points very near together, then ab in the limiting position will be perpendicular to oa, *i.e.* parallel to OA.

Thus the acceleration of a body moving in a circle is $\frac{v^2}{R}$ and is always directed along the radius. The force, which will produce this acceleration will be $\frac{m\,v^2}{R}$ where m is the mass of the body. This force which makes a body describe a circle is called **Centripetal force.** In the case of a stone whirled round at the end of a string, the centripetal force is supplied by the tension of the string.

The force, which when the string is let off, takes

*Angle between two lines is equal to the angle between their respective perpendiculars (Euclid).

the stone tangentially is called the **centrifugal force**.

Now if w is the circular measure of the angle described by the radius in a unit time, then the time of describing a complete circle will be $\frac{2\pi}{w}$ but it is also equal to $\frac{2\pi R}{v}$ ($2\pi R$=circumference).

Thus $\frac{2\pi}{w}=\frac{2\pi R}{v}$ or $v=Rw$.

Substituting this value of v in the expression for acceleration, we have $a=\frac{v^2}{R}=\frac{w^2R^2}{R}=Rw^2$.

SUMMARY

1 **Momentum** of a body is the property, which it possesses by virtue of its mass and velocity conjointly and it is always measured by the product of the mass and the velocity of the body.

Thus $M=mv$.

2 **Inertia** means the inability of matter to change its position.

3 **Force** is measured by the product of the mass and the acceleration it produces.

Dyne is the unit of force on the C. G. S. system. It is the force which would produce an acceleration of one centimetre per second per second, when acting on a mass of one gramme.

Poundal is the unit of force on the *F. P. S.* system; and is the force, which would produce an acceleration of one foot per second per second, when acting on a mass of one pound.

4. **Impulse** means the product of the force and the time for which it acts.

5 The change of momentum is proportional to the impulse which produces it.

6. **The principle of conservation of momentum.** It suggests that whenever a transference of momentum takes place between two bodies, the momentum gained by one body is lost by the other or the total momentum of any mechanical system measured in any direction is constant and is not affected by any interaction between the parts.

The force, which makes a body describe a circle is called **centripetal force** and that which tends to take it away tangentially is called **centrifugal force.** It is equal to $\frac{mv^2}{R}$ or mRw^2.

EXAMPLES

1. Calculate the momentum of a mass of 2 tons moving with a velocity of 30 miles per hour.

$$30 \text{ miles per hour} = \frac{30 \times 1760 \times 3}{60 \times 60} \text{ feet per second}$$
$$= 44 \text{ feet per second.}$$

$\therefore$ the required momentum $= 2 \times 2240 \times 44$.

2. A mass of 5 lbs. moving with a velocity of 10 feet per second is brought to rest, while it passes over 25 feet by the action of a constant force. What is the force?

Here a velocity of 10 feet per second is destroyed, when the body traverses 25 feet, therefore to find out the constant retardation, apply the formulæ $v^2 - u^2 = 2aS$

$$\therefore \quad 0^2 - 10^2 = 2.\ a.\ 25$$
$$\text{or} \quad 0 - 100 = 50a \text{ or } a = -2$$
$$\text{but} \quad F = m\ a.$$

Therefore the force $= 5 \times -2 = -10$ poundals.

2. (*a*) The driving wheel of a locomotive is 5 feet in diameter. What is the angular velocity of the wheel, when the engine is running at 30 miles per hour?

$$\text{Velocity of the locomotive} = \frac{30 \times 1760 \times 3}{60 \times 60} = 44 \text{ ft. per sec.}$$

We may find the revolutions of the wheel per sec. by dividing the velocity by the circumference of the wheel.

$$\therefore \text{ Number of revolutions} = \frac{44}{\pi d} = \frac{44}{5\pi} = \frac{44 \times 7}{5 \times 22} = 2{\cdot}8$$

$$\therefore \quad w = 2{\cdot}8 \times 2\pi = 17{\cdot}58 \text{ radians per sec.}$$

3. A body of mass 15 lbs. moving with a velocity of 30 feet per second is subjected to a constant force in a direction opposite to that of its motion and is brought to rest after it has described 12 feet. Find the magnitude of the force.

4. Find the momentum generated, when a force of 25 grammes weight acts upon a mass of 10 kilogrammes for

30 seconds. Find also the velocity generated.

5. A bullet fired horizontally from the top of a tower with a velocity of 500 feet per second, hits the ground in four seconds. Find the height of the tower and the distance from the foot of the tower, where the bullet strikes the ground.

6. A constant force acts for 5 seconds on a body of mass 50 grammes. After that the force ceases and the body describes 100 feet in the next 5 seconds. Find the magnitude of the force acting.

7. The mass of a gun is 2 tons and that of the shot is 14 lbs. The shot leaves the gun with a velocity of 800 feet per second, what is the initial velocity of recoil ?

8 How long must a force of 5 units act upon a body in order to give it a momentum of 300 units ?

9. By what number would the acceleration due to gravity be expressed, if a day and a mile were the units of time and length.

10. Compare the amounts of momentum in (i) 56 lbs. weight, which has fallen for 2 seconds from rest and (*ii*) a cannon ball of 12 lbs. moving with a velocity of 900 feet per second.

11. An iron ball of mass 5 lbs is dropped from the roof of a house 160 feet high, at the same instant a similar ball is thrown upwards with a momentum of 400 units When and where will the two balls meet? What will be the momentum of the first ball at that time?

12. A string will break under a load of 2 kilogms. A mass of 250 gms. is attached to the end of a piece of this string 2 metres long, and is rotated horizontally. Find the number of revolutions, which it can make without breaking the string.

CHAPTER V
GRAVITATION

23. Gravitation. That all bodies fall to the ground was known to the ancients; but *how* and *why*, they did never care to enquire. In 384 B. C. Aristotle, the Greek Philosopher taught that bodies fall at rates depending on their weights, *i. e.* the heavier a body, the faster it should fall. It is a wonder that this statement passed unchallenged till the time of Galileo (1590) *who asserted that all bodies whatever their weight fall at the same rate unless resisted by other forces.* He found that when two leaden weights (ratio 100 : 1) were dropped simultaneously from the top of the leaning tower of Pisa,* they reached the foot of the tower at the same time. Thus he showed that the rate of fall was independent of the weight of the substance and was the same for all bodies. By taking weights of different materials such as a pound of iron, of stone and of wood and making them fall in the same way, he showed that the motion was also independent of the substance of which the ball is made.

The statement that "Bodies fall with constant acceleration towards the earth" is true only when every disturbing cause is absent; but if the experiment be performed in air, a discrepancy may be observed. For example, if a lead weight and a feather be dropped simultaneously from a tower it will be observed that the former reaches the ground considerably in advance of the latter. This is due to

*Galileo's experiments can be repeated in the laboratory by suspending two iron weights of different size from two electromagnets at the same height and then releasing them simultaneously by breaking the circuit. The two iron weights are seen to strike the ground simultaneously.

the feather being retarded by the air more than the lead. The air offers resistance to a moving body which depends on the *area* of the body and not on its mass. This air resistance would be practically inappreciable in comparison to its weight, if the body is heavy and of small surface area ; but it may be appreciable in comparison to its weight, if the body is light and has a large surface area The air like water has a floating power though to a considerably less extent. To prove the truth of the above explanation, a glass tube about five feet long (fig 12) may be taken, having one of its ends completely closed and a brass stop-cock fitted to the other. A lead piece and a feather are introduced into the tube and air is pumped out. The stop-cock is then closed and the tube suddenly inverted. It is seen that both the feather and the lead fall with the same speed If now air be introduced by opening the stopcock and the experiment repeated, it is seen that the motion of the feather is retarded

Thus in vacuum, all things fall at the same rate. This combined with the deduction made from Newton's second law that force is equal to the product of mass and acceleration, drives us to the conclusion that the pulling force exerted on a falling body is proportional to its mass If the acceleration with which a body falls to the ground be denoted by g, then the pulling force is measured by its mass multiplied by g, and this pulling force is known as *weight*

Thus $W = m \times g$ **(9)**

FIG 12

The ratio of the weight of a body to its mass is spoken of as the intensity of gravity and is always denoted by g. If g is known, it becomes quite easy to convert gravitational units into absolute ones. At Lahore g is found to be 32·1 ft. sec.^{2} or about 978·4 cms. sec ^{2}

Gravitation. We have shown *how* bodies fall towards the ground, but we have made no attempt so far

to explain *why* bodies fall towards the ground. This question Newton asked himself, when sitting under an apple tree, and he saw the fruit fall down. By the power of his imagination and clear foresight of the principles of Astronomy, Newton discovered that the range of the *forces*, which made the apple fall towards the ground, was not limited and that the very same laws extended to the Sun, the Moon and the planets. After working out the very complicated problem of the motion of heavenly bodies along their orbits, Newton gave in finished form the doctrine of universal gravitation as follows, "*Every particle in the universe attracts every other particle with a force directly proportional to the product of their masses and inversely proportional to the square of their distance apart*" Thus if two particles of masses m and m_1 respectively be d centimetres apart, then the force of attraction is proportional to $\frac{m.m_1}{d^2}$, *i. e.* $F \propto \frac{m\,m_1}{d^2}$.

It is in consequence of this force* that planets describe orbits round the Sun. It may however, appear that in consequence of this force, the planets ought to fall into the Sun. This would certainly have happened, had the force of attraction been the only force acting upon them. But on account of the inertia and the impulse, which they received at the time of the creation of this universe, they tend to go away from the Sun. The resultant of the force of attraction and their original velocity makes them describe nearly circular orbits round the Sun.

24. Gravity. Gravity is a particular form of the more general phenomenon of Gravitation described above. It is the attraction exerted by the Earth on bodies on its surface. The acceleration with which bodies move towards the Earth is uniform at a point as has been shown above, and this follows directly from the law of gravitation. For if M were the mass of the Earth,

* The force of attraction supplies the necessary centripetal force to make them describe circular orbits

R its radius and m the mass of a given body, then the force acting on it due to the pull of the Earth would be given by $\frac{M \times m}{R^2} \times G$ where G* is a constant known as Gravitational constant.

The acceleration with which the body would move is obtained by dividing the force by the mass moved. Thus acceleration would be given by $\frac{M}{R^2} G$; an expression quite independent of the mass of the body and a constant depending only upon the mass of the Earth (an unalterable quantity), and its radius (4000 miles.)

The acceleration of a *falling body* varies as the inverse of the square of the radius of the Earth; but as the radius of the Earth is not uniform at all points on its surface, therefore we should expect the value of gravity to vary from point to point on the surface of the Earth and this is actually the case. The poles of the Earth (being flattened) are nearer to its centre than the equator, hence the acceleration of a falling body *i.e.*, g (the intensity of gravity) is greater there than at the equator and the weight of a body (which is the product of its mass and the value of g at the given place) is greater at the poles than at the equator. This is shown conclusively with the help of a spring balance. An ordinary balance fails to show this; because as the pull of the Earth increases on the object to be weighed, when it is shifted from the equator to the pole, similarly does it increase on the 'Standard weights' employed to weigh the object. Hence an ordinary balance measures mass, while a spring balance measures weight.

25. Motion of connected bodies. The accel-

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* G the mean value of gravitational constant $= 6{\cdot}56 \times 10^{-8}$
and the mean value of the Earth's density $= 5{\cdot}15$

ration of freely falling bodies due to gravity is generally denoted by g. If however, by any arrangement the *weight* is made to move another mass as well as its own, the acceleration will be less than g. Thus if we have two masses M and m, connected by means of an inextensible string passing over a smooth pulley as shown in fig. 13, then if M be greater than m, the weight Mg will move downwards, while m will move upwards with the same acceleration. Let the common acceleration of the two bodies be denoted by a and the tension of the string by T.

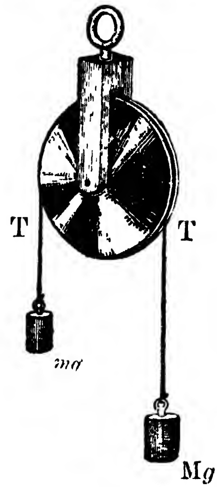


Fig. 13

Then the acceleration of $M = \dfrac{Mg-T}{M} = a$

$$\text{or } Mg - T = Ma \quad \ldots \quad (i)$$

and the acceleration of $m = \dfrac{T-mg}{m} = a$

$$\text{or } T - mg = ma \quad \ldots\ldots \quad (ii)$$

Adding the two equations together, we have

$$a(M+m) = (M-m)g$$

$$\text{or } a = \frac{M-m}{M+m}\,g \quad ..(iii)$$

To find the tension T of the string, we divide equation (i) by equation (ii),

$$\text{and get } \frac{Mg-T}{T-mg} = \frac{M}{m}$$

$$\text{or } Mmg - Tm = MT - Mmg$$

Transposing we have $T = \dfrac{2Mm}{M+m}\,g$ poundals if masses are expressed in lbs. ....(iv)

The pressure on the pulley is equal to twice the

tension, therefore it is equal to $\frac{4Mm}{M+m}g$ poundals (v)

This arrangement of two unequal weights over a pulley is of great advantage in determining the value of g by the application of equation (iii) for thereby the acceleration can be reduced to any convenient value, easily measurable Such an arrangement forms the principle of Atwood's machine.* This device is sometimes spoken of as a method of diluting the intensity of gravity.

26. The inclined plane.—Another mode of reducing the intensity of gravity is by means of an inclined plane which is a plane surface as AC (fig. 14) inclined to the horizontal at an angle θ.

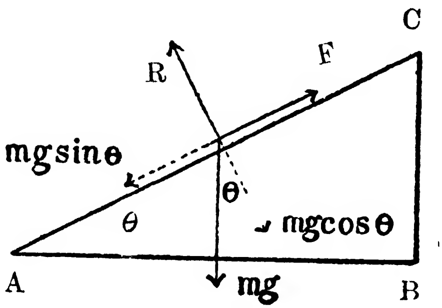


Fig. 14

Suppose we have a body resting on the inclined plane. Then the forces acting on the body are (i) Its weight equal to mg dynes acting vertically downwards. (ii) The reaction R between the plane and the body which *when the surface is smooth, always acts perpendicular to the plane*

The body cannot be at rest under the action of these two forces, for they are not in one and the same straight line; the body therefore moves down the inclined plane. To get the acceleration of this body down the plane, resolve the force mg due to gravity in two directions, one along the plane and the other perpendicular to it. The components are $mg \sin\theta$ and $mg \cos\theta$ (*for as is clear from the diagram, perpendi-*

*As this apparatus is not at all now used for finding the value of g, therefore it is omitted

cular to the inclined plane makes an $\angle\theta$ with mg). The latter component counterbalances the reaction, while the former, *i.e.* $mg \sin\theta$ accelerates the body downwards.

$\therefore$ the acceleration of the body would be

$$\frac{mg \sin\theta}{m} = \mathbf{g} \sin\theta \qquad \ldots(10)$$

By making θ small, the acceleration can be reduced to any value. Thus the acceleration of a body down an inclined plane is obtained by the product of the value of g and the sine of the angle of inclination of the plane to the horizontal. The inclination is sometimes denoted by the sine itself, *i.e.* $\frac{h}{l}$. Thus if in an inclined plane, the length is 100 and the height is 1, the inclined plane is spoken of as rising 1 in 100. This means that for every 100 units of length traversed along the inclined plane, the level is raised by one unit of length. To keep the particle in equilibrium on the plane, a force equal to $mg \sin\theta$ must be applied parallel to the inclined plane.

27. Methods of measuring the value of g.

(*i*) **The falling plate.** This is a direct method of finding g. A smoked plate of glass is supported by a thread hanging from a peg. A tuning fork carrying a stiff spike on one of its prongs is adjusted so as to touch the lower end of the plate, (fig. 15). The tuning fork is set vibrating and the thread is burnt. The plate drops and the spike traces out a wavy line on the smoked surface, the length of the wave is small at first and gradually increases (as shown in fig. 15). The period of the wave is equal to the period of vibration of the fork. The distance travelled in an exactly measured interval of time can therefore be read off from the tracing. Two consecutive measurements are sufficient to determine the value of g.

Suppose t is the time of vibration of the tuning fork.

s_1 is the distance between A and C
s_2 " " " C and E

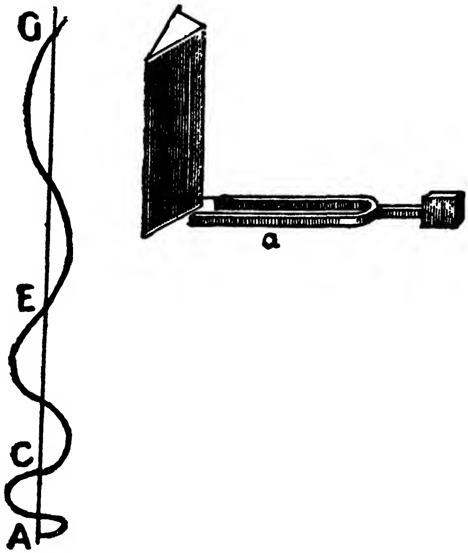


FIG 15

and u is the velocity of the plate, when A is in contact with the spike; then we have

$$s_1 = ut + \tfrac{1}{2}gt^2$$
$$\therefore\ 2s_1 = u \times 2t + gt^2$$
and $$s_2 + s_1 = u \times 2t + \tfrac{1}{2}g(2t)^2$$
$$\therefore\quad s_2 + s_1 - 2s_1 = gt^2$$
or $$g = \frac{s_2 - s_1}{t^2} \quad \text{.................. (11)}$$

This method however, is rather tedious. The easiest and the most convenient methed of finding the value of g is by simple pendulum.

(*ii*) **The simple pendulum.** A simple pendulum consists of a small heavy bob at one end of a weightless string, the other end of which is fixed. The bob swings to and fro in a vertical plane under the

action of gravity. Then, if the arc of swing be very small, it can be shown both experimentally and mathematically that the period of complete vibration

$$T=2\pi\sqrt{\frac{l}{g}} \qquad \ldots\ldots\ldots \quad \textbf{(12)}$$

Where l is the length of the pendulum, from the point of support to the centre of gravity of the bob and g is the value of gravity.

Both l and T* can be measured with a high degree of accuracy Knowing these, the value of g is given by the formula

$$g=4\pi^2\frac{l}{T^2}, \text{which may also be written as } \frac{l}{T^2}=\frac{g}{4\pi^2}.$$

Note —*From the above equation it follows that* $\frac{l}{T^2}$, *is constant for a simple pendulum*

The fact, that vibrations of a pendulum are isochronous, is of great utility in measuring time. Most of the clocks are regulated by means of a pendulum. It is the time-keeping part of the clock. A pendulum which makes one complete vibration in 2 seconds is called a seconds pendulum, because the bob passes its lowest point once every second.

27. (*a*) **The Escapement** The device by which the rate of rotation of the wheels of a clock carrying the hands is controlled by the swinging of a pendulum is called the *escapement.* It is shown in fig. 15 (*a*) Its second function is to constantly transfer energy from the falling weight or spring to the pendulum so that its vibrations may not be damped.

The escapement wheel tends to revolve rapidly due to the action of the spring. The *anchor A*, with two pallets *a* and *b*, holds the escapement wheel firmly and allows it to rotate, only when it (*i.e.* the anchor) is displaced. The anchor is connected by a small rod called the 'crutch' and 'the fork' to the pen-

* Time of vibration can be accurately determined by noting the time of 100 oscillations and then dividing by 100.

dulum rod. This vibrates along with the pendulum. When the pendulum is at its lowest point it displaces the anchor and the wheel rotates through the space of one tooth. A seconds pendulum passes through its lowest point *twice* in 2 *seconds* and therefore the wheel rotates through one tooth per sec. The crutch and the fork transfer energy to the bob of the pendulum and thus keep it swinging with constant amplitude There are various forms of the escapements but the principle underlying is the same.

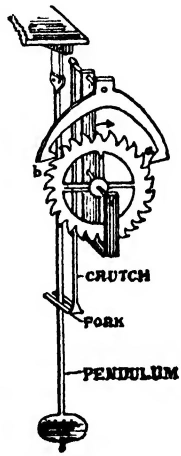


Fig. 15 (*a*)

28. Gravitational units of force. The *dyne* and the *poundal* are the absolute units of force in the *C.G.S.* and the *F.P.S.* systems respectively. The gramme weight and the pound weight are known as gravitational units, for their value depends upon the value of gravity at the given place. They are more convenient than the absolute units for ordinary use and they can be easily converted into absolute units, by multiplying them by the value of acceleration due to gravity, in the corresponding system of units. At Lahore a lb. wt. is equal to 32 poundals and a *gramme weight* is equal to 978·4 dynes.

SUMMARY

1. **Gravitation** is the general phenomenon by which every particle of matter in the universe attracts every other with a force proportional to the product of their masses and inversely as the square of the distance between them

$$F \propto \frac{m \times m'}{d^2}.$$

2. **Gravity** is the phenomenon of attraction of bodies towards the Earth

3. The force with which a body is attracted towards the Earth is known as its **weight.**

4. The weight of a body is proportional to its mass. It is equal to the product of mass and the acceleration, with

which a body moves towards the Earth, when free to do so.

$$w=mg.$$

5. The time of oscillation of a simple pendulum is given by the formula $t=2\pi\sqrt{\frac{l}{g}}$.

6. A **gramme weight** and a **pound weight** are the gravitational units of force on the two systems.

A **gramme weight**$=981$ dynes
A **pound weight**$=32$ poundals.

7. The acceleration, with which two bodies connected together would move, when hung up by the two ends of a string is equal to the product of g and the ratio of the difference of their masses to their sum:

i.e. $$a=\frac{P-Q}{P+Q}\times g.$$

8. The acceleration of a body down an inclined plane is equal to $g \sin \theta$ where θ is the angle of inclination.

EXAMPLES

1. Find the acceleration produced in a mass of 10 lbs. by a force of 4 lbs. weight.

The force$=4$ lbs. weight$=4\times32=128$ poundals
The mass$=10$ lbs.
Applying the formula $F=m\ a$. we get

$$128=10a$$

Or $$a=12{\cdot}8 \text{ ft. per sec. per sec.}$$

2. Convert the weights of (*i*) 5 lbs, (*ii*) 5 grammes, (*iii*) 4 ounces and (*iv*) two kilogrammes into the corresponding absolute units.

(*i*) 5 lbs weight $=5\times32=160$ poundals.
(*ii*) 5 gms. weight $=5\times981=4905$ dynes.
(*iii*) 4 oz. weight $=\frac{4}{16}\times32=8$ poundals
(*iv*) 2 kilgms. weight$=2\times1000\times981=1962000$ dynes.

3. What is the weight of 40 grammes at a place, where a falling body travels 245·25 cms. in the first second.

The distance described$=\frac{981}{4}$ cms.

By the formula $s=\frac{1}{2}at^2$, we have

$\frac{981}{4}=\frac{1}{2}a.\ 1$ or $a=\frac{981}{2}$ cms. per sec. per sec.

$\therefore$ 40 grammes would weigh $\dfrac{40\times\frac{981}{2}}{981}$ grms. weight

$=20$ grammes weight.

4. Find the length of a pendulum, which makes one complete oscillation per second and also of a seconds pendulum ($g=981$).

Suppose the length of such a pendulum $=l$.

Then $t=2\pi\sqrt{\frac{l}{981}}$

or $\quad l=\frac{981}{4\pi^2}=24{\cdot}84$ cms. $\qquad$ (*i*)

Again $2=2\pi\sqrt{\frac{l_1}{g}}$.

or $4=\frac{4\pi^2 l_1}{g}$ or $l_1=\frac{981}{\pi^2}=99{\cdot}36$ cms.

5. A pendulum beats seconds at a place, where $g=981{\cdot}17$, find its time of swing at a place where $g=978{\cdot}10$.

Here the length remains the same, therefore the periods are inversely proportional to the square roots of the value of g. Therefore the period of oscillation at the second place $=\sqrt{\frac{981{\cdot}17}{978{\cdot}10}}=1{\cdot}00156$ seconds.

6. Weights of five and ten grammes are connected by a string, which passes over a pulley; if the weights are allowed to fall, find the velocity, when the heavier weight has descended through a metre.

The acceleration, with which the two connected bodies would move $=\frac{10-5}{15}\times 981$ cms. per sec. per sec.

$=\frac{981}{3}=327$,, ,, ,, ,, ,,

Suppose v is the velocity after they have traversed 100 cms. then

$$v^2-0^2=2\times 327\times 100$$

Or $v=255{\cdot}7$ cms. per second.

7. Show that the velocity acquired down an inclined plane is that due to a fall through the same vertical height as that of the plane.

Suppose h=height, and l=length of the plane. Then if the body falls vertically downwards a distance h, then its velocity v would be given by the equation.

$$v=\sqrt{2gh}$$

for $v^2-u^2=2as$ or $v^2-0^2=2gh$.

If however, the same body were to roll down the inclined plane, then its acceleration would be $g\times\frac{h}{l}$ and the distance traversed would be equal to l. Hence its velocity will be given by $v^2=2g\frac{h}{l}l$ or $v=\sqrt{2gh}$

8. The bob of a pendulum consists of a jar containing mercury. If more mercury is poured in, will the time of oscillation be increased or diminished?

9. Find the value of g at a place, where the length of the seconds pendulum is 0 994 metre.

10. Masses of one and three pounds hang from the two ends of a fine string, suspended over a smooth pulley. At what rate will they be moving at the end of one second after they are set free.

11. A mass of one ton is acted upon by a constant force and acquires a velocity of 20 feet per second in 4 seconds. Find the force.

12. A body acted upon by a force describes in three successive seconds distances equal to 20, 30 and 40 feet respectively. What ratio does the force producing the motion bear to the weight of the body?

13. What force measured in lbs. weight acting on a body of mass 10 stones would move it through 48 feet in 8 seconds ?

14. What distance shall a mass of 20 lbs. starting with a velocity of 20 feet per second describe before coming to rest, if it is subjected to a retarding force equal to $^1/_4$ of its weight.

15. Two masses P and Q are connected together by a string passing over a pulley. P moves downwards 2 ft. in the first second. Find the ratio of masses; g=32.

16. A railway truck, tare 5 tons, slips a distance equal

to 100 feet down an incline, rising 3 in 25. Find its momentum.

17. A train starts from rest on a level line and moves through 1200 feet in the first minute. It then begins to ascend a uniform incline, up which it is found to run with uniform velocity. Find the inclination of this portion of the line on the supposition that the engine exerts a constant pull.

18 With what velocity must a particle be projected down a plane 12 feet in height and inclined to the horizon at an angle of 30°, so as to reach the bottom in one second.

19. With what velocity must a particle be projected up a plane 10 feet in height and inclined to the horizon at an angle of 30°, so as to reach the top in one second

20. If the weight of a certain mass be 15 lbs. at a place where the body falls 64 feet in two seconds What will be the weight of the same mass at a place, where the body falls through 176 feet in 3 seconds?

EXAMINATION QUESTIONS I

1. What are the two systems of units? Express the units of length, mass and time on these units.

2. What is meant by the composition and resolution of velocities? Enunciate and prove parallelogram law of velocities?

3. Define the terms motion, velocity acceleration, force, momentum, dyne, poundal, gramme weight and inertia.

4. State Newton's second law of motion and show how to deduce the measure of unit force from it.

5. Distinguish between a poundal and the weight of one pound. What is the experimental evidence for the statement that the weight of a body is proportional to its mass?

6. State the conditions, which must hold in order that three forces, which are not parallel may be in equilibrium. A picture weighing 3 lbs. is hung by a cord 10 inches long passing over a peg. Find the tension in the string if the ends of the cord are 8 inches apart.

7. A train moving 40 miles per hour has its speed reduced to 15 miles per hour in two minutes. What distance has it travelled in the meantime?

8. What velocity is acquired by a body falling down a

smooth inclined plane, rising 1 in 20, if its length be 320 feet?

9. An iron cage descends a mine. The tension in the rope equals the weight of 200 lbs. When at rest, it was 225 lbs. Find the time of descending 100 feet.

10. The velocity of a body is observed to increase by 4 miles per hour in every minute of its motion. Compare the force acting on it with the force of gravity.

CHAPTER VI
FRICTION

29. Friction. When a body, say a book, is lying on a table, then its weight acts vertically downwards and its re-action vertically upwards, the two forces are then said to be in equilibrium If however, a small force be applied parallel to the table, then no visible effect is produced. Thus the force, with which we attempt to move the book in the horizontal direction, is resisted by some force called into play. This latter force is called the *force of friction.*

It is seen that the minimum force required to produce motion in the book is less when the surface of the table is smooth and the book is light. Further it is observed that when the book is set into motion, its acceleration is not equal to the force applied, divided by the mass; but it is considerably less. We find that when one body is moved over another, with which it is in contact, a resistance is offered to its motion. This is called *Friction.*

Friction then, we see, is due to the roughness of the two surfaces, and as in nature there is no surface absolutely smooth, a certain amount of friction will always come into play even between the most highly polished surfaces.

Experiments on Friction. The laws of friction can be demonstrated experimentally with the help of the instrument shown in the figure. It consists of a board of hard wood and a small tray Q attached to a string, which passes over a pulley and supports a pan and weight P at the other end.

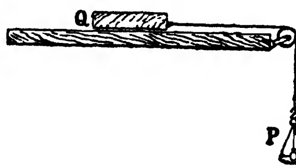


FIG. 16

Place the tray on the wooden board. It will remain at rest. Place a small weight in the pan; still the tray remains at rest. Thus at this stage the force of friction brought into play is equal to the pull of the pan and the small weight placed in it. If however, the weight on the pan be gradually increased, a point will be reached when the friction brought into play would be just sufficient to keep the tray at rest and a slight further increase in the weight would make the pan move. Thus the force of friction cannot increase any further. It has reached its maximum value and this value of friction is spoken of as the *limiting friction.* Now place a heavy weight in the tray and repeat the experiment Observe, that to produce motion, the weight in the pan has to be increased. Thus we see that the weight necessary to move the tray is directly proportional to its own weight and that of its contents, *that is*, to the reaction between the two surfaces. If now the tray be laid on its side instead of on its base, it will be seen that the same force is required to make the tray move as before. *Thus the force of friction is independent of the areas of the two surfaces in contact.* Further if the base of the tray be made up of glass instead of wood, it will be seen that the force of friction is considerably decreased, proving that the force of friction depends upon the nature of the surfaces in contact.

These experimental results, put in concise form, are known as laws of friction. They are as follows:—

1. The force of friction always acts in a direction, opposite to that in which motion would ensue.

2. The magnitude of the force is such as just to preserve equilibrium; but at no point can the amount necessary exceed the "*limiting friction.*" The friction then remains constant; and if the applied force be increased, the body will move. The net force acting on the body would be the difference between the applied force and the limiting friction.

3. The limiting friction is independent of the

areas of the two surfaces in contact, but depends only on their nature.

4. The limiting friction between two surfaces is proportional to the normal re-action between them.

5. When motion takes place, the frictional resistance is less than the limiting friction, but is always independent of velocity.

Co-efficient of friction. The ratio of limiting friction to the normal re-action between the surfaces is known as the co-efficient of friction.

Thus μ the co-efficient of friction$=\frac{F}{R}$.

The value of μ differs considerably for different materials, but it is always less than unity.

Angle of friction. Suppose the re-action of a body resting on a rough surface is denoted by R and the friction by F, then the resultant of these two forces will be a force P making an angle θ with the direction of R, such that its tangent$=\frac{F}{R}$

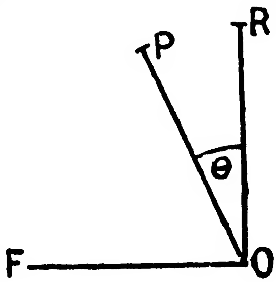


FIG. 17

i. e. $\frac{F}{R}=\text{tangent } \theta$

If F is the limiting friction (*i. e.* motion is just on the point of taking place), $\frac{F}{R}=\mu$.

In this case angle θ is denoted by λ and is termed the angle of friction.

$$\therefore \quad \mu=\text{tangent } \lambda \ldots\ldots\ldots \textbf{(13)}$$

The angle λ, whose tangent gives the coefficient of friction, is called the angle of friction.

If motion is not on the point of taking place; then F must be less than the limiting friction and the resultant or total reaction must make an angle θ with

the normal reaction such that θ is always less than λ.

Rough inclined plane Suppose a body is resting on an inclined plane AC Then the forces acting upon it are (*i*) its weight$=mg$, acting vertically downwards (*ii*) Its reaction R, acting perpendicular to AC, and (*iii*) the force of friction F acting parallel to AC.

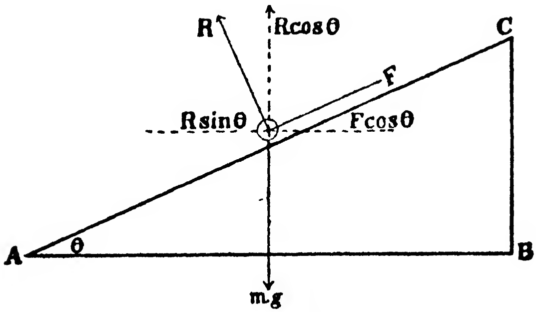


FIG. 18

Then resolving horizontally, we have

$F \cos \theta = R \sin \theta$

or $\frac{F}{R} = \tan \theta$, where θ is the angle of inclination.

This method is sometimes employed to find the value of F and λ (for λ will be equal to θ, when motion is about to ensue).

The following is a table of approximate values for the co-efficients of friction:—

Wood upon wood	·5
„ „ „ lubricated	·2
Wood upon polished metal	·3
„ „ „ lubricated	·12
Metal upon metal	·18
„ „ „ lubricated	·12

SUMMARY

1. When one body is to be moved over another with which it is in contact, a resistance to motion is offered and this resistance is called friction.

The maximum value of friction is spoken of as *limiting friction.*

2 Laws of friction :—

(*i*) The force of friction always acts in a direction opposite to that in which motion would ensue.

(*ii*) The magnitude of frictional force is such as just to preserve equilibrium, but at no point the amount necessary can exceed the limiting friction.

(*iii*) The limiting friction is independent of the area of the two surfaces in contact, but depends on their nature

(*iv*) The limiting friction between two surfaces is proportional to the normal reaction between them.

(*v*) When motion takes place the frictional resistance is less than the limiting friction and is independent of velocity.

3 Co-efficient of friction The ratio of limiting friction to the reaction between the surfaces is called the co-efficient of friction.

4. Angle of friction The angle, whose tangent gives the co-efficient of friction, is called the angle of friction.

EXAMPLES

1 A mass of 1 cwt rests on a rough inclined plane of angle 30° If the co-efficient of friction be $1/\sqrt{3}$, find the greatest and the least forces, which acting parallel to the plane can just maintain the mass in equilibrium

The force acting downwards due to gravity $= mg \sin \theta$

The force due to friction $= \mu R = m \dfrac{g\sqrt{3}}{2} \times \dfrac{1}{\sqrt{3}} = \frac{1}{2} mg$,

where $R =$ reaction $= mg \cos \theta$

Thus the force of friction and of gravity are equal and therefore the body would remain at rest without exerting any force and would be just on the point of slipping downwards.

If the body is to be just on the point of moving upwards, the forces to be balanced are (*i*) $\frac{1}{2}$ mg due to gravity and (*ii*) $\frac{1}{2}$ mg due to friction , therefore total force required to keep the body in equilibrium shall be the sum of these two and thus equal to the weight of the mass itself, *i.e* 112 lbs weight

2 A weight W rests in equilibrium on a rough inclined plane, being just on the point of slipping down On applying a force W parallel to the plane, the weight is just

on the point of moving up. Find the angle of the plane and the co-efficient of friction.

Suppose θ is the angle of inclination and μ the co-efficient of friction.

Then from the first condition we have

$W \sin\theta$ (*i.e.* force due to gravity)$=\mu . R$ (*i.e.* force due to friction).

or $\quad W \sin\theta=\mu . W \cos\theta$ *

From this we get $\mu=\dfrac{\sin\theta}{\cos\theta}$ $\qquad$ (*i*)

From the second condition, $W \sin\theta+\mu . W\cos\theta=W$ for both the forces (*i*) due to gravity and (*ii*) due to friction are together counterbalanced by W.

Substituting $\dfrac{\sin\theta}{\cos\theta}$ for μ from equation $\qquad$ (*i*)

We have $2\sin\theta=1$

or $\quad \sin\theta=\frac{1}{2}$ or $\theta=30^\circ$ $\quad$ .. $\qquad$ (*ii*)

Substituting the value of θ in equation (*i*) we have

$$\mu=\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}=\frac{1}{\sqrt{3}}.$$

3. Calculate the force required to maintain a train of 140 tons running on a horizontal plane at a uniform speed, the co-efficient of friction being $\dfrac{1}{160}$.

4. A body slides down an inclined plane rising 1 in 2 and 18 feet long. If it starts from rest, find its velocity when it reaches the lower end of the plane: (*i*) when the plane is smooth and (*ii*) when it is rough, $\mu=\sqrt{3}/4$.

5. A body will just rest on a plane inclined to the horizon at 30°. Find its acceleration, when the plane is inclined at 45°.

6. A train of which the mass is 200 tons, can be drawn by an engine at a uniform speed of 30 miles an hour up an incline of 1 in 200, or at 40 miles an hour up an incline of 1 in 400. Assuming the frictional resistance to be independent of the velocity, calculate the frictional resistance n lbs. weight per ton.

---

* Note that the reaction $R=W\cos\theta$.

CHAPTER VII

PARALLEL FORCES

30. Parallel Forces. Forces are said to be parallel, when the lines, along which they act, are parallel to one another. Parallel forces thus, can act only on extended bodies. They cannot act at a point.

Parallel forces are said to be *like*, when they act in the same direction and *unlike*, when they act in opposite directions.

Moment of a force. The moment of a force about a given point or line is its tendency to produce rotation about that point or line, regarded as fixed ; it is always measured by the product of the *force* and the *perpendicular* drawn from the point or line, on the *line of action* of the force.

The moment of a force about a point is generally regarded as *positive*, if it tends to set up rotation in a counter-clockwise direction ; and negative, if the direction is clockwise.

It is a common experience, that to shut or open a door, it is always more convenient to apply the force away from the hinges. The greater the distance of the point of application of the force from the hinges, the lesser the force required to turn the door and *vice versa*.

31. Resultant of Parallel Forces. The resultant of two *like*, parallel forces is a force :—

(*i*) Whose magnitude is the sum of the two component forces *i.e.* $R=P+Q$.

(*ii*) Whose direction is the same as that of the two component forces.

(*iii*) Whose line of action is parallel to either of the

two component forces.

(*iv*) Whose point of application is a point C between A and B, such that the moments of P and Q about that point are equal and opposite, *i.e*

$P \times AC = Q \times BC.$

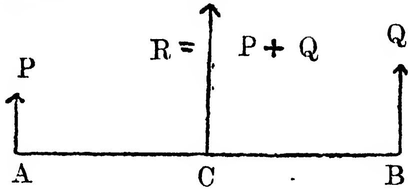


FIG 19

The resultant of two *unlike*, parallel forces is a force:—

(*i*) Whose magnitude is the difference of two forces P and Q, *i.e* $R = P - Q$.

(*ii*) Whose direction is the same as that of the greater of the two forces

(*iii*) Whose line of action is parallel to either of the two components.

(*iv*) Whose point of application is a point C on AB produced on the side nearer to B, (*i.e.* the point of application of the bigger force) such that the moments of P and Q about that point are equal and opposite, *i e.* $P \times BC = Q \times AC$.

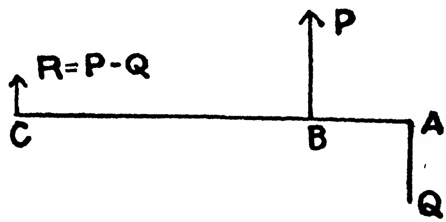


FIG. 20

Hence we see that in order that the translatory effect of the resultant be the same as that of the components, conditions (*i*), (*ii*) and (*iii*) should be fulfilled and in order that the rotatory effect of the resultant be the same as that of the components, condition (*iv*) must be satisfied. To sum up then, *the resultant of parallel forces is the algebraic sum of its components and acts in the direction of the numerically bigger component and at a point such that the moment of the resultant equals the algebraic sum of the moments of all the components about any point.*

Experiment. The above statement is experimentally proved in the following simple way:—Take a half-metre rod AB of steel, find its centre of gravity C by balancing it over a wedge, find its weight by means of a spring balance and suspend the rod from the two spring balances as shown in the figure. Suspend at the point C a weight, which together with the weight of the rod, is equal to 12 lbs.

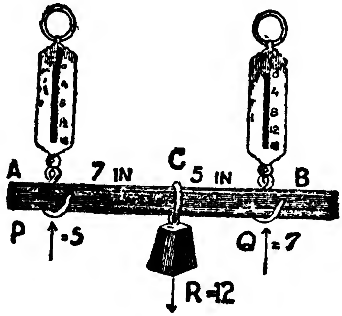


FIG 21

Then the forces acting on the rod are parallel and are in equilibrium for the rod remains at rest, but the total downward force is equal to 12 lbs. and therefore this must be equal in magnitude and opposite in direction to the resultant of the two upward forces P and Q Take the readings of P and Q See that $P+Q=5+7=12$ lbs. Measure the distances AC and BC. It is found that BC is 5 inches and AC is 7 inches.

$\therefore$ the moment of P about $C=5\times7=35$

and the moment of Q about $C=7\times5=35$

The law of moments, which is universally true, is most useful in the case of parallel forces, for it affords a ready method of finding the *position* of the resultant arithmetically If the forces acting on a body be w_1, w_2, w_3, etc. at perpendicular distances x_1, x_2, x_3, etc. respectively from one end O, then the distance of the resultant from the same end is given by x, where

$$x=\frac{w_1x_1+w_2x_2+w_3x_3+\ldots\ldots}{w_1+w_2+w_3+\ldots}.$$

Provided $+$ sign is given to forces acting in one direction and $-$ sign to those acting in the opposite direction.

32. Couples. The case when two equal but unlike parallel forces act on a body is very peculiar, for theoretically, since the forces are equal and unlike their resultant must be zero; but since the forces do not act in the same line, they will have a moment about any point in the plane and *this moment is always equal to either of the forces multiplied by the perpendicular distance between them.* It is impossible, therefore, to find a single force, which shall satisfy the two conditions of equilibrium. *Thus a system of two equal unlike parallel forces which do not act through the same point is known as a couple and this cannot be replaced by a single force.*

The perpendicular distance between the two forces is called the 'Arm of the couple.'

The effect of a couple is always to rotate the body and this rotatory effect is always equal to its moment. The system of forces applied by the two hands to the lever of a duplicator press constitutes a couple, the hands pressing at the two ends of the lever with equal and opposite force.

The algebraic sum of the moments of the forces which constitute a couple, about any point in the plane of the couple, is constant and is equal to the moment of the couple.

(*i*) Let the point about which moments are required be O (between A and B), then the sum of the moments due to two forces, each equal to P, acting at A and B, is equal to:—

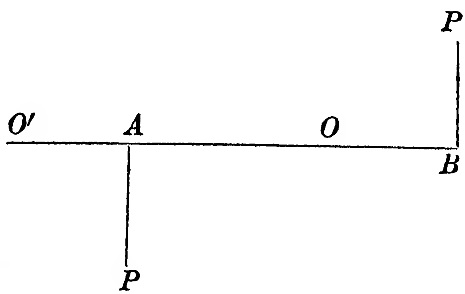


Fig. 22

$$P \times AO + P \times BO = P(AO + BO)$$
$$= P.\, AB$$

(*ii*) Let the point, about which moments are required be O' (outside BA).

Then the sum of the moments due to two forces, each equal to P, acting at A and B, is equal to

$$P.\ BO' - P.\ AO' = P.\left\{(BA + AO') - (AO')\right\}$$
$$= P.\,BA$$

Moments tending to rotate the body in the counter-clockwise direction are called *positive* while those tending to rotate it in the clockwise direction are taken as negative.

NOTE.—Two couples impressed on a rigid body balance, if their moments are equal and opposite.

Thus to balance a body acted upon by a couple, it is necessary to impress *another couple of equal but opposite moment on the body.*

33. Centre of gravity. The composition of parallel forces is strikingly illustrated by the force of gravity. A rigid body is made up of a very large number of small particles and every particle is pulled towards the centre of the Earth, with a force equal to the product of its mass and the value of gravity, *i. e.* mg. These forces acting on the various particles of a body due to gravity are parallel for all practical purposes, provided however, that the body is of finite size. The pulling forces exerted on the various particles, though in reality converging towards the centre of the Earth, may yet be considered as parallel, since the centre of the Earth is very far off as compared to the distances with which we have to deal. Thus the whole pull on a body due to the Earth, known as its weight, is in reality the resultant of an infinite number of parallel forces.

The resultant of various parallel forces due to gravity passes through a point fixed relatively in the body, and is called the **centre of gravity of the body.**

To find the centre of gravity of a body by calculation. In the case of symmetrical bodies, it is possible to find the centre of gravity of the body from its geometrical form. Thus the centre of gravity of a

sphere is its centre, similar is the case with a cube or an ellipsoid. In some cases however, even if the body be less uniform, it is possible to calculate its position, for it is simply the point through which the resultant of a number of parallel forces passes. Thus if m_1, m_2, etc. be the masses at distances x_1, x_2, etc. from any point in a line, the distance x of their centre of gravity from the same point is given by

$$x = \frac{m_1x_1 + m_2x_2 + \ldots\ldots\ldots}{m_1 + m_2 \quad + \ldots \ldots \ldots}$$ as proved above in the case of resultant of parallel forces,

Thus the centre of gravity of :—

a uniform rod	is its middle point.
a circular ring	is its centre.
a circular disc	is its centre.
a sphere	is the centre of the sphere.
a parallelogram	is the point of intersection of its diagonals.
a triangle	is the point of intersection of its median lines.
a solid cone	is $^1/_2$ of the way up the axis of the cone.

SUMMARY

1. **Parallel forces.** Forces are said to be parallel when the lines along which they act are parallel to one another. They are said to be like, when they act in the same direction; and unlike, when they act in opposite directions.

2. A system of two equal, unlike, parallel forces, which do not act through the same point is known as a **couple** and this cannot be replaced by a single force.

3. The **moment of a couple** is the product of one of the forces and the perpendicular distance between them.

4 The **centre of gravity** of a body is the point relatively fixed to the body, such that through it, passes the resultant of all the parallel forces due to the Earth's attraction in the body.

EXAMPLES

1 Two like parallel forces of 200 and 400 grammes weight act upon the extremities of a body 18 centimetres long. Find the position and magnitude of the resultant (the weight of the body being neglected).

$R=P+Q$; $\therefore$ 200 $+$400 $=$600 grammes weight.

Suppose x=distance of the position of action of R from the force of 200 grammes weight:

$$\text{Then } 200x=400(18-x)$$
$$\text{or} \qquad x=12 \text{ centimetres.}$$

2. Find the resultant of parallel forces 2, -4, 8 and -6 acting at equal distances of 6 inches along a bar the weight of which may be neglected.

The resultant of forces 2 and 8 would be 10 and shall act at a point X so that $8\times XC=2(12-XC)$

$$\text{or} \quad XC=2{\cdot}4 \text{ inches.}$$

Similarly the resultant of forces -4 and $-6=-10$ and shall act at a point Y so that $-6\times(12-BY)=-4\times BY$ or $BY=7{\cdot}2$ inches or $CY=1{\cdot}2$ inches.

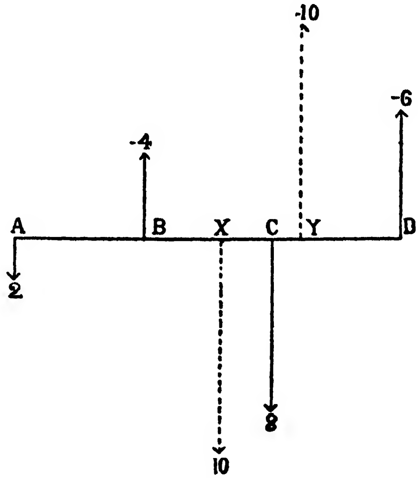


FIG. 23

Hence the distance between $XY=2{\cdot}4''+1{\cdot}2''$
$=3{\cdot}6$ inches.

The two forces are equal in magnitude and opposite in sign. Thus they constitute a couple the moment of which is equal to $\dfrac{10\times3{\cdot}6}{12}=3$.

3. A man and a boy carry a load of 100 lbs. by means of a pole 8 feet long. Where must it be attached to the pole, that the boy may bear one-fourth of the weight,

(*i*) neglecting the weight of the pole, (*ii*) taking the weight of the pole as 40 lbs.

(*i*) The total weight to be borne by the man and the boy$=$100 lbs.; and as the boy is to bear 1/4 of the total weight, therefore he must bear 25 lbs. and the man 75 lbs.

Let x be the distance of the point from the boy and $8-x$ from the man.

$$\text{Then } 25\times x=75\times(8-x)$$
$$\text{or } x=6 \text{ feet.}$$

(*ii*) The total wt. in the second case$=100+40=140$ lbs.

The boy is to carry $\dfrac{140}{4}=35$ lbs. and the man 105 lbs.

Let x be the distance of the point of suspension of the load from the boy. The weight of the pole (40 lbs.) acts through its centre. Taking moments about the position of the boy for equilibrium, we have $100x+40\times4=105\times8+35\times0$ or $x=6{\cdot}8$ feet from the boy or $1{\cdot}2$ feet from the man

4. A beam, whose length is 8 feet and weight 20 lbs. has weights of 4 lbs. and 8 lbs. suspended from its extremities. Find the position of a point about which it will balance.

5. An iron bar weighs 5 lbs. per foot, it balances about a point 2 feet from one end, when a weight of $6^1/_4$ lbs. is suspended from that end. How long is the bar?

6. A light bar (the weight of which may be neglected), 5 feet in length, is placed upon a fulcrum, 1 foot from one end. If a weight of 10 lbs. is placed at this end, what weight must be placed at the other end for equilibrium?

7. A straight uniform rod of 9 feet has weights of 20 and 30 lbs. attached to its ends, and rests in equilibrium when placed across a fulcrum, distant 4 feet from 30 lbs. weight. Find the weight of the rod.

8. A uniform beam 10 feet long, weighing 80 lbs., is suspended from two points in a horizontal ceiling, 16 feet apart, by strings each 5 feet long attached to its ends. Find the tension in each string.

9. A uniform plank 24 feet long rests on the top of the two walls 12 feet apart. The walls are of the same height and the plank projects equally beyond each. A man starts walking from the centre towards one end of the plank. If his weight be 8 stones and that of the plank 4 stones, find how far beyond the wall he can walk without tipping up the plank.

CHAPTER VIII

WORK AND ENERGY

34. Work. Whenever a force acts upon a body in such a way as to produce motion, work is said to be done.

From the above definition, it follows that a man does work, if he lifts up a body from the ground and that he does no work if he does not succeed in moving the body; although he may be exerting a considerable force. Similarly a man supporting a weight on the palm of his hand at a fixed height from the ground does no work, although he exerts force in order to prevent it from falling to the ground. **Thus work is done by a force only when its point of application moves.**

Work is done *by* a force, when the body upon which the force acts, moves in the direction of the force; and work is done *against* the force, when the body on which the force acts moves in the direction opposite to that of the force.

No work is said to be done either by or against a force, if the direction of motion of the body is *perpendicular to the direction of the force.*

Thus in the case of a train moving on a level line, no work is done either by or against the force of gravity but when it is moving uphill, work is done by the engine *against* the force of gravity and when it is moving downhill, work is done *by* the force of gravity.

35. Measurement of work. The work done by or against a force is measured by the product of the *force* into the *distance* through which its point of application has moved in *the direction of the force.*

$$\mathbf{W = F \times S}. \qquad \ldots\ldots\ldots\ldots \mathbf{(14)}$$

When however, the body on which the force is acting does not actually move in the direction of the force, as for example, a stone resting on a smooth table AB (fig. 24) is pulled by a string AD in the direction AC with a force P, but the stone slides along the table to the point B, the work done is still equal to the product of the *components* of the force in the direction of motion and the distance through which the body moves.

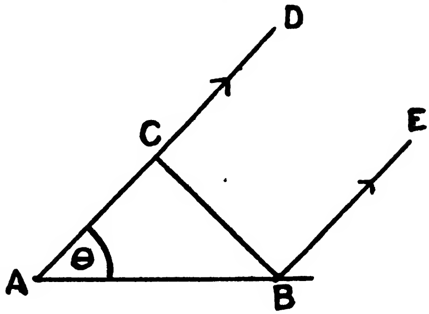


FIG. 24

i.e. $W = AB \times P \cos\theta = P \times AC$, where C is a point on the line AD, where the perpendicular from the point B on AD falls and AC is the distance which the particle has moved parallel to the line of action of the force.

Thus the work done, W = the component of the force in the direction of displacement $\times$ distance

or *W = force $\times$ component of displacement in the direction of the force.*

From the above it follows, that the work done against gravity in raising a mass m through a height h is independent of the path taken by it. The work done in raising a body through a perpendicular distance h is given by

$$\mathbf{W = mgh} \qquad \textbf{(14a)}$$

Units of work. In the *F.P.S.* system, the absolute unit of work is called the **foot-poundal** and it is the work done by a force of one poundal, when its point of application moves through one foot. The practical unit of work (in this system) is the **foot-pound** and is equal to the work done by a force of one pound weight, when its point of application is moved through one

foot. It is approximately equal to 32 foot-poundals.

In the *C.G.S.* system, the absolute unit of work is called the **erg** and is the work done by a force of one dyne, when its point of application moves through one centimetre. The gravitational unit of work on the *C.G.S.* system is called the **gramme-centimetre** and is the work done in raising a mass of one gramme through one centimetre against the force of gravity. It is equal to 981 ergs. The *Practical unit* of work is called the **joule** and is equal to 10^7 ergs. The French Engineers use the kilogramme-metre as their practical unit of work. It is equivalent to the work done by a weight of a kilogramme when raised through a metre. It is equal to 981,00,000 ergs.

36. Power.—It is the *rate* at which an agent does work. When uniform, it is measured by the amount of work done in one second.

In the *F.P.S.* system, the common unit used by the Mechanical Engineers, is known as the **Horse-Power**. It was supposed to represent the rate at which an average horse was capable of doing work.

The rate of working is said to be one horse-power, when 550 *foot-pounds of work are done per sec.* or 33,000 *foot-pounds of work are done per minute.*

In the *C.G.S.* system, the prevalent unit of power is called the **Watt** and is *equal to the power of an agent which can do work at the rate of one joule* (10^7) *ergs per second.* This unit of power is extensively used by Electrical Engineers.

746 Watts = One Horse-power.

37. Energy*. The energy of a body is its capacity of doing work. Every body in motion, for example a bullet shot from a gun, is capable, under suitable

*Energy should not be confused with power. Energy refers to the total quantity of work, which a body can possibly do without any restriction of time, during which the work is done Power refers to the rate at which work is done and has nothing to do with the total quantity of work done.

2. It should not be confused with work either, but it may be called possible work.

conditions, of doing work. A body at rest may also be capable of doing work under suitable circumstances. Thus a stone lying on a roof will, if slightly pushed over to one side, acquire a great velocity during its fall and thus be capable of doing work. Similarly, a stretched India-rubber tubing and an elastic spring are capable of doing work when released.

Two kinds of energy:—(i) **Potential and** (ii) **Kinetic.**

Potential Energy is the energy possessed by a body by virtue of its position or configuration. Thus the energy of a stone on the roof of a building, a stretched spring and hot steam in the boiler of an engine are examples of bodies possessing potential energy.

Kinetic Energy is the energy possessed by a body by virtue of its mass and velocity. The energy of a moving bullet and of water in motion are examples of kinetic energy.

Measurement of Kinetic Energy. Work is done when a body is set in motion; and conversely, a moving body is capable of doing work, when brought to rest by a resistance. Suppose a force F acts upon a body of mass m, which is initially at rest. The force F would produce an acceleration of a cms. per sec. per sec., given by the equation

$$F=ma \ldots\ldots\ldots (i)$$

Suppose the force F continues to act till the body has traversed a distance equal to S. At that time the body shall have a velocity v given by the equation:—

$$v^2=2aS \ldots\ldots (ii)$$

Substituting the value of a from equation (i)

we have $v^2= 2.\frac{F}{m}.S$

$$\textbf{or } \tfrac{1}{2}\,\mathbf{mv^2=F.S.} \ldots\ldots \mathbf{(15)}$$

The right-hand expression denotes the work done by the force, when its point of application has been shifted through the distance S, but the whole of this work has been expended in producing a velocity v in the body ; therefore the left-hand expression gives the

value of kinetic energy of the body.

Suppose now, that the above body of mass m moving with velocity v is retarded by the action of some force F. This force will produce a negative acceleration of $-a$, given by the equation.

$$\frac{F}{m}=-a \quad \ldots (i)$$

and the body in consequence of this will be brought to rest after travelling a distance S, given by the equation:—

$$0^2-v^2=2\times -a\times S \qquad (i)$$

$$\text{or} \quad v^2=2\times \frac{F}{m}\times S \qquad (ii)$$

$$\therefore \quad {}^1/_2\, \mathbf{mv^2=FS} \qquad \textbf{(15a)}$$

The right-hand side expresses the work done against the retarding force: the left-hand side must denote the work done by the moving body and therefore its kinetic energy.

Thus the kinetic energy of a moving body of mass m and velocity v is always equal to $^1/_2\, mv^2$. If m is expressed in grammes and v in centimetres per second, the energy is measured in ergs; and if m is in pounds and v in feet per second, it is measured in foot-poundals.

38. The Conservation of Energy. "The total energy of any material system is a quantity, which can neither be increased nor diminished by any action between the parts of the system; though it may be transformed into any of the forms, of which energy is susceptible" (Clark Maxwell). The energy of a body may change its form from Potential to Kinetic, and *vice versa*. Thus a bullet shot upward from a gun starts with kinetic energy, but with no potential energy. As it goes up, its kinetic energy goes on decreasing, while its potential energy goes on increasing correspondingly, since its height becomes greater and greater till at its highest point, when the bullet is instantaneously at rest, the kinetic energy becomes zero, and the

whole of it is transformed into potential energy. The height to which the bullet rises can be easily found by dividing its kinetic energy at start by its weight.

$$\text{Thus} \quad {}^1\!/_2\, mv^2 = mg\ h$$

$$\text{or} \quad h = \frac{{}^1\!/_2\, mv^2}{mg} \ldots \qquad \ldots.(\mathbf{16})$$

Thus the energy of the bullet is conserved during its flight; at the bottom it is totally kinetic and at the top it is totally potential; but the amount is the same throughout the motion. The principle is not capable of experimental proof in the strict sense of the word; for it is not possible for us to follow with any close certainty the various interactions, which may occur between material bodies. The most convincing proof of the principle is probably the fact, that many other principles founded on it stand the test of experiment. The discovery of the above principle has perhaps been the greatest and the most important of all the discoveries of the last century. To sum up then, the principle of conservation states that whenever work is done or energy produced, it is not created out of nothing; but it is actually manufactured out of previously existing energy.

To show that the sum of kinetic and potential energies of a falling body is constant and is equal to the potential energy possessed by the body at the highest point.

Let a body of mass m be initially situated at a height h from the ground, its potential energy will then be $mg.\ h$.

Suppose it falls through a distance x. Its potential energy will be $mg(h-x)$ and its kinetic energy will be ${}^1\!/_2 mv^2 = {}^1\!/_2 m \times 2xg = mgx$.

Therefore sum of both kinds of energy will be $mg(h-x)+mgx=mgh$.

This is the potential energy, possessed by the body at its highest point.

39. Dissipation of Energy. The transformation of energy from available or useful form to unavailable

or useless form is spoken of as Dissipation of Energy.

We have mentioned above, that energy can neither be created nor destroyed; but what happens to the energy of a moving bullet, when it strikes against a hard target? Experience shows that the bullet is stopped and is heated Thus its kinetic energy is no doubt transformed into an equivalent amount of heat energy; which warms the air and cannot be utilized for any useful purpose. In this case the energy of the bullet is said to be dissipated, for it is transformed to a low level form, wherefrom it is not possible to make use of it. If we were to follow the phenomena of natural interactions of various bodies in the universe more closely, we would always see that energy of all forms is finally degraded to heat, though it may pass through many stages before it does so.

As a consequence of this transformation, the average temperature of the universe must be slowly increasing. Some of the scientists have gone even so far in this direction, as to declare that "the mechanical energy of the universe will be more and more transformed into universally diffused heat, until the universe will no longer be a fit abode for living beings" We may however, console ourselves by the idea that this would not take place at least for several generations to come.

Forms of Energy The following are the various forms in which energy manifests itself: 1. Motion 2. Strain. 3. Vibration. 4. Heat 5. Radiation. 6. Electrification. 7. Electricity in Motion. 8 Chemical separation. 10 Gravitative separation. Sometimes vital energy is added to these.

SUMMARY

1. **Work.** Whenever force acts upon a body in such a way as to produce motion, work is said to be done.

2. **Work**$=F.S$, *i. e* work is measured by the product of the force and the distance through which its point of application is shifted in the direction of the force.

3. **Erg** is the absolute unit of work in the *C. G S.* system; and is the work done by a force of one dyne, when its

point of application moves through one centimetre.

4. **Foot-poundal** is the absolute unit of work on the *F.P.S.* system; and is the work done by a force of one poundal, when its point of application moves through one foot.

5. **Gramme-c. metre and foot-pound** are the gravitational units in the *C.G.S.* and *F.P.S.* systems respectively. They are 981 and 32 times the corresponding absolute units.

6. **Power.** The rate at which an agent does work is called Power.

Horse-Power is the unit of power in the *F.P.S.* system; and is equal to the rate of doing work at 33,000 foot-pounds per minute or 550 foot-pounds per second.

Watt is the unit of power in the *C.G.S.* system; and is equal to the rate of doing work at one joule or 10^7 ergs per second.

7. **Energy of a body** is its capacity of doing work. It is of two kinds: (*a*) Potential energy, *i.e.* the energy which a body possesses by virtue of its position and mass. It is usually equal to mgh. (*b*) Kinetic energy, *i.e.* the energy, which a body possesses by virtue of its mass and velocity. It is measured by $\frac{1}{2}mv^2$.

8. **Conservation of Energy.** Energy can neither be created nor destroyed.

9. **Dissipation of Energy.** Transformation of energy from useful to useless form is known as dissipation of energy.

EXAMPLES

1. Find the energy of a bullet of mass 25 grammes moving with a velocity of 100 cms. per second.

$$K.E. = \tfrac{1}{2} \times 25 \times 100^2 = 125000 \text{ ergs.}$$

2. A stone weighing 2 lbs. is dropped from a tower 20 feet high. Calculate its *K.E.* when it reaches the ground.

(*a*) The work done by a weight of two pounds in falling through a distance of 20 feet

$$= 2 \times 32 \times 20 = 1280 \text{ foot-poundals.}$$

As work = kinetic energy, therefore $K.E. = 1280$ foot-poundals.

(*b*) The velocity of the stone on reaching the ground is given by :—

$$v^2 = 2 \times 32 \times 20$$

$$\text{and } K.E. = \tfrac{1}{2}mv^2 = \tfrac{1}{2} \times 2 \times 2 \times 32 \times 20$$

$$= 1280 \text{ foot-poundals.}$$

3. Find the work done and the horse-power developed by a locomotive, when it draws a train of 100 tons up an incline of 7 in 64 at a speed of 30 miles an hour, the friction being 15 lbs. weight per ton.

$$30 \text{ miles per hour} = \frac{30 \times 1760 \times 3}{60 \times 60} = 44 \text{ feet per sec.}$$

The forces to be overcome are (*i*) $15 \times 100 = 1500$ lbs. wt. due to friction, and (*ii*) $\frac{100 \times 2240 \times 7}{64} = 24500$ lbs. wt. due to gravity;

∴ Work done in one second $= F.S = 26000$ lbs. wt. $\times 44$ ft.
$= 1144000$ ft. lbs.

This represents an *H.P.* of $\frac{1144000}{550} = 2080$.

4. A bullet of mass 20 grammes moving with a velocity of 100 metres per second is brought to rest, after penetrating 10 cms. in the target. Find the average resistance offered by the target.

The *K.E.* of the bullet $= \frac{1}{2}mv^2 = \frac{1}{2}.\ 20.\ (10000)^2$
$= 10 \times 10^8$ ergs.

But the energy = the work done by the retarding force.

$F.\ 10 = 10 \times 10^8$ ergs
or $F = 10^8$ dynes.

5. A train of 75 tons is running at 30 miles per hour. What force is required to stop it in 220 yards?

The $K.\ E. = \frac{1}{2}\ mv^2 = \frac{1}{2} \times 75 \times 2240 \times 44^2$ foot-poundals; and $K.\ E.$ = Work done by the retarding force.

∴ $F \times 660 = \frac{1}{2} \times 75 \times 2240 \times 44 \times 44$
or $F = 246400$ poundals.

6. If a train moving at 40 feet per sec. up an incline of 1 in 64, slips a carriage; how far will the carriage move up, before beginning to run back?

The initial velocity of the carriage = that of the train = 40 feet per second.

The acceleration downwards is $32 \times \frac{1}{64} = \frac{1}{2}$ ft. per sec. per sec.

∴ the distance *S*, which the carriage will move up before beginning to run back is given by:—

$0^2 - 40^2 = 2 \times -\frac{1}{2} \times S$
or $S = 1600$ feet.

7. A reservoir contains water at a height of 100 feet above the ground. What is the potential energy of the water in foot-poundals per gallon?

The potential energy = force × height.

∴ the energy of one pound of water = 3200 ft.-poundals

∴ the potential energy per gallon = 10×3200
$= 32000$ foot-poundals

for 1 gallon = 10 lbs.

8. A bullet of 100 grammes is discharged with a velocity of 100 metres per second from a rifle, the barrel of which is one metre in length. Calculate the energy of the bullet, when it leaves the muzzle and the mean force exerted by the powder

The KE of the bullet is $\frac{1}{2} \times 100 \times (10{,}000)^2$
$= 5 \cdot 0 + 10^9$

This energy must be equal to the work done by the force of powder; ∴ $F.\ 100 = 5 \cdot 0 \times 10^9$
∴ $F = 5 \cdot 0 \times 10^7$ dynes.

9. A man of mass 120 lbs. walks up a ridge rising 1 in 8 at the rate of 4 miles an hour. Find his rate of doing work.

The velocity of the man is $4 \times \frac{1760 \times 3}{60 \times 60} = \frac{88}{15}$ feet per sec

The force due to gravity $= 120 \times \frac{1}{8}$ lbs. weight
$= 15$ lbs. weight

∴ the work done in one second $= 15 \times \frac{88}{15}$
$= 88$ foot-pounds,

or $\frac{88}{550} = \frac{4}{25}$ Horse-Power.

10. Assuming that a person walking on level ground does work equivalent to the raising of his own weight vertically upwards through one-twentieth of the distance walked. Find (in foot-tons) the average daily work done by Hamid in his walk of 5000 miles in 100 days, his weight being 9 stones and 2 lbs.

11. A mass of 12 kilogrammes is raised to a height of 5 metres. Find its energy.

12. What is the $K.E.$ of a mass 1 cwt., after it has fallen from rest for a second.

13. A shot travelling at the rate of 200 metres per

second is just able to pierce a plank 4 cms. thick. What velocity is required to pierce a plank 12 cms. thick?

14. A railway carriage 6 tons heavy contains 100 passengers, each weighing 10 stones. Find its *K.E.* when it is moving at the rate of 60 miles an hour.

15. Find the horse-power exerted by an engine, which draws up an incline of 1 in 200, a train weighing 150 tons at the rate of 15 miles an hour, the frictional resistance being 16 lbs. per ton.

16. A man cycles up a hill, whose slope is 1 in 20, at the rate of 8 miles an hour. The weight of the man and the machine is $187\frac{1}{2}$ lbs. What work per minute is he doing?

17. A railway carriage of mass 2 tons, moving at the rate of 3 miles an hour strikes against the buffers of a train, the springs of which yield 4 inches. Find the average force exerted by them.

18. A rifle bullet, whose mass is 25 grammes leaves the muzzle of a rifle with a velocity of 640 metres per second. Find its kinetic energy State carefully the units in which your answer is expressed.

19. A stone of mass 6 kilogrammes falls from rest at a place, where $g=90$ cms. sec.^{2} What will be its kinetic energy at the end of 5 seconds?

20. A mass of 3 lbs. is shot vertically upwards so as to rise to a height of 25 feet. Find its original kinetic energy?

CHAPTER IX

EQUILIBRIUM

40. Equilibrium. When any number of forces acting on a body are so balanced, that they produce no acceleration of any kind, the forces as well as the body are said to be in equilibrium. The conditions which the forces then satisfy, are called the conditions of equilibrium.

From the above definition, *equilibrium only means zero acceleration but not zero velocity necessarily.* For a body to be in equilibrium, all what is required is that either the body may be *at rest* or it may be moving with *uniform velocity in a straight line.* Thus a train resting on a railway line is in equilibrium and so is a train moving with uniform velocity in a straight line. Equilibrium is zero acceleration; and as we see, the acceleration is zero in both the cases cited above.

Conditions of Equilibrium for two forces. The two forces acting on the body should (*i*) be of *equal magnitude and* (*ii*) *act in opposite directions along the same straight line.*

A body resting on a table is in equilibrium; for the two forces acting on it, namely its weight acting vertically downwards and the reaction of the table acting vertically upwards are equal in magnitude but opposite in directions along the same straight line. Similar is the case of equilibrium of a picture hanging from a nail. In order that the picture be in equilibrium, it is essential that the tension of the string must act vertically upwards in the same straight line, in which the weight acts vertically downwards.

41. Conditions of Equilibrium for three forces: (*a*) when the forces are not parallel. When three forces acting on a body are in equilibrium, the following conditions must be fulfilled :—

(*i*) The three forces must be in the same plane, (*ii*) their lines of action must all pass through the same point and (*iii*) it must be possible to represent them in magnitude as well as in direction by the three sides of a triangle taken in order.

These conditions can be deduced easily from the conditions of equilibrium of two forces cited above. Thus if P, Q and R be the three forces in equilibrium, the lines of action of P and Q must meet at some point O; then by the parallelogram law of forces, the resultant of P and Q must pass through O. Now the force R which balances P and Q must balance their resultant: and this it can do not only by its being in the same straight line as the resultant but also by its being equal and opposite to it. Hence the line of action of R must pass through O, *i.e.* the lines of action of P, Q and R must meet in a point, (condition *ii*). And R must be in the same plane as the plane of the parallelogram with P and Q as adjacent sides; for it is to be in the same straight line as their resultant, (condition *i*). Also R must be equal to the third side of the triangle, two of whose sides represent the other forces, *i.e.* P and Q, (condition *iii*).

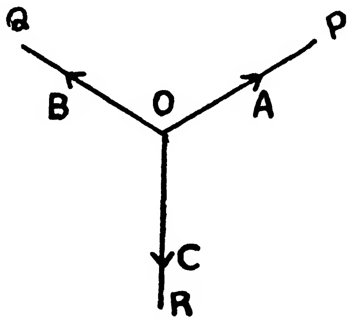


FIG. 25

(*b*) **When the forces are parallel.** When three forces acting on a body are parallel in direction, the following conditions must be fulfilled:— (*i*) The forces must be in the same plane; (*ii*) the *algebraic* sum of the forces P, Q and R must be equal to zero, *i.e.* forces acting in one direction must be numerically equal to

those acting in the opposite direction; and (*iii*) The *algebraic* sum of the moments of all the forces about any point in the plane must vanish.

These conditions can be easily verified; for as one force R is to counterbalance the resultant of P and Q, R must be in the same plane as P and Q and must be equal to their sum. In short, conditions (*i*) and (*ii*) are essential for no motion of translation; while condition (*iii*) must be satisfied for no motion of rotation.

Condition of Equilibrium of any number of forces:

(*a*) **When they are acting on a particle.** Any number of forces, greater than three, may not necessarily be in the same plane; nor meet in a point in order to be in equilibrium. All that is required is, that it should be possible to represent the various forces acting on a point by the sides of a closed polygon, drawn parallel to the respective forces, and taken in order.

When a polygon is complete, it means there is no resultant. For the line required to complete a polygon represents the resultant; but as no line is required to complete a closed polygon, hence there is no resultant.

(*b*) **When they are acting on a rigid body and are not parallel.**

(*i*) The sum of the components of all the forces, in any one plane, in two directions at right angles to each other, must be zero.

(*ii*) The sum of the moments of all the forces about any point in the plane should vanish.

The proof of these is left as an exercise to the student. It must however be noted, that condition (*i*) is essential for no rectilinear acceleration and condition (*ii*) for no spinning acceleration. When both the conditions are fulfilled, equilibrium would be complete.

(*c*) **When they are acting on a rigid body but are parallel.** All the conditions are the same as those enumerated for the equilibrium of three parallel forces.

42. Conditions of equilibrium of a body resting on a plane surface, horizontal or inclined, on which no slipping can take place. The condition required is that the vertical line drawn from the centre of gravity must necessarily pass within the base of the body. In these cases, the body is to be in equilibrium under the action of two forces, namely:—

Its weight acting vertically downwards and the re-action of the plane acting perpendicularly to the plane. In order that these forces be in equilibrium, they should either be *equal and act in opposite directions along the same straight line*; *or they must produce equal moments in opposite directions about the point P.* These conditions *can* only be fulfilled if the vertical line from the centre of gravity passes within the base; and if it does not do so, as shown in figure 27, then the body will topple down.

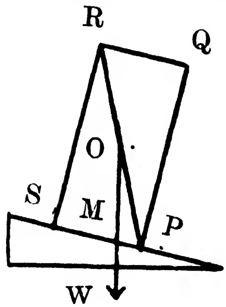


FIG. 26

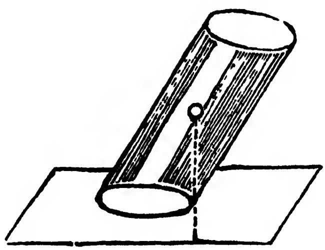


FIG. 27

SUMMARY

1. A body is said to be in equilibrium, when either it is at rest or is moving with a uniform velocity in a straight line.

2. **Conditions of equilibrium** :—

(*a*) for two forces—

(*i*) they must be of equal magnitude and must act in opposite directions along a straight line.

(*b*) for three forces, not parallel in direction—

(*i*) They must be in the same plane.

(*ii*) Their lines of action must all pass through the same point.

(*iii*) It should be possible to represent them by the three sides of a triangle taken in order.

(*c*) for three forces, parallel in direction—

(*i*) They must be in the same plane.

(*ii*) Their *algebraic* sum must be zero.

(*iii*) The sum of their moments about any point must vanish.

(*d*) for any number of forces—

(*i*) The sum of the components of all the forces in any two directions, in a plane, at right angles to each other, must be zero

(*ii*) The sum of the moments about any point must vanish.

3. Equilibrium is of three kinds: (*i*) Stable, (*ii*) Unstable and (*iii*) Neutral.

EXAMPLES

1. A uniform rod AB is at rest, with one end A against a smooth vertical wall. To its middle point C is attached a string, the other end of which is fastened to a point C in the wall Prove that the reaction of the wall is along AB and show that the rod is horizontal.

Figure 28 shows the forces acting on the rod AB, one end of which is against the smooth wall and to the middle point of which is fastened the string. The reaction due to a smooth wall is always perpendicular to it.

For equilibrium, the three forces must meet at a point; therefore the reaction must pass through the centre of gravity G of the rod, for the tension and the weight act at that point.

Thus the reaction must act along the rod which must be perpendicular to the smooth wall.

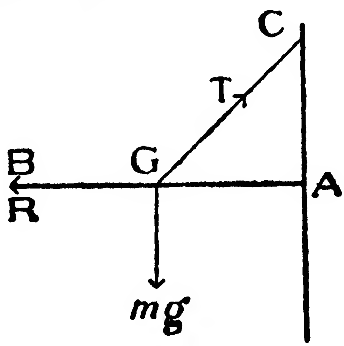


FIG. 28

2. In the above example find the reaction, if the length of the string$=AB$, the length of the rod itself, and it weighs 10 lbs.

We have $CG=AB=L$ or $AG \quad = {}^1/_2 CG$

$\therefore$ The angle $GCA=30^\circ$ and $CA=\frac{\sqrt{3}}{2}$

Now as the forces are in equilibrium, it should be

possible for us to represent them by the sides of a triangle. CAG is the triangle, such that its three sides are respectively parallel to the three forces.

$$\therefore \quad \frac{AC}{AG}=\frac{10}{R}$$

$$\text{or} \quad \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\frac{10}{R} \text{ or } R=\frac{10}{\sqrt{3}} \text{ or } \frac{10\sqrt{3}}{3} \text{ lbs.}$$

3. A bridge 10 feet long and weighing 2 tons, rests on two supports at its ends. What forces are exerted on each support, when a trolly weighing $^1/_4$ of a ton is half way across?

In this case, the total force acting downwards$=2+{}^1/_4$ $=2^1/_4$ tons.

$\therefore$ the upward pressure exerted by the two supports must also be equal to $2^1/_4$ tons. As the weight acts at the centre of gravity and the supports are equidistant, therefore equal force is exerted by each.

Hence the force exerted$=1\frac{1}{8}$ tons

4. A lamp weighing 10 lbs. hangs from the end of a horizontal rod 10 ft long, sticking out perpendicularly from a wall. The other end of the rod is hinged to allow motion in a vertical plane A wire is attached to the middle of the rod and to a hook in the wall, 5 ft above the hinge. Find the tension of the wire, the rod weighing 5 lbs. (*P. U.* 1931)

EXAMINATION QUESTIONS II

1. Define momentum and kinetic energy, and state clearly the relation between force, work, energy and power.

2. Explain how you can measure a force. State clearly the difference between pounds weight and poundals. What are the corresponding French units, and what are pounds and grammes?

3. Distinguish between Kinetic and Potential energy and show that the sum of Kinetic and Potential energies of a falling body is constant and equal to its potential energy at start.

4. State the principle of the Conservation of Energy as employed in Mechanics, and illustrate it by some examples. By the aid of this principle, show that the velocity acquired by a body in falling down a smooth inclined plane, depends only on the vertical height and is independent of its length.

5. The erg and foot-pound are both units of work.

A horse-power is 33,000 foot-pounds per minute. How many ergs per hour would this be (1 lb.=453·6 gms. and 1 inch =2·54 cms.)?

6. Explain by means of the motion of a pendulum, the meaning of the terms "Potential energy" and "Kinetic energy." When has the pendulum kinetic energy, when potential and when both?

7. Define a couple. What sort of action has a couple when applied to a body? Give an example of the application of a couple to a body. When are two couples said to be equal?

8. State the conditions of equilibrium of three forces acting on a body.

9. Give the laws of friction. A cubical block rests on an inclined plane with four edges horizontal. The angle of inclination is slowly increased. If the coefficient of friction is $\frac{1}{\sqrt{3}}$, determine the angle when the block just slides. Also determine the angle when the block turns over, if prevented from sliding by a small obstruction in front.

(*P. U.* 1931)

CHAPTER X

MACHINES

43. Machine. Any contrivance by which force exerted at one point and in a particular direction, is available for doing external work at any other point and in some other direction, is known as a machine. When a labourer has to draw a bucket of water from a well, he finds it convenient to pull the rope, which passes over a pulley, than to haul the same up without the use of the pulley.

Power or Effort. The effective force exerted by a body which loses energy, on a machine is often called the power. The choice of the word is very unfortunate, for power has already been used in quite a different sense. Rankine has suggested the better term "Effort"; and it is always better to use the term effort in preference to power.

Weight or Resistance. The force, which the machine exerts over some other body, the resistance of which is to be overcome, is often called the 'weight.' As 'Weight' has already been defined in a different sense, therefore the term 'Resistance' is now extensively used in preference to weight.

Mechanical advantage. When the "Effort" applied and the "Resistance" to be overcome by a machine keep the same in equilibrium, then the ratio of the **"Resistance"** to the **"Effort"** is known as the *mechanical advantage* of the machine.

It must however, be noted that energy can neither be created nor destroyed and hence a machine is only an intermediate agent, by which energy is transferred from one body to another. The quantity of energy

gained by one body should be theoretically equal to that lost by the other; but in practice some energy is always dissipated away as heat, hence the energy or work actually gained from a machine is always less than that *put into* it.

Efficiency. The ratio of the useful work done by a machine to the total work done on the machine is called its *efficiency*. It is always a proper fraction and never equal to unity, except in the case of a perfect machine.

Perfect Machine. A perfect machine is such that its various parts are frictionless, weightless and rigid; and the chains or ropes are flexible. Thus a perfect machine is a physical impossibility.

Velocity Ratio. The ratio of the distance moved through by the "effort" to the distance moved through by the "resistance" is known as velocity ratio.

Principle of Virtual Work or Virtual Velocities. According to this principle, if a system be in equilibrium under the action of several forces, then no work is done, when the whole system suffers a little displacement.

Thus in this case, from the conditions of equilibrium of several forces we deduce, that when a body is in equilibrium, no expenditure of energy is needed to produce a small displacement. Work is done by some forces and against others; if there be equilibrium these two amounts of work are equal. This principle is of great help in finding the conditions of equilibrium of a system and is called the principle of *virtual work*: on account of the fact that the displacement need not really occur and may be considered purely imaginary.

44. Function of a Machine. As it has been established already that no extra work can ever be obtained by the use of a machine, the question naturally arises, "Where then lies its utility"? Work is always the product of two factors, force and distance; and all that we can do with the help of a machine is that the ratio of the two factors can be changed without in any way

altering the product. Just as the number 24 can be split up into pairs of factors 24 and 1, 12 and 2, 8 and 3, or 6 and 4; similarly the factors F and S of a constant quantity of work may be varied at will. This is the sole use of a machine. Thus a smaller force acting through a longer distance, can by the use of a machine, move a bigger force through a shorter distance. This is sometimes expressed by the pithy saying that "what is gained in power by any machine, is lost in speed (or time or distance)." Thus the mechanical advantage of a machine, is always the inverse of the ratio of the distance travelled by the "resistance" to the distance travelled by the "effort", the machine being supposed to be perfect.

Forms of Machines Any machine, such as a pump, a locomotive or a sewing machine, consists of a number of simple parts. It is desirable to classify them.

The following are the types of simple machines:—

1. **The lever.**
2. **The wheel and axle.**
3. **The pulley.**
4. **The inclined plane.**
5. **The wedge.**
6. **The screw.**

45. The lever. It is a rigid rod or bar, straight or curved, which can freely turn only about a fixed point, called the *Fulcrum*. The distance between the fulcrum and the point of application of the effort is called the *Power arm* or the *Effort arm* and that between the fulcrum and the point of application of the resistance is called the *Weight arm* or the *Resistance arm*. The ordinary crowbar is an example of a straight lever. There are three kinds of levers:—

Class 1 In this case, the fulcrum lies between the point of application of the force and the resistance. For equilibrium we have, if R be the re-action of the fulcrum, a and b be the lengths of

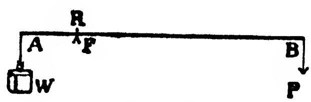


FIG. 29

the arms FA and FB respectively:—

$$R = W + P \qquad \ldots\ldots (i)$$

where P is the effort and W the resistance

$$P.\,BF = W.\,AF \text{ or } Pb = Wa$$

$$\therefore \frac{\mathbf{P}}{\mathbf{W}} = \frac{a}{b} \qquad \ldots\ldots (ii)$$

Thus, the mechanical advantage is the ratio of the power arm to weight arm. It will be greater or less than unity, according as b is $>$ or $<$ a.

Examples of this class of lever are a crowbar, a see-saw and a pair of scissors.

Note. Work done by the effort $=$ Work done against the resistance.

Class II. In this case, the point of application of the resistance lies between the fulcrum and the point of application of the force. Force and resistance act in opposite directions. For equilibrium we have:—

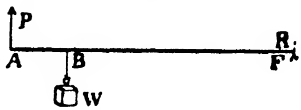


FIG. 30

$$R = W - P \ldots\ldots \quad (i)$$

$$P.\,FA = W.\,BF$$

$$\text{Hence } \frac{W}{P} = \frac{FA}{BF} = \frac{a}{b} \ldots\ldots (ii)$$

where $FA = a$ and $BF = b$.

Mechanical advantage in this case is always greater than unity, for a is always greater than b. Examples of this class of lever are a pair of nut-crackers and an oar.

Class III. In this case, the point of application of the effort lies between the fulcrum and the point of application of the resistance. Here force and resistance act in opposite directions.

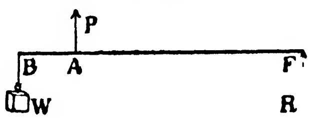


FIG. 31

For equilibrium we have as before,

$$R = P - W \ldots\ \ldots\ldots (i)$$

$$P.\,FA = W.\,BF$$

$$\therefore \frac{W}{P}=\frac{FA}{FB}=\frac{a}{b} \quad (ii)$$

The mechanical advantage is less than unity for a is always less than b. Examples of this class of lever are a pair of tongs and a human fore-arm. In the latter case, the fulcrum is the elbow joint, the effort is applied by a muscle attached to the arm between the elbow and the hand when the weight is held in the hand.

Bent levers. When a lever is not straight but is bent, the condition of equilibrium can still be obtained by equating the moments of the effort and of the resistance about the fulcrum.

46. The wheel and axle. It consists, as shown in figure 32, of a wheel of large diameter and a cylinder of small diameter, capable of rotation about a common fixed axis.

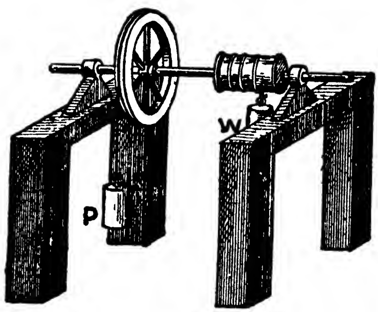


FIG. 32

The force is applied by means of a string coiled on the wheel and the resistance or the weight to be raised is attached to another rope, which is coiled on the cylinder in a direction *opposite* to that of the string on the wheel. In this way a weight can be raised by pulling downwards, the rope passing over the wheel.

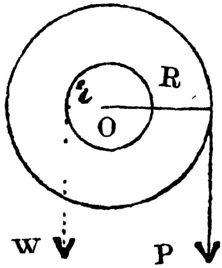
The mechanical advantage of a wheel and axle can be found out, both by the law of moments and by the principle of work.

(*a*) Let R denote the radius of the wheel and r that of the axle, then for equilibrium, we have by taking moments about the axis,

$$P.R = W.r$$

$$\text{or} \quad \frac{W}{P}=\frac{R}{r} \quad \ldots\ldots\ldots\ldots\ldots (i)$$

(*b*) Let us suppose that the wheel makes one complete revolution, then a length equal to $2\pi R$ uncoils itself from the wheel and a length$=2\pi r$ is coiled round the axle. Thus the distance through which the force P moves is $2\pi R$ and that through which the weight W is raised is $2\pi r$;



$\therefore \quad P.2\pi R = W.2\pi r,$

i.e. in-put of work=the out-put

or $\dfrac{W}{P} = \dfrac{R}{r}$......(*ii*)

FIG. 33

The device is sometimes known as a windlass or capstan. It is often used in a slightly modified form on wells for drawing water.

47. Pulleys. A pulley is a small circular disc or wheel, with a groove cut in its outer edge round which a string can pass. The disc can rotate about the axis which passes through its centre and the ends of the axis rest on the block, within which the pulley turns.

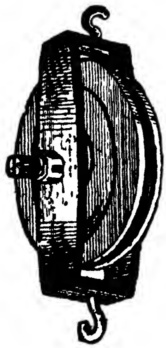


FIG. 34

Fixed Pulley. A pulley is said to be fixed, when the block in which the pulley can move is fixed as shown in figure 34; and it is said to be movable when otherwise. *The fixed pulley is useful only in changing the direction of the force.* For taking moments round the axis O (fig. 35), whatever the direction in which the rope is pulled, we have for equilibrium,

$P \times OA = W \times OB$

But $OA = OB$ (being radii of the same circle).

$\therefore \quad P$ must be equal to W. (*i*)

Movable Pulley. Before investigating the conditions of equilibrium, it will be advisable to understand the assumptions usually made:

1. The supports of the pulley are supposed to be frictionless.

2. All parts are supposed o be perfectly smooth.

3. The tension in all parts of same string is supposed to be the same.

4. The strings are all supposed to be parallel, unless otherwise expressed.

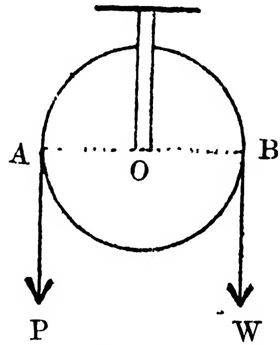


FIG. 35

Single movable Pulley. When the two strings are parallel as shown in fig. 36, the forces acting are (i) the tension of the two strings each equal to P, the force applied, acting upwards and (ii) the weight acting vertically downwards; therefore for equilibrium we have :—

$$2P = W$$

or $\frac{W}{P} = 2$(the mechanical advantage).

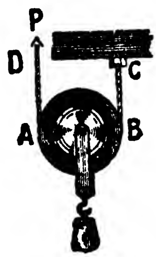


FIG. 36

When however, the two strings are not parallel, but each is inclined to the vertical at an angle θ; then for equilibrium, resolving both vertically, we must have :—

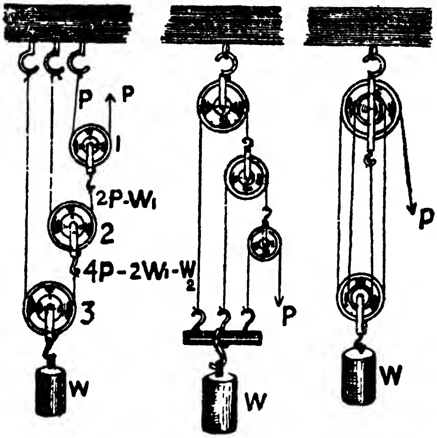
$$2\,P \cos\theta = W$$

or $\frac{W}{P} = 2\cos\theta$.

If however, the weight of the pulley be not neglected and be assumed as equal to w, then the last equation becomes $\frac{W+w}{P} = 2\cos\theta$.

Systems of Pulleys. There are various systems under which movable pulleys may be arranged. Of these we shall describe only three, known as the *first*, *second* and *third* systems of pulleys.

The First System. The first system as shown in



(*i*) (*iii*) (*ii*)

FIG. 37

fig. 37(*i*), consists of a number of pulleys (3 only shown here), each of which is suspended by a separate string passing round it, with one end attached to a fixed support and the other, fastened to the block of the next pulley. The weight to be raised is attached to the lowest pulley and the force is applied to the string passing round the highest pulley. Sometimes the force is applied after passing the string round a fixed pulley.

Mechanical advantage. Let w_1, w_2 and w_3 be the weights of the pulleys numbered 1, 2 and 3 respectively fig. 37 (*i*), W the weight to be raised, P the force applied, then the tension of the second string is clearly equal to $2P - w_1$; for the forces acting on the first pulley are P and P acting upwards; and w_1 plus T acting downwards. The tension of the third string would, by similar reasoning, be equal to $2(2P - w_1) - w_2$. Now this tension in the two arms supports the weight W plus w_3 of the third pulley.

$\therefore$ We have $W + w_3 = 8P - 4w_1 - 2w_2$

or $W=8P-4w_1-2w_2-w_3$

or $W=2^3P-2^{3-1}w_1-2^{3-2}w_2-w_3$

or in general $2^n P=W+2^{n-1}w_1+2^{n-2}w_2+\ldots+w_n$

If however, the weights of the pulleys be neglected, then we have $2^n P=W$,

or $\frac{W}{P}=2^n$, the mechanical advantage,

where n is the number of movable pulleys.

The Second system of Pulleys. In this system, as in fig. 37 (*ii*), there are two blocks of pulleys, one attached to the beam and the other to the weight. In this case the same string passes round all the pulleys.

Mechanical advantage. As the same string passes round, the tension is equal to the effort or force applied. Thus $n P=W+w_1$, where w_1 is the weight of the lower block and n=number of strings passing through it.

If w_1 be negligibly small, then we have $\frac{W}{P}=n$.

The Third system of Pulleys. In this system Fig. 37 (*iii*), each end of the string is attached tothe bar, which carries the weight. The uppermost pulley is fixed and a string passing round it supports the second pulley and so on. The force is applied to the last string.

Then the tension of the first string$=P$

that of the second string$=2P+w_1$.

and that of the third string$=4P+2w_1+w_2$

$\therefore$ $W=7P+3w_1+w_2$

Neglecting the weights of the pulleys, we have

$W=7P$ or $W=(2^3-1)P$

$\therefore$ $\frac{W}{P}=2^n-1$, **the mechanical advantage.**

48. The inclined plane. It is a plane inclined to the horizontal at an angle θ. When the force acts parallel to the plane as shown in fig. 14, page 53; then if P be the force and W the weight to be raised, we have by the principle of work, $Pl=Wh$. For when P moves from A to C, W is raised

from B to C,

$$\therefore \quad \frac{\mathbf{W}}{\mathbf{P}}=\frac{l}{h}=\frac{1}{\sin\theta}$$

When however, the force P acts parallel to the horizontal plane as shown in figure 38, we have as before

$$P.b = W.h$$

or $\dfrac{\mathbf{W}}{\mathbf{P}}=\dfrac{\mathbf{b}}{\mathbf{h}}=\dfrac{1}{\tan\theta}$

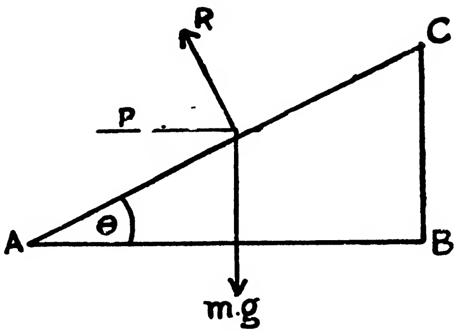


Fig. 38

48. (*a*) **The wedge.** It is a double inclined plane. Its practical forms are very numerous, such as knives, chisels, wood-cleavers etc.

To find the mechanical advantage, we see (Fig. 39) that the force P is applied parallel to BC, while the resistance W acts perpendicular to AC and $A'C$.

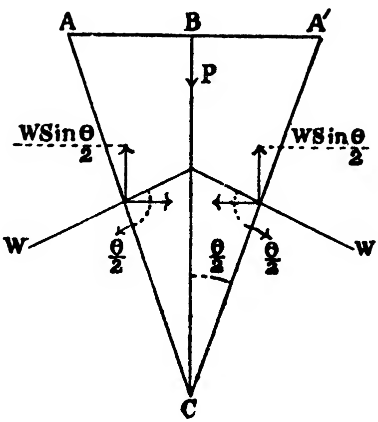


Fig. 39

Resolving in the horizontal and vertical directions

we must have

$$P=2W\sin\frac{\theta}{2}$$

$$\text{or } \frac{W}{P}=\frac{1}{2\sin\frac{\theta}{2}}$$

In practice it is difficult to find the mechanical advantage as the force P is applied by an impact.

49. The screw. A screw consists actually of a combination of a lever and an inclined plane. It may be regarded as made up of a right angled triangle such as ACB Fig. 40,

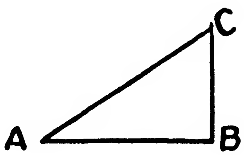


FIG. 40

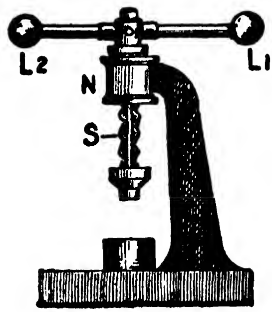


FIG. 41

wrapped round the axis of a cylinder; and the projecting spiral thread may be considered to coincide with the position of the hypotenuse AC. This cylinder passes through another of same radius and having a hollow groove in it, so that the projecting thread of the screw just fits into the hollow of the groove. This hollow cylinder is called the '*nut*'.

The distance between two consecutive threads is known as the **pitch** of the screw. The nut being fixed, the screw can be moved by means of the lever L_1 L_2.

The mechanical advantage can be easily found by the 'principle of work'. Suppose P is the force applied at L_1 and W the weight to be raised or resistance to be overcome, as the case may be, and let d be the pitch of the screw. When one complete revolution is given, the work done by the force $=P\times 2\pi\times l$, for $2\pi\times l$ is

equal to the circumference, *i. e.* the distance through which the force is shifted, where l is the arm of the lever.

The work done against the weight is equal to Wd; for the weight is raised by the distance, through which the screw is raised and this is equal to the pitch of the screw. Thus $W.d = P.2\pi l$,

$$\text{or } \frac{W}{P} = \frac{2\pi l}{d} \quad \ldots \quad (i)$$

Thus the mechanical advantage can be considerably increased by either making l very large or the pitch very small.

49. (*a*) **The Screw-Jack.** It is a machine for lifting heavy loads such as a derailed engine or a punctured motor lorry. It is like an ordinary screw; but with this difference that the screw is fixed, while the nut is movable and the power to rotate is applied by a system of cog-wheels. Fig. 42 represents a small screw-jack used by motorists. The vertical screw is *rigidly fixed* to the horizontal cog-wheel and is threaded through a steel block, which acts like a movable nut, and on this the weight W to be lifted is placed. The teeth of the horizontal cog-wheel are 'engaged' by the teeth of a vertical bevel-wheel, which is rotated by a lever. The horizontal wheel is always of much greater diameter than the bevel-wheel, so that the former contains many more teeth on its circumference than the latter. Thus one revolution of the bevel-wheel rotates the horizontal wheel (*i.e.* the screw) only through a small fraction of a rotation. Hence if the horizontal wheel has n times as many teeth as the bevel-wheel; n revolutions of the bevel-wheel will be required to give

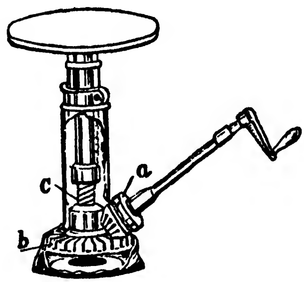


FIG. 42

one rotation to the screw. If in addition, the bevel-wheel is rotated by a lever of arm l and the screw has a pitch p; the mechanical advantage will be

$$\frac{W}{P}=\frac{\text{Distance moved through by the applied force}}{\text{Distance through which the weight is raised}},$$

where P=the applied force

$$i.\ e.\ \frac{2\pi \times l}{\frac{1}{n}\times p}=\frac{2\pi l n}{p}.$$

Thus if the pitch be $\frac{1}{10}$ of an inch,
n (the ratio of the no. of teeth in the horizontal wheel to those in the bevel-wheel) be 5,
and l be 12 inches.
The mechanical advantage will be $12\times5\times10\times2\pi$
$=1200\pi$.

SUMMARY

Machine. Any contrivance by which force exerted at one point becomes available for doing external work at some other point is called a machine.

Effort. Force applied is called the *effort* or *power*

Resistance. The resistance overcome is called the resistance or weight.

Mechanical advantage. It is the ratio of the resistance or weight overcome to the effort applied.

Efficiency. It is the ratio of useful work done by a machine to the total work done on it.

Mechanical advantage in the case of

(*i*) Lever $=\dfrac{\text{length of effort-arm}}{\text{length of resistance-arm}}$

(*ii*) Wheel and axle$=\dfrac{\text{Radius of wheel}}{\text{Radius of axle}}$

(*iii*) (*a*) First system of Pulleys$=2^n$
(*b*) Second ,, ,, ,, $=n$
(*c*) Third ,, ,, ,, $=2^n-1$

(*iv*) Inclined plane $=\dfrac{\text{length}}{\text{height}}$, when the force acts parallel to the inclined plane; and $\dfrac{\text{base}}{\text{height}}$, when the force acts parallel to the base.

(iv) a. Wedge$=\dfrac{1}{2\sin\frac{\theta}{2}}$

(v) Screw$=\dfrac{2\pi l}{d}$

(vi) Screw-Jack $=\dfrac{2\pi l n^{1}}{p}$

EXAMPLES

1. A mass of 10 lbs. weight, rests on a plane inclined at 30° to the horizontal. What force acting parallel to the plane will be required to keep it in equilibrium.

Suppose P is the force, then

$$\frac{W}{P}=\frac{l}{h}$$

When the inclination is 30°, then h is half of the length,

$$\therefore \quad \frac{W}{P}=\frac{l}{\frac{l}{2}} \text{ or } P=\tfrac{1}{2}W=5 \text{ lbs. weight.}$$

2. The pitch of a screw is 1 mm. and the effort is applied at the end of a lever one metre long. It is found that a force equal to the weight of 100 grammes must be applied to raise a mass of $3{\cdot}1416\times10^5$ grammes. Find its efficiency.

$$\frac{W}{P}=\frac{2\pi l}{d}=\frac{2\pi\times100}{\frac{1}{10}}=2000\pi$$

or $\quad P=\dfrac{W}{2000\pi}$, if the machine were perfect,

or $W=100\times2000\pi=6{\cdot}2832\times10^5$

Thus the given effort, if the machine were perfect, ought to raise $6{\cdot}2832\times10^5$ grammes; but it raises only $3{\cdot}1416\times10^5$ grammes.

$$\therefore \text{ efficiency }=\frac{3{\cdot}1416\times10^5}{6{\cdot}2832\times10^5}=\tfrac{1}{2} \text{ or } 50\ \%.$$

3. A man weighing 100 lbs. is supported in a well by means of a wheel and axle, the radii of which are 20 and 8 inches respectively. What force must be applied to (i) keep the man at rest in the middle of the well and (ii) let him down with uniform velocity?

$$\frac{W}{P}=\frac{R}{r}$$

$$\frac{100}{P} = \frac{20}{8} \text{ or } P = 40 \text{ lbs. wt. in both the cases.}$$

4. The average force that has to be applied at the end of a pair of nut-crackers in order to crack a nut is equal to 15 lbs. weight. If the nut-cracker be 8 inches long and the nut placed at 2 inches from the hinge; find the force in poundals, which will crack it.

Nut-cracker is a lever of second kind.

$$\frac{W}{P} = \frac{a}{b} \text{ or } \frac{W}{15} = \frac{8}{2} \text{ or } W = 60 \text{ lbs. weight}$$

$$\text{or } 60 \times 32 = 1920 \text{ poundals.}$$

4. (*a*) A motor-lorry, tare half-ton, and loaded with 19 5 tons of luggage is to be raised up by a screw-jack of which the pitch is $\frac{1}{8}$ inch, the crank of the handle 7·5 inches and the horizontal cog-wheel has 4 times as many teeth as the bevel-wheel. Find the force necessary for the purpose.

$$\frac{W}{P} = \frac{20 \times 2240}{P} = \frac{2 \times \pi \times 7{\cdot}5 \times 4}{\frac{1}{8}} = 300\pi$$

$$\therefore \quad P = \frac{448}{3\pi} \times \frac{7}{22} = 53{\cdot}6 \text{ lbs. wt.}$$

5. An inclined plane rises 3 in 5, what is its mechanical advantage?——when the force is (*i*) parallel to the plane and (*ii*) horizontal.

6. In a certain machine, it is found that the force-arm must be shifted through a distance of 18 inches to raise the weight by a distance of 3 inches. What effort will be re quired to raise a mass of 2 stones?

7. If a weight of 5 lbs. keep at rest, a weight of 8 lbs. by means of a single movable pulley; calculate the weight of the movable pulley.

8. A straight uniform lever, whose weight is 16 lbs., balances about a point one foot from its middle, when weights equal to 6 lbs. and 10 lbs. are suspended from its ends. Find the length of the lever.

CHAPTER XI

BALANCE

50. Balance. A balance is an instrument used for comparison of the masses of two bodies; but before we proceed with the construction and theory of the balance, it will be advantageous to recapitulate: "What is mass and how it can be measured."

Mass. *The mass of a body is defined as the quantity of matter contained in it.* Newton's second law gives us a method of measuring it and implicitly lays down that if various bodies be acted upon by the same force, then their masses are inversely proportional to the accelerations produced in them.

Weight. The above method of comparing masses is very cumbrous and inaccurate. In practice to compare masses, *we make use of the fact that the weights of bodies are proportional to their masses.* The simplest proof lies in the fact that all bodies fall to the ground in vacuum, with the same acceleration. Suppose we have two bodies A and B of masses m_1 and m_2 and weights w_1 and w_2 respectively. Then their accelerations would be proportional to $\frac{w_1}{m_1}$ and $\frac{w_2}{m_2}$, but we know by experiments that they are the same.

Therefore $\frac{w_1}{m_1}=\frac{w_2}{m_2}$ or $m_1 : m_2 :: w_1 : w_2$.

The Balance. The ordinary balance is an instrument of common use and every body is familiar with it.

Figure 43 represents an ordinary laboratory balance. It consists of a lever of the first kind with equal arms, from the ends of which two equal and

similar pans are suspended. Near the fulcrum a vertical pointer moves over a horizontal scale, and remains at the centre when the beam of the balance is in the horizontal position. To make this adjustment small screw weights S and S' (fig. 43) are provided at the ends of the arms.

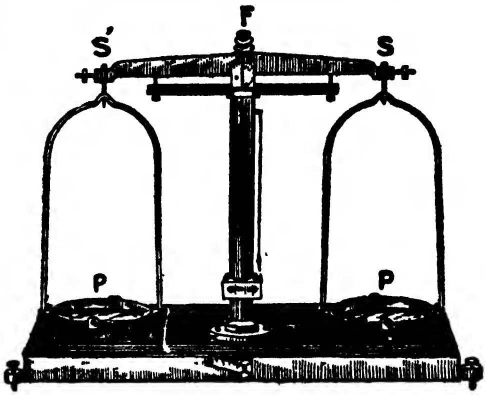


FIG. 43

To ascertain the mass of a body, it is placed in the left-hand pan of the balance and the standard weights in the right-hand pan; then the mass of the body is equal to the mass of standard weights, when the beam is horizontal.

51. Requisites of a good balance. The following are the three chief requisites of a good balance: (*i*) *Truth*, (*ii*) *Sensitiveness* and (*iii*) *Stability*.

(*i*) **Truth.** A balance is said to be true, if the beam is in the horizontal position, whenever equal masses are placed in the scale-pans.

(*ii*) **Sensitiveness.** A balance is said to be sensitive, if the beam turns through an appreciable angle from its horizontal position, for a very small difference of weights in the two pans.

(*iii*) **Stability.** A balance is said to be stable, when the beam, if disturbed from its equilibrium-position, readily comes back to it.

(*i*) **Conditions for Truth:—**

(*a*) *The centre of gravity G should be vertically below the fulcrum*; for it is only then, that the weight w of the beam shall have no moment about the fulcrum, when the beam is in the horizontal position.

(The student should himself draw a figure to illustrate it.)

(*b*) *Scale-pans should be of equal weights, and*

(*c*) *The arms should be of equal length.*

For, let us suppose that S and S' be the weights of the scale-pans, a and b the arms of the balance; then, if the pans are empty and the beam horizontal, we have for equilibrium, by the law of moments,

$$S.a=S'.b \quad \ldots \quad (i)$$

Further, suppose equal weights P and P are placed one in each pan, then if the beam is to remain horizontal, we must have

$$(P+S)a=(P+S')b$$

or $Pa+Sa=Pb+S'b$

therefore $Pa=Pb$ for $Sa=S'b$ by equation (*i*),

or a must be equal to b ——(condition c)

Substituting this in equation (*i*)

we have $S=S'$———(condition *b*)

(*ii*) **Conditions for sensitiveness:—**

(*a*) *The weights of the beam and the pans should be small,*

(*b*) *The distance of the centre of gravity from the fulcrum should be small, and*

(c) *The arms should be long.*

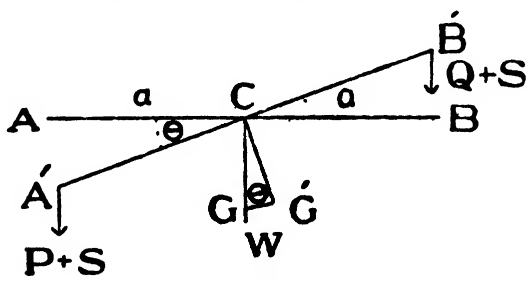


Fig. 44

Let $A'CB'$ be the position of the beam, when weights P and Q are put in the two pans. Then for equilibrium; taking moments about the fulcrum C, and bearing in mind that moment is the product of the force and the *perpendicular distance* between the line of action

of the force and the fulcrum , we have

$(P+S)a \cos\theta=(Q+S)a \cos\theta+wh \sin\theta$ .... (i)

where a=length of each arm

S=weight of each scale-pan

h=distance of $C. G.$ below the fulcrum C

w=weight of the beam

and θ=angle of tilt.

Transferring, we have from equation (i) above

$a \cos\theta\,(P-Q) = w\,h \sin\theta$

$$\text{or } \tan\theta = \frac{a\,(P-Q)}{w\,h} \qquad (ii)$$

In order that the beam be tilted through a large angle for a small difference in P and Q; it is evident, from the law of moments, that a the length of each arm should be long, w the weight of the beam should be small and h the distance of the centre of gravity from the fulcrum should also be small.

(*iii*) **Conditions for Stability**:—(*a*) *The distance of the centre of gravity from the fulcrum should be large.* In order that the beam, when slightly displaced, should return to its position of equilibrium, it is essential that the restoring moment of w, the weight of the beam, about the fulcrum should be great, but this depends upon w and h, therefore w and h should be large. It should be noticed however, that no useful purpose is served by increasing w only; for if this alone be increased, then the mass to be moved, *i. e* w is increased and so also the impressed force The acceleration would not be changed at all, therefore for stability, h the distance of the centre of gravity from the fulcrum should be large.

It will be noticed however, that the condition for stability is opposed to one for sensitiveness Fair sensitiveness and good stability can be secured by making the arms of the beam long and the distance of the centre of gravity from the fulcrum not very low.

Note.—In order that the sensitiveness of a balance may not vary, it is essential as well as desirable that the fulcrum and the points of support of the scale-pans

should all be in one and the same straight line.

52. To find the real weight of a body by means of a false balance. (*i*) When the falseness of a balance is simply due to inequality of the weights of the scale-pans, the true weight of the body is obtained as follows :—

Weigh the body first in one scale-pan, then in the other Add together the two apparent weights and divide by two

For if S and S' be the weights of the scale-pans, W the true weight of the body, x and y its apparent weights then, because the arms are equal, we must have

$$W+S=x+S' \text{ in one case,}$$

$$\text{and } W+S'=y+S \text{ in the second case.}$$

By addition we get, $2W+S+S'=x+y+S+S'$

$$\text{or } 2W=x+y$$

$$\text{or } W=\frac{x+y}{2}.$$

(*ii*) If however, the falseness of a balance is due to the inequality of the arms only, then the true weight of the body is obtained as follows :—

Weigh the body first in one scale-pan, then in the other Multiply the two apparent weights and take the square root

For if a and b be the arms of the balance, W the true weight of the body, x and y its apparent weights ; then we must have for equilibrium.

$$W.a=x.b \text{ in the first case} \qquad (i)$$

$$\text{and } W.b=y.a \text{ in the second case} \qquad (ii)$$

when the body and the weights have been interchanged.

Multiplying the above two equations, we have

$$W^2.ab=xy.ab$$

$$\text{or } W^2=xy$$

$$\text{or } W, \text{ the true weight } =\sqrt{xy}$$

Generally for rough measurements, shop-keepers take the arithmetic mean, *i. e.* $\frac{x+y}{2}$, as the true weight instead of the actual geometric mean $\sqrt{xy}$. The

difference between the two quantities is very small, when x and y differ by small amounts. It must however, be noted that the arithmetic mean is always greater than the geometric mean [See Hall and Knight's Algebra, section 65]. Therefore a shopkeeper would be giving less quantity to his customers than the true measure if he arrives at the same by taking the arithmetic instead of the geometric mean of the two false measures of the same quantity, as bankers generally do, when giving gold to their customers

The ratio of the arms of a Balance is found in the following way:-

We have, with the notation of equations (i) & (ii) P. 114,

$$\frac{Wa}{Wb}=\frac{x\times b}{y\times a} \text{ or } \frac{a}{b}=\frac{x}{y}\times\frac{b}{a}, \text{ or } \frac{a^2}{b^2}=\frac{x}{y}$$

$$\text{or } \frac{a}{b}=\sqrt{\frac{x}{y}}.$$

Thus the ratio of a to b is known

(ii) If however, the falseness of a balance is due to more than one condition required for truth, not having been satisfied, the real weight of the body is found as follows:—

Place the body to be weighed in one scale-pan and balance it with sand or lead shots in the other pan. Remove the body and balance the shots by means of standard weights These standard weights would then give the true weight of the body.

For let w and w' be the weights of the two pans, l_1 and l_2 the arms of the beam and W the true weight of the body.

Then we have

$(W+w)l_1=(X+w')l_2$ .. (i)

where X=weight of the lead shots.

Suppose now standard weights equal to Z are placed instead of the body and equilibrium is again restored, then we have

$$(Z+w)l_1=(X+w')l_2$$

or $(Z+w)l_1=(W+w)l_1$

or $Z=W.$. (ii)

Note.—This is the most accurate method of finding the true weight of a body and is known as the method of double weighing.

53. The common steel-yard. The steel-yard sometimes known as the Roman steel-yard consists, as shown in figure 45, of a beam AB movable about the fulcrum F near the end B, which carries a pan to support the object to be weighed. A movable standard weight P is hung from the arm FA, which is graduated so as to give the weight of the object placed in the pan in terms of P.

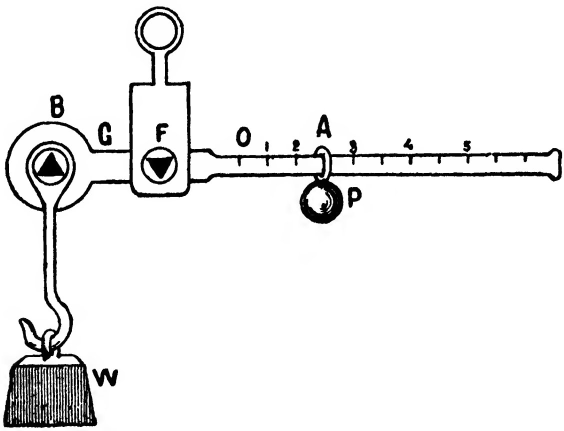


FIG. 45

Graduation of a steel-yard. The pan is unloaded and the weight P placed at a point O so that the steel-yard is horizontal. In this position, the weight P balances the combined weight W of the steel-yard and of the pan, which acts at G their common centre of gravity.

Then $W \times GF = P \times OF$ ....(i)

Then from O, distances equal to BF are marked off as 1, 2, 3, etc. along FA. The positions of these marks denote the weight of the body in the pan B in terms of P,

when the latter is so placed as to keep the steel-yard horizontal.

For, let us suppose that a body of weight w is placed in B and P stands at the nth division, when the beam is in the horizontal position. Then we have to show that $$w = n \times P$$

Taking moments about F, we have

$$w \times BF + W \times GF = P \times FA \ldots\ldots\ldots\ldots (ii)$$

or $w \times BF + W \times GF = P \times FO + P \times OA$ (for $FA = FO + OA$).

Subtracting equation (i) from (ii), we get

$$w \times BF = P \times OA$$

or $w \times BF = P \times n. BF$ (*for* OA *is* n times BF)

$$\therefore \quad w = n \times P$$

The steel-yard is used for weighing heavy loads; since by making BF small, a single small weight may be made to weigh even loaded wagons and carts. Moreover, it is quick in action.

54. Spring Balance. The spring balance, as shown in fig. 46, consists of a spiral spring contained in a brass case. The spring is fixed at the upper end but is free to move downwards and carries a hook. To the lower end of the spring is attached a pointer, which moves over the scale graduated on the face of the case in lbs. weight or in grammes weight. The instrument is supported by means of a stout upper hook and the force to be measured is exerted on the lower hook. The position of the pointer gives the measure of the force. It should be noted that a spring balance actually measures *forces* and *not masses*; but as at a given place the masses of bodies are directly proportional to their weights, therefore it may be used for comparing masses at the same place. It should be remembered that a spring balance cannot be used to compare masses at different places. For suppose a given mass produces an extension of one inch at the equator where $g = 31{\cdot}8$, the same mass shall

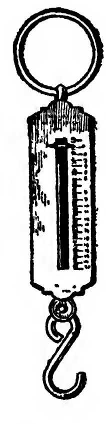


Fig 46

produce an extension of $\frac{32\cdot2}{31\cdot8}$ inches near the pole where $g=32\cdot2$, since the force acting is proportional to g. Thus a smaller mass would in the latter case be required to produce the same extension. Hence a spring balance actually measures forces and *not* masses; but at the same place it may be used to measure masses.

SUMMARY

1. The three chief requisites of a good balance are —(*i*) Truth, (*ii*) Sensitiveness and (*iii*) Stability.

2. Conditions for truth —

(*i*) The centre of gravity should be vertically below the fulcrum,

(*ii*) Arms should be of equal length, and

(*iii*) Scale-pans should be of equal weights.

3. Conditions for sensitiveness —

(*i*) The weight of the beam should be small,

(*ii*) Arms should be long,

(*iii*) The centre of gravity should not be much below the fulcrum, and

(*iv*) The knife-edges should be sharp.

4. Conditions for stability: —(*i*) The centre of gravity should be well below the fulcrum, (*ii*) The arms should be small, and (*iii*) The beam should be heavy.

5. When a balance is false on account of the inequality of scale-pans, the true weight of a body is equal to the arithmetic mean of the two measures, *i.e.* $W=\frac{x+y}{2}$.

6. If however, the falseness of a balance is due to inequality of the arms, then the true weight of a body is equal to the geometric mean of the measures in the two pans, *i.e.* $W=\sqrt{xy}$

7. If the falseness is due to more than one cause, then the true weight is found by double weighing.

EXAMPLES

1. A balance has a piece of wax attached to one of its scale-pans. A body when placed in one pan appears to weigh 10 grammes and only 8 grammes in the other. What is its real weight and what is the weight of the wax?

The weight $W = \frac{x+y}{2}$ or $\frac{10+8}{2} = 9$ grammes.

$\therefore$ the weight of wax $= 10 - 9 = 1$ gramme.

2. A body whose weight is 10 lbs. appears to weigh 11 lbs. when placed in one pan of a balance with unequal arms. What will be its apparent weight when placed in the other pan?

True weight $W = \sqrt{xy}$

or $10 = \sqrt{11y}$

or $100 = 11y$ (by squaring both sides)

$\therefore \quad y = \frac{100}{11} = 9\frac{1}{11}$ lbs.

3. The arms of a false balance are in the ratio of 10 to 11. What will be the loss to a tradesman, who places articles at the end of the shorter arm, if he is asked for two lbs of goods, priced at 1 shilling per lb?

Instead of 2 lbs the tradesman gives x lbs

then $2 \times 11 = 10x$ or $x = 2{\cdot}2$ lbs.

Therefore he suffers a loss equal to ·2 shilling

4. How would you test a trader's scale and weights, if you are provided with a standard seer?

5. An object is placed in one scale-pan of an ordinary balance and it is equipoised by 10 lbs. The object is then put into the other scale-pan and now it takes $10^1/_2$ lbs. to balance it. When both scale-pans are empty, the scales balance. What is the matter with the balance and what is the true weight of the object?

6. Show that if a man sits in the pan of a weighing machine and pushes upwards at the beam at any point between the scale-pan and the fulcrum, he will appear to weigh more than before.

7. If the arms of a false balance are 8″ and 10″ long, what price is paid for tea at 3*s*. per lb., if it is weighed out from (*i*) the longer and (*ii*) the shorter arm of the balance?

CHAPTER XII

PROPERTIES OF MATTER

55. Properties of Matter. We have already attempted to define, 'What matter is'; but it is a strange anomaly for a Physicist that though he can well enumerate the various properties of matter, he is at a loss to give a clear, comprehensive idea of its nature. We know many of the properties of matter, as for instance the mutual attraction of two pieces of matter, the law of gravitation is known, but the mechanism by which this universal attraction is exerted, is still a matter of speculation. It would be advisable to enumerate some of the properties of matter.

(*i*) *Matter is indestructible*, that is, it can neither be created nor destroyed It can simply change form.

(*ii*) *Matter possesses inertia*, *i. e.* force must be applied to matter to move it or bring it to rest.

[*Newton's I law of motion*].

(*iii*) *All particles of matter exert mutual attraction on one another.* [*Newton's law of universal gravitation.*]

(*iv*) *Matter can be divided into very minute particles.* It is common experience that a lump of iron can be divided into a very large number of minute particles, iron filings. But it must not be understood that matter can be divided *infinitely*. There is a limit to its divisibility. Before the time of Johnston Stoney, who demonstrated the existence of electrons (mass$=\frac{1}{2000}$ of an hydrogen atom), Chemist's atom was considered to be the ultimate limit of divisibility of matter. Recently however, Sir J. J. Thomson has carried this limit still further to electrons.

(*v*) *Matter is the sole vehicle of energy.* It is impossible for energy to exist by itself. It must exist in connection with matter; and it is due to this fact that properties of an elastic solid are attributed to the hypothetical medium called the Ether, which is supposed to fill the whole space beyond the atmosphere.

According to the modern conception, matter is energy and one is convertible into another.

(*vi*) *Matter is porous.* All bodies are compressed by the application of force to a more or less extent and if matter were continuous, we could never produce any change in its volume. Bacon in 1640 proved the porosity of lead by compressing water in a sphere of lead; and at a later date Florentine proved the same fact about silver in a similar manner.

(*vii*) *Matter is elastic.* It has the property of offering resistance to forces, tending to change its shape or volume.

So far we have been considering matter as perfectly rigid and insusceptible of strain;* but in practice, we know that there is nothing perfectly rigid in this world and every body is strained, more or less, by the action of forces.

Thus, if we take a ball of rubber and an iron sphere of equal size, we realize that a comparatively small force is needed to change the shape or volume of the rubber ball, while a considerable force is needed to produce any appreciable effect in the iron sphere. Further, if the force applied be not too large, both iron and rubber do regain their original size and shape as soon as the force is removed. Thus iron and India-rubber are both elastic substances. Hence *an elastic substance is one, which fully regains its shape or volume as soon as forces producing the change of shape or volume are removed.*

Elasticity is thus of two kinds:—(*i*) *Volume elasti-*

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* Strain means change of size or shape. Change of size strain is called *Compression* or *Dilatation* and change of shape strain is called *Distortion*.

city or the property of resisting forces tending to change the volume of a body and (ii) *Elasticity of form or Rigidity*, *i.e.* the property of offering resistance to forces tending to change the shape of a body.

Elasticity of form or 'Rigidity.' In the preceding example of an iron sphere and a rubber ball, we have seen that an iron sphere can resist comparatively greater force than a rubber ball. Hence iron is said to be more rigid than rubber ; and this property is called *rigidity*. A perfectly rigid body would be one in which no force, however great, would produce any *change of shape*. In nature no perfectly rigid body exists ; but substances like hard rock, metals, glass and many other solids possess rigidity to such a high extent, that it is customary to regard them as perfectly rigid for all practical purposes. Only solids possess elasticity of shape. Liquids and gases do not possess the same ; for they can be readily transferred from one vessel to another and they always assume the shape of the vessel in which they are poured.

Elasticity of volume. The property of a substance of offering resistance to forces tending to change its volume is called its volume elasticity.

Incompressible body. A body, the volume of which cannot be altered by any force, however great, is called an Incompressible body.

No known body is perfectly incompressible ; for by the application of high stress, water has been made to pass through gold and lead, mercury through most metals and gases through iron. For practical purposes almost all solids and substances like water and alcohol, which although have no rigidity, are nevertheless regarded as incompressible.

56. Young's modulus of elasticity. If a long wire of any metal be suspended from a point and a scale-pan be attached to its lower end ; it is observed that on applying a force to the free end by putting weights into the scale-pan, the wire increases in length. The

expansion produced is proportional to the stretching force; and this fact is known as **Hooke's law.** The statement is true only within narrow limits; for if the stretching force be gradually increased, it is found that at a certain point known as the *elastic limit*, the increase in length is relatively greater than the corresponding increase in force and a continued increase in the force beyond this limit results in breaking the wire. The increase in length l within elastic limits is however,

(1) in proportion to the stretching force F,

(2) in proportion to the original length L, and

(3) in inverse proportion to the area A of the cross-section of the bar.

Thus $l \propto \dfrac{F.L}{A}$

or $l = \dfrac{F.L}{A} \times \dfrac{1}{E}$, where E is a constant quantity, depending only on the material of the bar and is called Young's modulus.

The above equation can be written in its more logical form as $E = \dfrac{F}{A} \times \dfrac{L}{l}$ or $\dfrac{F}{A} / \dfrac{l}{L}$.

Suppose further that $l = L$ and $A =$ unity, then we have $E = F$. Thus the Young's modulus, which is the ratio of the stretching force per unit area of cross-section of wire to the extension per unit length, is the force in dynes which would stretch a bar of the material one square cm. in sectional area to twice its original length, if its elastic properties remain perfect during the operation. *

57. States of matter. According to their behaviour towards forces tending to alter their shape, substances are divided into two main classes (*i*) *Solids* and (*ii*) *Fluids.*

Solids. A solid is a body, which can offer a permanent resistance to small forces tending to alter its shape.

* In practice, it is impossible to double the length of a wire without breaking it.

Fluids. A fluid is a body, which can offer no permanent resistance to forces tending to change its shape.

Thus a piece of stone, wood or cork shall retain its shape indefinitely, if the external forces acting upon it are very small; hence the above substances are examples of *typical solids.* On the other hand, alcohol and carbon dioxide take up the form of the vessel in which they are poured and thus offer practically no resistance to forces tending to change their shape and are examples of *typical fluids.* The behaviour of fluids is very varied. Water and alcohol take no time to change their shape; while treacle and honey take a considerable time. An extreme case of this kind is furnished by hard pitch, which offers high resistance to forces applied for a short time; but the same piece of pitch, when placed in a funnel, would begin to flow, on account of the continued downward force of gravity. It is thus incapable of offering a permanent resistance to forces tending to change its shape, though the resistance is very prolonged.

Fluids are further subdivided, as regards their behaviour towards forces tending to change their volume, into two classes (*i*) *Liquids* and (*ii*) *Gases.*

Liquids. *A liquid is a fluid, which offers very great resistance to forces tending to change its volume; as water, alcohol etc.*

Gases. *A gas is a fluid, which offers very small resistance to forces tending to change its volume; as hydrogen, oxygen etc.*

Further, liquids have free surface, while gases have none. Thus, imagine a cylinder about twelve inches long fitted with an air-tight piston and filled with water to a height of about six inches. First, let us suppose that the piston is just touching the surface of water, *i. e.* it is six inches above the bottom. We shall see that a considerable force may be applied to the piston without producing any appreciable diminution in the volume of water. Now if the piston be moved

upwards, we shall see that water-level would remain where it was and the space between the piston and the water-level would be empty, but for the water-vapour. If however, instead of water, we have a gas in the cylinder, we shall see that a small force in the first instance would produce some appreciable compression; and secondly if the piston be moved upwards the gas-level would not remain where it was; but the gas shall actually fill the whole space between the piston and the bottom of the cylinder.

The existence of a definite free surface is the most important point of distinction between liquids and gases; for though gases can be easily compressed, yet a gas under high pressure may offer a high resistance to further compression.

Thus summing up, we may say that

'Solids have both size and shape;
Liquids have size but no shape;
Gases have neither size nor shape.'

58. Viscosity. *The temporary resistance, which fluids offer to forces tending to change their shape, is called viscosity.* It is measured by the tangential force per unit area required to maintain a relative velocity of 1 cm., between two parallel planes 1 cm. apart in a fluid. This measure is called co-efficient of viscosity and is denoted by n. Pitch, treacle and honey have very high viscosity; while water and alcohol have very little viscosity. The difference between viscosity and rigidity is one of *duration.* A rigid body can offer a permanent resistance to a force tending to change its shape, while a viscous fluid can only offer temporary resistance to such forces. However weak the force may be, the fluid must eventually yield to it. The rate at which a fluid yields depends upon its viscosity. A fluid having no viscosity is spoken of as a *perfect fluid or mobile fluid.* In nature there is no perfect fluid as there is no perfectly rigid body. Gases, alcohol and ether are considered as perfect fluids.

To show that water possesses viscosity, take a vessel as shown in figure 47. This arrangement is adopted to keep the level of the liquid constant throughout the experiment. The overflow tube *T*, passing through the centre of the vessel has a larger bore than the inlet tube *I*; and the capillary side-tube *C*, smaller bore than the inlet tube. Thus the surplus liquid finds its way through the overflow tube *T*: and the level is always maintained to the head of the overflow tube.

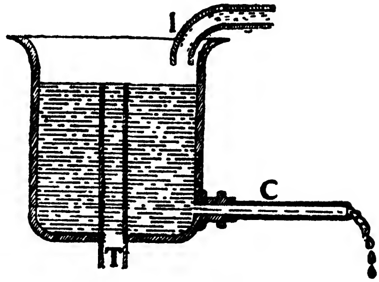


FIG. 47

The shape of the liquid as it flows along the long capillary tube is constantly changing. If there were no force resisting the change of shape of the liquid, the vessel would have emptied itself in no time. As a matter of fact, if the tube be sufficiently narrow the liquid flows slowly and takes a considerable time, due to the force *resisting its change of shape, called Viscosity.*

The quantity of a liquid which flows per second along the capillary tube is directly proportional to

(1) the difference of pressures at the two ends of the capillary tube,

(2) the fourth power of the radius of the tube,

(3) the co-efficient of viscosity, and

(4) is inversely proportional to the length of the tube.

The viscosity of liquids decreases with the rise of temperature, while that of gases increases.

59. Ether. This Chapter on Properties of Matter would be incomplete, if we were to omit totally any reference to ether, a substance much talked of, but least known till now. It is in fact a proverbial trap into which Physicists wish to fall, when they fail to give a satisfactory explanation of any phenomenon.

Truly speaking, it is the parting line between Physics and Metaphysics.

Light and heat pass from the Sun to the Earth, but not instantaneously. They take some time to traverse the space. Thus for some time both these forms of energy must exist in the interplanetary space. We have stated already, that matter is the vehicle of energy. This leads us to imagine that matter of some sort must fill the whole interplanetary space. This matter (*i. e.* the one which exists throughout the universe) has been christened as Ether. Nothing is known about this ; it must be totally different from the ordinary matter, the existence of which we can perceive by our senses.

In order to explain the varied natural phenomena, ether is endowed with properties apparently enigmatical. To explain the propagation of radiant-energy, it is supposed to possess all the properties of an highly elastic solid, *i.e.* one capable of executing transverse vibrations.

To explain the laws of universal gravitation, it is supposed to be a continuous incompressible fluid, for it seems inconceivable that Sun could attract a planet without some connecting medium and similarly does it appear improbable that electric and magnetic forces could be exerted without some connecting link. To explain the unimpeded revolution of planets round the Sun, ether is endowed with the properties of a fluid more perfect than the most perfect gas known. In fact, it is impossible to reconcile all these views and yet this mysterious ether possesses all these. It must then be Aerial ether, the nature of which is beyond our comprehension.

SUMMARY

Properties of Matter:—

1. It is indestructible.
2. It has inertia.
3. It exerts mutual attraction.
4. It is divisible.
5. It is the vehicle of energy.

6. It is elastic.

7. It is porous.

States of Matter:—

(1) Solids (2) Liquids and (3) Gases.

Solids resist forces tending to change their size and shape.

Liquids resist forces tending to change their size but not shape.

Gases do not resist forces tending to change their size or shape.

HYDROSTATICS

CHAPTER XIII

THRUST, PRESSURE AND GENERAL PROPERTIES OF LIQUIDS

60. Thrust. Suppose we have a heavy weight lying on a table. As the weight is at rest, it is evident that the forces acting on it must be in equilibrium. The forces acting are two, *i. e.* its weight acting vertically downwards and the reaction of the table, which must act vertically upwards, *vide* conditions of equilibrium of two forces. The total force with which the body presses against the table or the table against the weight, is known as a *Thrust*; and the two forces which are equal and opposite are together spoken of as **stress.**

Tension. Suppose two men are pulling at the ends of a rope in opposite directions. In this case, each cross-section of the rope is acted upon by two forces tending to lengthen it. Such a pair of forces is spoken of as a **Pull or Tension** instead of a *Thrust.*

Shearing stress. Suppose two forces are applied in opposite directions at the two ends of an iron rod tending to twist it. In this case, each cross-section of the rod tends to slide parallel to the next one below it; and the forces have a tendency to change the shape of the body to which they are being applied. Such a pair of forces is said to constitute a **shearing stress.**

Fluids are incapable of exerting any permanent resistance to continued shearing stress, however small; while solids offer permanent resistance to all forms of stresses.

Pressure. The thrust exerted per unit area of the surface is called *pressure.* Thus if T be the total thrust and a the area on which it acts, then $\frac{T}{a}$ gives the pressure. If the thrust on every point of the surface be the same, then the thrust is said to be uniformly distributed over the surface; but thrust per unit area of the surface is known as *pressure.* Thus $P=\frac{T}{a}$, or conversely, we have the total thrust T equal to the product of the uniform pressure P and the area a, on which it acts.

Pressure may be measured in dynes per square centimetre; but in practice, it is measured in grammes weight per square centimetre in the *C.G.S.* system. Similarly in the *F.P.S.* system, the pressure may be measured in poundals per square foot; but in practice, English Engineers measure the same in lbs. weight per square inch.

Pressure at a point. If the thrust is not uniformly distributed, the pressure will be different on different parts of the surface. In such a case, the pressure at any point is the ratio of the thrust on a very small area containing the point, to the small area itself. Mathematically pressure at a point is $\frac{T}{a}$, where a is indefinitely small.

61. (a) **The force exerted by a Liquid at rest, on any surface in contact with it, is always at Right Angles to that surface.**—Suppose the pressure exerted by the liquid is not perpendicular to the surface; but is in the direction DB inclined to the normal BR at an angle α. Then by Newton's third law, the *reaction* which the surface exerts on the fluid, must be in the direction BD. This reaction can be

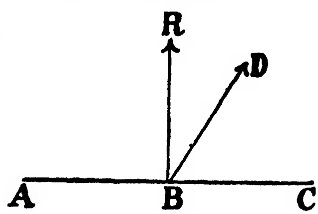


FIG. 48

resolved into two components, one along the normal BR and the other parallel to the surface BC. The latter component BC tends to make the liquid particles slide over the surface and thus constitutes *a shearing force.* According to definition, a liquid has no rigidity and since it cannot resist this force, motion ought to take place; but this is contrary to the supposition that the liquid is at rest. Hence there can be no component of the reaction along the surface. The whole force therefore, must act perpendicular to the surface and hence the thrust is *normal* to the surface.

The normal component tends only to compress the liquid, but liquids offer very high resistance to such forces. They are in fact assumed to be incompressible.

(*b*) **Pressure at a point in a liquid is the same in all directions.** The direct mathematical proof is too cumbersome to be given here and the experimental proof requires very complicated apparatus. We may however, take it as true for the results of the experiments are in exact accord with theoretical deductions. However to give an idea, imagine an extremely small cavity in a liquid, it is evident that the liquid would rush in from all sides to fill it. Suppose now the cavity has been filled up; the liquid bounding it is kept up in position on account of the pressure exerted on its various sides by the liquid in the cavity. These forces thus form a system in equilibrium among themselves and therefore all of them must be equal.

62. Transmission of pressure: Pascal's law. *When pressure is applied to any part of the boundary of a fluid; an equal and uniform pressure is transmitted over the whole of the fluid.*

Suppose, we have a vessel filled with water as shown in fig. 49, having a number of openings of equal cross-section and each closed with an air-tight piston. In order to keep the pistons in position, a force shall have to be applied on each of them. If now the force on any one of them be

increased, it would be found that the force applied on each piston must be increased correspondingly, in order to keep them all in position. Thus we see that pressure is transmitted equally throughout the liquid, and this is known as **Pascal's Law.**

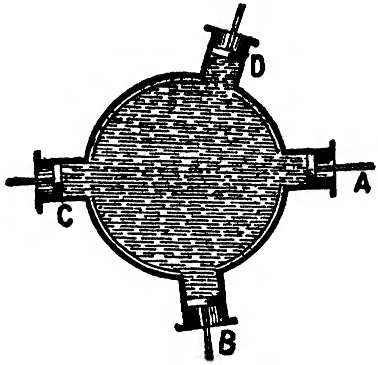


Fig. 49.

63. The Bramah Press. Pascal's law finds an important application in the construction of Hydraulic Press. It consists as shown in fig. 50 of a force-pump from which a pipe *CE* opens into a strong cylinder *A*, in which a big solid cylinder works through a water-tight collar. The piston of the pump is much smaller

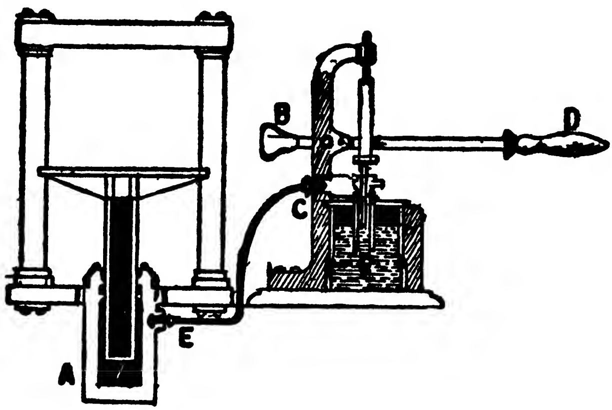
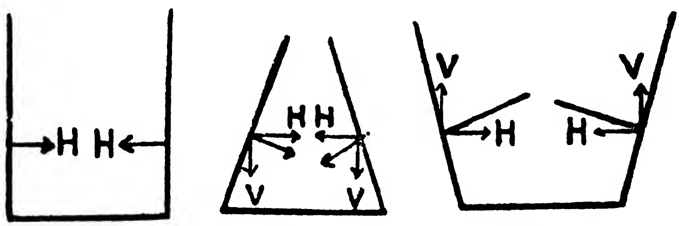


Fig. 50

in sectional area than the solid cylinder and is moved by a lever *BD.* If this piston is forced down, then by the principle of transmissibility of fluid pressure, the same pressure is transmitted to the whole of the fluid.

Hence if a force F is applied to the piston of the pump, it will produce a pressure equal to $\frac{F}{a}$, where a is the sectional area of the piston. The same pressure will act on the big cylinder vertically upwards; and if its area be b, the thrust on it will be equal to $\frac{F}{a} \times b$ Therefore for equilibrium, we have $W = \frac{F}{a} b$ or $\frac{W}{F} = \frac{b}{a}$, where W is equal to the weight placed on the bigger cylinder. Thus Bramah Press is a hydrostatic machine, the mechanical advantage of which is, area of the bigger cylinder divided by the area of the smaller piston. If the area of the bigger cylinder be to the area of the smaller piston as 100 : 1, then by applying a force of 1 ℔. weight, resistance equal to 100 ℔s. weight can be kept in equilibrium.

64. Pascal's vases. These are vessels open at both ends, of different shapes, but of equal bases and heights, as shown in figs. 51. They *are used to illustrate that the thrust on a given surface due to a given liquid is dependent only on the depth of the liquid.*



(*i*) (*ii*) (*iii*)

Fig. 51

To verify this, hold the vase with a clamp, fit a circular disc against the lower edge of the vase and fasten the string passing through the middle of the disc to the hook of the left-hand pan of a balance. Put such weights in the right-hand pan of the balance, so that the disc is firmly held against the lower edge of the vase.

Pour water into the vase, till the disc gets detached; note the level of the liquid at that instant. Repeat the experiment with the other two vases, keeping the same weights in the right-hand pan and note that *in each case the disc gets detached when the liquid-level in the vase reaches the same height. This shows that the thrust on a given surface due to a given liquid is proportional to the height of the liquid-column* and is independent of the volume *of the liquid.* By using liquids of different densities and vases of different bases, it may be shown that the thrust is equal to the product of the area, the density of the liquid, its depth and the intensity of gravity.

Thus $T = g \rho h a$.

The above experiment at first sight appears to be a puzzle; for the amounts of liquids contained are different and yet they exert the same thrust. For this reason, the experiment is spoken of as **Hydrostatic Paradox.** There is however, nothing paradoxical. In Fig. 51 (*i*), the liquid pressure is normal to the sides of the vessel as shown by the arrow-heads and it has no component either in the upward or downward direction; and the thrust, in gravitational units, is equal to the weight of the liquid, *i.e.* equal to height × area × density, and pressure is equal to height × density per sq. cm.

In Fig. 51 (*ii*), the liquid pressure as before, is normal to the sides of the vessel, which not being vertical, its components will be H and V in the horizontal and vertical directions respectively. The horizontal component H does not produce any effect on the base; while the vertical component gives rise to extra pressure on the base. The total thrust will evidently be greater than the actual weight of water.

In Fig. 51 (*iii*), the vertical component acts in the upward direction and supports the weight of some of the liquid. Thus the thrust on the base will be less than the weight of the liquid.

65. In a liquid at rest under the action of gravity, the pressures at any two points in a horizontal plane are the same.

Let A, B be the two points in a horizontal plane; about AB as axis construct a cylinder of indefinitely small section. Consider the equilibrium of the liquid contained within this cylinder; the forces acting on the liquid are:—

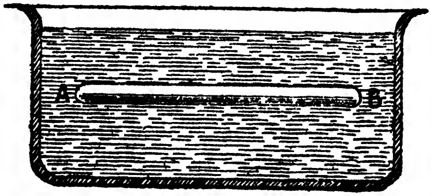


FIG. 52

(*i*) Its weight acting vertically downwards;

(*ii*) Thrusts on the curved surfaces, everywhere perpendicular to them; and

(*iii*) Thrusts on the ends at A and B along AB.

The thrust at the end A=the thrust at the end B, (*for the liquid is supposed to be at rest*).

But area of A=area of B;

therefore the pressure at A is equal to the pressure at B.

65. (*a*) **In a liquid at rest under the action of gravity, the difference of pressures at two points in it (one of which is vertically below the other), varies as the difference of their depths.**

Let P and Q be the two points; about PQ as axis construct a cylinder of indefinitely small section a. Let p be the pressure at P and q at Q; and let d be the density of the liquid. Consider the equilibrium of the liquid contained within the cylinder; the forces acting on it are:—

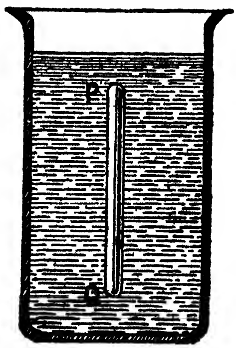


FIG. 53

(*i*) Thrusts on the curved surfaces in the horizontal direction;

(*ii*) Its weight acting downwards;

(*iii*) The thrust at the end P downwards$=p.a$; and

(*iv*) The thrust at the end Q upwards$=q.a$.

∴ Equating the vertical forces, we have for equilibrium

$$p.\,a + (h' - h)\,a \times d = q \times a,$$ *where* h' and h are respectively the depths of Q and P from the surface of the liquid. Or $q - p = (h' - h)\,d$.

Thus the pressure at a depth h below the free surface of a liquid of density d, is equal to the product of the density and the depth. That is:—

$p = h.d$ in gravitational units,

or $p = h.d.\,g$ in absolute units.

Or we may say that the pressure due to a liquid is always equal to the weight of a column of liquid, whose height is equal to the given depth and whose area of cross-section is unity.

If however, the free surface of a liquid is subjected to a pressure P_1 from some external source, such as atmospheric pressure, then this will be uniformly transmitted according to Pascal's law. The actual pressure at any point would be the sum of the separate pressures (*i*) due to the external pressure and (*ii*) due to its own weight.

Thus $P = P_1 + h\,\rho$, in gravitational units.

Experiments to Illustrate Liquid Pressure.

(*i*) If water be allowed to flow from holes made at various depths in a tall vessel as shown in figure 54, it is seen that the water flows more rapidly from the lower holes than from those above. This illustrates that pressure is greater at greater depths.

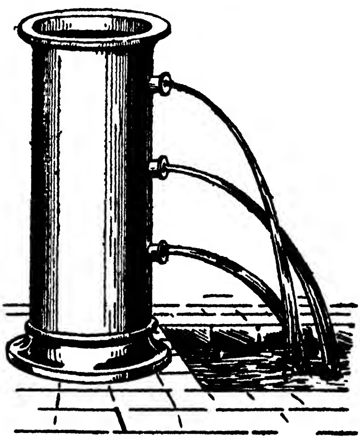


FIG. 54

(*ii*) Fit a smooth glass plate against one end of a tube and hold it against the bottom with a string, as shown in fig. 55.

Lower the whole to a certain depth into a vessel of water and release the string. The glass plate remains in tact; for the upward pressure is more than sufficient to support its weight.

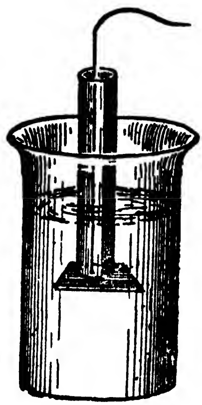


FIG. 55

Now pour coloured water gently into the tube. Notice that the glass plate falls, when the level of the coloured liquid inside reaches the level of the water outside. This shows that the pressure of a liquid is proportional to its depth.

(*iii*) *Hydraulic lift*.—A wide cylinder A has a water-tight piston B and is connected to a long vertical tube EO (fig. 56) of small cross-section; and the whole is filled with water. If h be the difference in the level of the liquid in the two sides, d be its density, then the upward thrust on B will be equal to $h\,d\,a$.

Thus a long narrow column of liquid may be used to lift heavy weights.

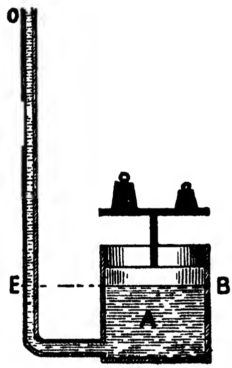


FIG. 56

66. Surface Tension. Every molecule of liquid pulls every other molecule towards itself. A molecule such as A situated well within the mass of a liquid is attracted by the neighbouring molecules equally in all directions, whereas a molecule such as B (fig. 57) situated near or on the surface is attracted towards the inside of the liquid and perpendicular to the surface. Thus the surface molecule has a tendency to move inside the liquid so that it may have the smallest possible surface for the given volume. *This tension, which constantly tends to contract a liquid, is known as surface tension. It is the force per unit*

length, and it acts at the surface only.

From the above it follows as a corollary that a mass of liquid would assume a spherical form; for a sphere is the geometrical figure, which has the smallest area for a given volume; but in most cases, it does not happen due to the action of gravity, which tends to spread a liquid evenly. If the force of gravity be eliminated somehow, then a given mass of liquid assumes a spherical form. This is illustrated by the following experiment, which we owe to Plateau.

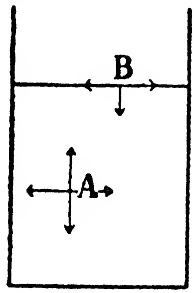


FIG. 57

Experiment.—Prepare a mixture of alcohol and water so that its density is the same as that of linseed oil. Drop a little amount of the oil below the surface of the mixture by a pipette. The oil will float as a perfect sphere in the mixture.

Liquids preserve their spherical form, when their mass is small and the *cohesive force* strong, as is noticed in the case of mercury drops, dewdrops and rain drops. The surface layer of a liquid behaves like a thin stretched membrane under uniform tension in all directions. An iron needle when slightly greased, if placed very carefully on the surface of water in a dish will float on its surface, though that is eight times as heavy as water. This is explained by saying that the portion of the liquid immediately below the needle is depressed and tends to flatten out; the vertical component of the surface tension round the edges of depression supports the weight of the needle. If however, the water had wetted the needle as it would have done if the needle were not greased, water would have risen about the needle and the surface tension in the liquid would have pulled it down.

The fact that the surface of liquid is in a state of tension is admirably explained by soap films.

Take a wire *FCDE* bent in the form shown in the figure and a thin wire *AB*. Get a soap film enclosed in the portion *ABDC*. On raising the wire from out of the soap solution *AB* is seen to move towards *CD*, which shows that a soap film contracts. *The total force acting on AB divided by twice the length AB, gives surface tension, because a film has two sides.*

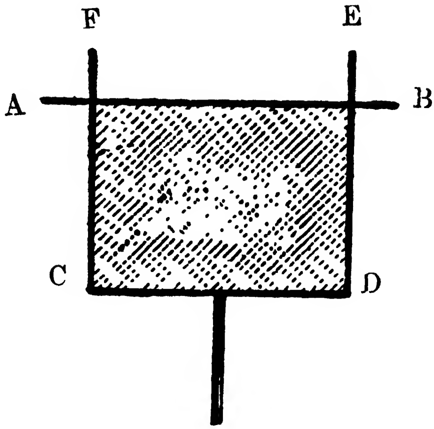


FIG. 58

SUMMARY

1. **Thrust** is the total force acting on a surface.
2. **Pressure** is the force acting on a unit area.
3. **Tension** consists of a pair of forces in opposite directions. The pressure at a point in a liquid is the same in all directions about that point.
4. **Pascal's law.** When pressure is applied to any part of the boundary of a fluid, an equal and uniform pressure is transmitted over the whole fluid.
5. Pressure due to a column of liquid is equal to **hdg.**
6. **Surface tension.** The force per unit length acting on the surface of a liquid, tending to make it occupy less surface area, is called *surface tension.*

EXAMPLES

1. What is the pressure of water at a depth of 80 feet ?

Pressure=height × density in lbs. weight or $80 \times 62\,^1/_2$ pounds wt.=5000 lbs. wt. per sq. foot.

2. The small piston of a Bramah press is half an inch and the large one 8 inches in diameter ; the pump is worked by a handle 5 feet long, the fulcrum being one inch

from the point of attachment of the piston. What is the greatest weight that can be raised by such a machine, by applying a force 1 cwt. at the end of the handle?

$$\frac{\text{The area of the larger piston}}{\text{The area of the smaller piston}}=\frac{\pi\left(\frac{4}{12}\right)^2}{\pi\left(\frac{1}{48}\right)^2}=256.$$

The force applied is equal to 112 lbs. weight. It would exert a pressure equal to 60 × 112 lbs. on the small piston, for one arm of the lever is 1 inch and the other is 60 inches.

∴ the greatest weight that can be raised is equal to 256 × 60 × 112 lbs. = 768 tons.

3. A rectangular lock gate is 10 feet wide and the water outside stands at a height of 10 feet above the bottom of the gate Calculate in tons weight, the thrust upon it.

The area of the surface of the lock gate in contact with water $=10\times10=100$ square feet.

The pressure at the centre of the surface $=5\times\frac{125}{2}$

∴ the total thrust in tons weight $=\frac{100\times5\times125}{2240\times2}$

$=13'95$ tons weight.

4. The moving piston of an hydraulic lift has a diameter of 1 foot. What head of water will be required to raise a mass of 1120 lbs?

5. Calculate the pressure at the depth of a sea, $^1/_2$ mile deep, specific gravity of sea-water being 1'025.

6. Find the total pressure on the bottom of a tank 10 feet square, 5 feet deep, full of water. Find the pressure on a side of the same tank.

7. The area of the smaller piston of a Bramah press is $2^1/_2$ square inches and the area of the larger piston is 200 square inches. What weight can be raised by applying a force of 10 lbs. weight?

8. Prove that if a cubical box be filled with water, the total pressure to which it is subject is equal to three times the weight of the water which it contains.

9. The pressure in the water pipe at the basement of a building is 34 lbs. weight to the square inch, whereas at the third floor it is only 18 lbs. weight to the square inch. Find the height of the third floor.

10. A tube 20 feet long with one end open is filled with water and inverted over a vessel containing water. What is the pressure in the water at the top of the column?—the height of water barometer being 33 feet.

CHAPTER XIV
PRINCIPLE OF ARCHIMEDES
FLOATING BODIES

67. Buoyancy of liquids. Whenever any object is dipped into water, then it occupies a certain portion of the space formerly occupied by water, which means that a certain quantity of water equal to the volume of the object is displaced. Thus whenever any solid is immersed either wholly or partly in a liquid, then liquid equal to the volume of that portion of the solid which is immersed, is displaced.

(*i*) Take an ordinary test tube of thin glass and dip the same in water, with its closed end downwards. You will feel an upward pressure and the tube will, if you were to release your hand, come out of water. Keep the test tube dipped in water and pour water slowly in it; you will find that the upward force lessens gradually till it vanishes altogether, when the level of the liquid inside and outside the tube is the same, assuming the walls of the tube to be thin.

Suppose now that the glass tube has been annihilated, then the water contained in the tube will remain at rest as all water at rest is. This water however, is in equilibrium under the action of two forces: (*i*) its weight acting downwards and (*ii*) the upward thrust of the surrounding liquid ; but this latter is equal to that experienced by the tube before its annihilation. Therefore the upward thrust on the test tube is equal to the weight of its own volume of water, *i. e.* equal to the weight of the water displaced by it, and acts vertically upwards. This result is known ***as the Principle of Archimedes.***

The thrust upon a solid immersed in a liquid

is equal to the weight of the liquid displaced and acts vertically upwards through the centre of the displaced liquid.

(*ii*) Further suppose a cylinder PQ of area of cross-section a is made to float in a liquid of density d as shown in fig. 53, page 135. Let the depth of P below the liquid-level be denoted by H and the length of the cylinder by h; then the downward pressure of the liquid on P will be

$$H a d$$

and the upward pressure on Q will be $(H+h)\ ad$; therefore the resultant upward pressure on the cylinder will be $(H+h)\ ad-H\ ad=h.a\ d$; but $h \times a=$ the volume of the cylinder and $h.a.d$ is the weight of the liquid displaced by the body. *Therefore the loss of weight of a body when weighed in any liquid is equal to the weight of the liquid displaced by the body.*

Experimental verification of Archimedes principle. A solid cylinder, which fits exactly into a hollow cylinder, as shown in fig. 59, is suspended from one pan of a hydrostatic balance, with the hollow cylinder above and is counterpoised by putting weights or lead shots in the other pan. A beaker of water is then brought under the solid cylinder so that the same is thoroughly immersed in it. The weights are found to be too great. Water is poured into the hollow cylinder and when it becomes exactly full, the beam is found to be horizontal again. Thus it is evident that the upward thrust or the effect of buoyancy of water is counterbalanced by the weight of an equal volume of water.

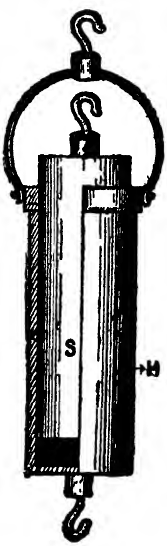


FIG. 59

68. Conditions for the equilibrium of a floating body. We have seen that when a body is immersed in any liquid, an upward thrust equal to the weight of an equal

volume of the liquid acts upon the body. If the body is denser than the liquid, its weight would be greater than the upward force and so it will sink; but if the body is lighter than water, its weight would be less than the weight of an equal volume of the liquid and so it would come up and float on the surface of the liquid.

To sum up then, the conditions necessary for a body to float are :—

(*i*) *The weight of the liquid displaced must be equal to the weight of the floating body.*

(*ii*) *The centre of gravity of the liquid displaced must be in the same vertical line, as the centre of gravity of the floating body.*

These conditions follow easily from the conditions of equilibrium of two forces; but for stability of equilibrium of a floating body, to these two conditions must be added the third, namely, *the metacentre must be above the centre of gravity.*

Metacentre. Suppose a body is floating in equilibrium in a fluid as in figure 60. Let G be its centre of gravity and C* that of the displaced liquid. Join CG and consider this line as fixed in the body. Disturb the body slightly in such a way that the mass of the displaced liquid remains constant. Let C' be the centre of gravity of this in the latter condition. Draw a vertical line through C' and let it intersect the line CG at the point M: then the point M is called the metacentre, for the displacement that has taken place.

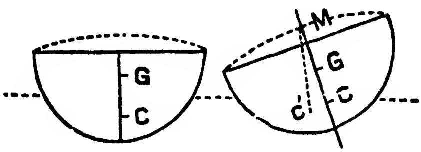


FIG. 60

The reason why the metacentre should be above the centre of gravity for stable equilibrium is, that the couple

* The point C is sometimes called the centre of buoyancy or the centre of pressure.

brought into existence on account of the small displacement of the body, tends to bring it to its former position, only when the metacentre is above the centre of gravity, while this couple tends to displace it still further if M is below G. The student should himself draw figures to show whether a body would be in stable equilibrium or otherwise. It is for this reason that mariners load their ships with ballast, when they have no heavy goods to carry.

SUMMARY

1. **The Principle of Archimedes.** The upward thrust on a solid immersed in any liquid is equal to the weight of the liquid displaced and acts vertically upwards through the centre of gravity of the displaced liquid.

2. **Conditions of floating bodies**:—(1) Weight of displaced liquid=weight of the body itself and (2) The centre of buoyancy must be in the vertical line drawn from its centre of gravity.

3. For stable equilibrium, the metacentre must be above G.

EXAMPLES

1. A glass-sinker weighs 25 grammes in air and 15 grammes in water. Find its volume

The loss in weight=10 gms.

∴ its volume =10 c.c.

2. A mass of iron 20 c.c's weighs 156 gms. in air. It is suspended in water by means of a thread. Calculate the tension in the string.

The volume of iron piece=20 c c's

∴ the weight of water displaced=20 gms.

Hence the tension=156−20=136 gms. weight.

3. A piece of iron weighing 275 grammes, floats in mercury (density 13·6) with $\frac{5}{9}$ of its volume immersed. Determine the volume and density of iron.

$$\tfrac{5}{9}\, v \times 13{\cdot}6 = 275$$

$$\text{or} \qquad v = \tfrac{9}{5} \times 275 \times \frac{1}{13{\cdot}6} = 36{\cdot}4 \text{ c.c's}$$

$$\therefore \quad \text{the density of iron} = \frac{275}{36{\cdot}4} = 7{\cdot}5$$

4. State the conditions, which must be fulfilled in order that a body may float in equilibrium in a liquid; and show that if a body of volume v and density S floats in a liquid of

density S', the volume of the part immersed will be $\frac{vS}{S'}$.

5. The density of ice is 0·918 and that of sea-water is 1·03. What is the total volume of an iceberg, which floats with 350 cubic yards exposed.

6. A solid weighs 100 grammes in air and 64 in water. Find its specific gravity.

EXAMINATION QUESTIONS III

1. What is a machine? Prove that in the case of a machine, "What is gained in power, is lost in Speed."

2. State the principle of work as applied to machines. Get an expression for the mechanical advantage of a screw by its aid.

3. What are the most important properties of matter? Define Young's modulus and state Hooke's law.

4. Enunciate the principle of Archimedes and write down the conditions of equilibrium of floating bodies. What is the specific gravity of a metal, a cubic foot of which will just float in glycerine of sp. gravity 1·25, when attached to 6 cubic feet of cork (sp. gravity=0·24)?

5. Explain what is meant by a liquid and the principle of transmission of liquid pressure. Illustrate your answer by examples.

6. Find the pressure at a depth of 12 feet below the surface of a lake. (*i*) in lbs. weight per square foot and (*ii*) in dynes per sq. cm., the atmospheric pressure being 15 lbs. wt. per sq. inch.

7. Define specific gravity of a substance and write down the conditions of equilibrium of floating bodies.

8. Get an expression for finding the specific gravity of a solid, which is lighter than water.

9. A U-tube contains water in its lower part, 32 cms. of kerosine oil of specific gravity 0·8 in one limb and a vegetable oil of specific gravity 0·95 in the other. The water-level on the kerosine oil side is 5 cms. lower than on the other. How long is the column of vegetable oil?

10. A beaker of water is placed on a balance-pan and counterpoised. A piece of iron weighing 15 gms. suspended from a thread is lowered into the beaker, until it is completely submerged without touching the beaker. What weight must be added to restore equilibrium and in which pan? Sp. gravity of iron=7·5.

11. Two liquids sp. gravities ·6 and ·75 are mixed (*i*) in equal volumes and (*ii*) by equal weights. Find the specific gravities of the mixtures in the two cases.

12. A lump of silver weighing 160 gms. is suspended by a string in water. If the density of silver be 10·5, what is the tension in the string?

CHAPTER XV

MEASUREMENT OF SPECIFIC GRAVITIES

69 (*a*) **By Hydrostatic Balance.** *To find the specific gravity of a solid which sinks in water.*—

Weigh the solid in air and let it be $= W_1$ gms.

Weigh the solid in water and let it be $= W_2$ gms.

Therefore weight of water displaced $= W_1 - W_2$,,

By the principle of Archimedes, this is the weight of an equal volume of water;

$$\text{Therefore specific gravity} = \frac{W_1}{W_1 - W_2}.$$

Precautions.—Students shall find detailed precautions in any book on Practical Physics, but most important of them are:—

(*i*) The weight of suspension-wire or thread should be taken into account.

(*ii*) A horse hair or a fine wire should be used in preference to thread, for the latter absorbs moisture and so varies in weight on absorption of water.

(*iii*) The bubbles of air which sometimes stick to the solid reduce its apparent weight on account of buoyancy and should be carefully brushed off.

(*iv*) Correction for temperature of water is necessary. Because 1 c.c. of water weighs 1 gramme at 4° C.; at other temperatures, it weighs less, therefore to get correct value of specific gravity, the above result must be multiplied by the density of water at the given temperature.

To find the specific gravity of a liquid.

Weigh a heavy solid in air and let it be w_1 grammes

,, the ,, ,, water ,, w_2 ,,

,, ,, ,, ,, liquid ,, w_3 ,,

Then $\dfrac{w_1 - w_3}{w_1 - w_2}$ = Specific gravity of the liquid.

The numerator gives the weight of an equal volume of liquid, while the denominator gives the weight of an equal volume of water.

To find the specific gravity of a lighter body (cork).

As cork floats in water, therefore a heavy body called the sinker is used to immerse it completely in water. The following observations are made:—

(*i*) Weigh the cork in air and let it be $=w_1$ gms.
(*ii*) Weigh the sinker in water and let it be $=w_2$ „
(*iii*) Weigh the cork and sinker (together in water) and let it be... ... $=w_3$ „

$$\therefore \text{ Specific gravity of cork} = \frac{w_1}{w_1-(w_3-w_2)},$$

for w_3-w_2 gives the weight of the cork in water.

70. By specific-gravity bottle. *To find the specific gravity of a liquid.*—

(*i*) Weigh the bottle empty and let it be... $=w_1$ gms,
(*ii*) Weigh the bottle filled with water and let it be . ... $=w_2$ „
(*iii*) Weigh the bottle filled with the liquid and let it be ... . $=w_3$ gms.

$$\therefore \text{ The specific gravity of liquid } = \frac{w_3-w_1}{w_2-w_1}$$

i. e. $\dfrac{\text{Weight of a given volume of liquid}}{\text{Weight of an equal volume of water}}$.

To find the specific gravity of a solid (iron nails).

(*i*) Weigh the bottle empty and let it be $=w_1$ gms.
(*ii*) Weigh the bottle + $^1/_3$ filled with iron nails and let it be ... . $=w_2$ „
(*iii*) Weigh the bottle + $^1/_3$ filled with iron nails + water filled to the brim and let it be ... . $=w_3$ „
(*iv*) Weigh the bottle wholly filled with water and let it be .. . . $=w_4$ „

Then $w_2-w_1=$ weight of solid in air; and
$w_3-w_4=$ weight of solid in water;

for the difference in (*iii*) and (*iv*) weighing is simply this, that in (*iii*) a certain volume of the bottle is occu-

pied by the solid, while in (*iv*) that volume is occupied by water.

$$\therefore \text{ Specific gravity} = \frac{w_2 - w_1}{(w_2 - w_1) - (w_3 - w_4)}.$$

NOTE.—When the substance is soluble in water such as common salt, then instead of water, a liquid in which the substance is insoluble. may be used instead of water; and the result obtained would be the specific gravity of the solid with respect to the liquid used. To get the specific gravity with respect to water, the result so obtained should be *multiplied by the density of the liquid itself*. In this way the specific gravity of solubles is obtained.

71. By hydrometers. There are two kinds of these instruments: Constant Weight and Constant Volume types. In the constant-weight type instrument, the *weight* of the displaced liquid is always the *same*; while the instrument sinks to different marks. In the constant-volume type, the *Volume* of the displaced liquid is the same; and the instrument is made to sink to the same mark in different liquids.

FIG. 61

Constant-weight type. The common hydrometer, usually consists of a uniform thin straight stem, ending in a hollow glass tube *A*. which is connected through a narrow constriction to a bulb containing mercury, so adjusted as to make the instrument float vertically. The stem contains a paper-scale inside its hollow, graduated differently in instruments of different makes.

Suppose V be the external volume of the hydrometer upto the zero mark, near the bottom of the uniform stem, and a the area of cross-section of the stem itself. Let the instrument sink to a mark h_1 above 0 when floating in a liquid of density S_1 and to a mark h_2 when floating in a liquid of density S_2. Since the instrument is in equilibrium under both the

conditions, we must have from the conditions of equilibrium of floating bodies :—

$S_1(V+h_1a)=W=S_2(V+h_2a)$, *i. e.* the weights of displaced liquids must be the same in both the cases and equal to the weight of the instrument itself.

Beaume's hydrometer. (*i*) *For heavy liquids*:—The instrument is of the shape described above, but is so graduated that the *density is given by the formula* $d=\dfrac{146}{146-reading}$. The method of making it is as follows :—

The instrument is made to float in water and the scratch made to denote the water-level is marked zero. It is then made to float in saline solution, density 1·2, made by mixing very nearly 24·34 parts of salt with 75·66 parts of water and the scratch made to denote the solution-level is marked 24·34. The distance between 0 mark and 24·34 mark is divided into 24·34 equal parts and the whole of the stem is thus graduated.

Theory.—Let V be the volume of the instrument upto 0 mark, then we have

$$V\times 1=\text{weight of instrument}=(V-24{\cdot}34)\times 1{\cdot}2$$
$$\text{or } V=146$$

Thus $146\times 1=$density of the liquid $(146-\text{reading})$.

$$\text{or density}=\frac{146}{146-r}.$$

(*ii*) *For light liquids, the density is given by* $\dfrac{146}{136+reading}$.

The method of marking is as follows:—The instrument is first made to float in saline solution, 10 parts of salt to 90 of water, density 1·0735, and the scratch made to denote the solution-level is termed 0. It is then made to float in water and the scratch made to denote water-level is marked 10. The distance between the two is divided into 10 equal parts and the whole of the stem is then graduated in equal divisions.

Theory.—Let V be the volume of the instrument upto the zero mark, then we have

$$(10+V)\times 1=\text{weight of the instrument}=V\times 1{\cdot}0735.$$

$$\text{or } \cdot 0735\, V = 10$$
$$\text{or } V = 136$$

Now suppose that in a liquid of density d, the instrument sinks upto mark r, then we have

$$(136+10) \times 1 = (136+r) \times d$$

$$\text{or } d \text{ the density} = \frac{146}{136+r}.$$

NOTE :—The heavy liquids instrument is graduated from top to bottom; while the light liquids one, from bottom to top.

To find the specific gravity of a solid by Nicholson's hydrometer. The instrument is a constant-volume hydrometer and consists as shown in fig. 62 of a scale-pan A, connected by a stout wire to a hollow metallic tube B, which carries a second scale-pan C. The latter is loaded with lead shots in order to make the instrument float vertically. To find the specific gravity of a solid, the instrument is made to float in water and weights are put on the upper pan so as to sink it to the given mark M. Let these weights be equal to w_1. The weights are removed and the substance, the specific gravity of which is required, is put on the upper pan along with some weights, so that the instrument sinks again to the same mark M. Let these weights be equal to w_2. Then the weight of the substance in air is clearly equal to $w_1 - w_2$ for $w_1 = w_2 +$ the substance. The substance is now put in the lower pan and weights $= w_3$ placed in the upper pan to sink the instrument to the same mark again, then the weight of the substance in water $= w_1 - w_3$ for $w_1 = w_3 +$ weight of the substance in water.

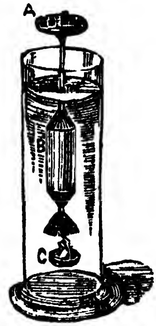


FIG. 62

$$\therefore \text{ Specific gravity} = \frac{w_1 - w_2}{(w_1 - w_2) - (w_1 - w_3)}$$
$$= \frac{w_1 - w_2}{w_3 - w_2}.$$

Note.—If the solid is lighter than water, it is tied to the lower pan by a fine wire.

Precaution.—To get good results, only a small quantity of solid should be used.

To find the specific gravity of a liquid by Nicholson's hydrometer. The instrument is first weighed in air. Let this weight be equal to H.

It is then made to float in water and weights equal to w_1 are put on the upper pan so as to sink it to the given mark M. Then the weight of water displaced by it is equal to $H+w_1$. Next the instrument is made to float in the given liquid and let w_2 be the weights required in this case, to sink the instrument to the same mark again; then the weight of the displaced liquid, which is equal in volume to the displaced water, is equal to $H+w_2$.

$$\therefore \text{ Specific gravity} = \frac{H+w_2}{H+w_1}.$$

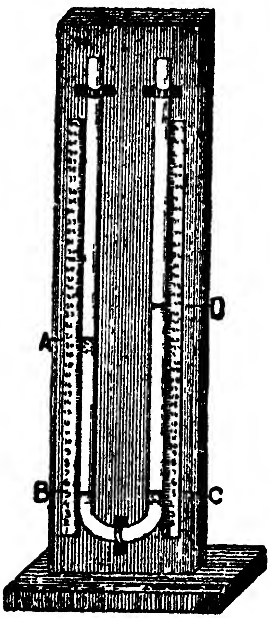


Fig. 63

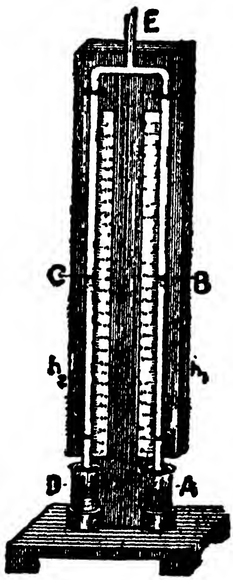


Fig. 64

72. By balancing columns. To find the specific

gravity of a liquid by U-tube.

Let $ABCD$ figure 63 be a U-tube. Let its one limb AB contain oil and the other water, and let B be the common surface of the two liquids. Let the horizontal line from B intersect the other limb at C.

Then the pressure at $B = A + h_1\rho_1 g$
and „ „ „ $C = A + h_2\rho_2 g$

$$\therefore \; h_1\rho_1 = h_2\rho_2$$

where A = Atmospheric pressure, and ρ_1 and ρ_2 the densities of the two liquids.

$$\therefore \; \frac{h_1}{h_2} = \rho_2 \text{ for } \rho_1 = 1, \text{ being the density of water.}$$

To find the specific gravity of a liquid by inverted Y-shaped tube.

When the two liquids mix, the U-tube method cannot be employed. Inverted Y-shaped tube method is then resorted to. The arrangement of the apparatus is shown in fig. 64, P. 152. On sucking the air through E, liquids rise in A and D; E is then firmly clamped and the heights of liquids are measured from their surfaces in the beakers. Let these be h_1 and h_2.

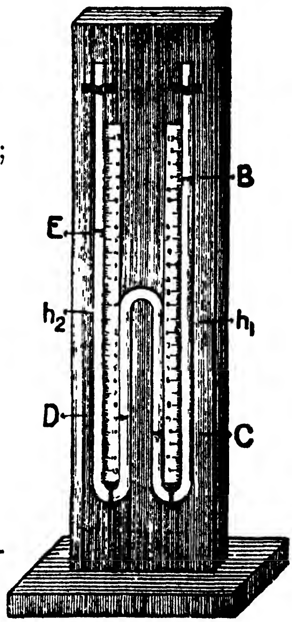


Fig. 65

Then we have the atmospheric pressure

$$A = \text{Pressure at } B + h_1\rho_1 g$$
$$= \text{Pressure at } C + h_2\rho_2 g;$$

but pressure at B and C is the same.

$$\therefore \; h_1\rho_1 = h_2\rho_2$$

or $\rho_2 = \frac{h_1}{h_2}$, for ρ_1 being the density of water is equal to unity.

To find the specific gravity of a liquid by W-shaped tube:—

The arrangement of the apparatus is shown in figure 65 page 153. Water is poured in one limb and the given liquid in the other. Gas is enclosed in the curved limb and on pouring more quantities of the liquids in the respective limbs, the gas is still further compressed. Let the columns stand as shown.

Pressure at C=Pressure at D (being of the enclosed gas).

But the pressure at $C=A+h_1\rho_1 g$

and at $D=A+h_2\rho_2 g$

$\therefore\ h_1\rho_1=h_2\rho_2$

or $\rho_2=\frac{h_2}{h_1}$, for ρ_1 being the density of water is equal to unity.

SUMMARY

1. **Specific gravity** is the ratio of the mass of a body to that of an equal volume of water at 4°C.

2. **Specific gravity** by Beaume's hydrometer for **heavier** liquids is obtained by the formula $\frac{146}{146-r}$ and for **lighter** liquids by $\frac{146}{136+r}$.

3. **The specific gravity** of liquids is obtained by balancing columns, by dividing the water-column by the given liquid-column, which balances it.

EXAMPLES

1. A specific-gravity bottle weighs 14·2 grammes when empty; 39·7 gms. when filled with water and 44·5 grammes when filled with a solution of common salt. What is the specific gravity of saline solution?

It is equal to $\frac{44{\cdot}5-14{\cdot}2}{39{\cdot}7-14{\cdot}2}=\frac{30{\cdot}3}{25{\cdot}5}=1{\cdot}188.$

2. A sinker weighing 30 grammes is fastened to a piece of cork weighing 8 grammes and the two together just sink when placed in water. Find the specific gravity of the sinker taking that of cork as 0·25.

Weight of cork=8 grammes and its specific gravity =0·25; therefore the weight of water displaced by cork is given by

$$S.G.=\frac{\text{weight in air}}{\text{weight of displaced water}}.$$

$$\text{or } 0{\cdot}25 = \frac{8}{\text{weight of displaced water}}$$

or weight of displaced water=32 grammes.

But the weight of water displaced by both together is 30+8 for they together just sink.

Hence weight of water displaced by the body=38−32 =6 grammes.

$$\text{Hence its specific gravity} = \frac{30}{6} = 5.$$

3. The density of the iron weights used with a Nicholson's hydrometer is 7·8. What weights in its lower pan would produce the same effect as 20·4 grammes in the upper pan?

$$\text{We have specific gravity} = \frac{\text{weight in air}}{\text{weight in air—weight in water}}$$

or $7{\cdot}8 = \dfrac{W}{W - 20{\cdot}4}$, for weight in water=20·4, by the condition of the question.

Hence W=23 4 grammes.

4. A solid body floats in water with half its volume immersed. When it floats in a mixture of equal volumes of water and another liquid, one-third of it is immersed. Find the specific gravities of the solid and the liquid.

5. A piece of cork 200 grammes in weight and specific gravity 0 25 is placed in a vessel full of water. How much water will overflow?

6. A cylinder density 2·8, one square centimetre in cross-section and 6 centimetres in length is dropped in a vessel containing mercury and water. Find its position of equilibrium.

7. A body weighs 110 grammes in air, 100 grammes in water and 90 grammes in a given liquid. Find the specific gravity of the liquid.

8. Find the length and specific gravity of a cylinder, which floats in water with 2 inches of its vertical axis out of it; and also in a liquid of specific gravity 1·5 with 6 inches, out of it.

9. Determine the sectional area of a wire, a piece of which, 1 metre long, weighs 24 grammes in air and 20 grammes in water. Find its density.

10. How much copper is contained in a rupee, if it

weighs 180 grains in air and $163\frac{1}{3}$ grains in water, taking copper as 9 times and silver 11 times as heavy as water?

11. A solid whose specific gravity is 1·85 is weighed in a mixture of alcohol (specific gravity ·82) and water. It weighs 28·8 grammes in air and 14·1 grammes in the mixture. Find the proportion of alcohol present.

12. A copper piece weighing 16 grammes is thrown in a graduated jar, the level of the water rises through 1·8 c.c. Calculate the specific gravity of copper.

CHAPTER XVI

THE ATMOSPHERE

73. All that has been said about liquids, applies almost equally well to gases. The only difference between gases and liquids is that the former have no free surface, while the latter have; and that the density of gases depends upon the pressure, while that of liquids is constant. Otherwise the principles of Archimedes and Pascal apply to gases as well as to liquids. Further, gases are subject to gravity like all other material media and thus exert on all bodies *pressures* depending on their depths and their densities. Although the density of air is very small, being only ·00129 gramme per c.c., yet the depth of the surface of the Earth below the surface of the Atmosphere is so great, that the pressure due to the air is very considerable. This pressure is termed *atmospheric pressure.* It is not perfectly constant and changes from place to place and from time to time depending on the local conditions of the weather. Its average value is about 15 lbs. *weight* per square inch or 1033 *grammes weight* per square centimetre.

It may appear paradoxical at first sight that such a large pressure should exist around us, without our being even conscious of the same. A little thought shows however, that the net upward pressure on a body due to the atmosphere is equal to the weight of the gas displaced by the body; just as a body immersed in a liquid experiences upward thrust equal to the weight of the liquid displaced by it.

The atmospheric pressure is exerted with perfect uniformity on all sides; and the only way to feel it,

is to remove, oy any device, the pressure from one side. The other side shall then experience the uncompensated pressure. Thus if air be drawn completely from any vessel, its outer side shall experience a very great pressure. Due to this, vacuum bulbs sometimes get cracked.

The easiest way to demonstrate atmospheric pressure is to fill a test tube with water to the brim; and then to put gently over it, a sheet of stiff card-board. On inverting the test tube, it is noticed that neither the cardboard nor the water falls down. The weight of both things is thus supported by the upward thrust of the atmosphere.

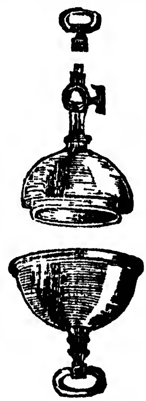


Fig. 66

The classical experiment to demonstrate atmospheric pressure is that of Magdeburg hemispheres. An air-tight receiver is formed by fitting two hemispheres closely, as shown in fig. 66. When there is air inside, the two can be separated without any exertion. If the air inside be exhausted, considerable force is required to separate the two. In Von Guericke's original experiment, a team of 16 horses was needed to pull the two hemispheres apart.

74. Measurement of Atmospheric Pressure.

Barometer, the instrument used for measuring the atmospheric pressures, consists of a tube about 36 inches long and closed at one end. It is filled with clean dry mercury ; and the open end is closed with thumb in such a way, as not to leave any air in it. On inverting it in a small trough of mercury and removing the thumb, it is noticed that the mercury in the tube stands about 30 inches above the mercury surface in the trough. The space

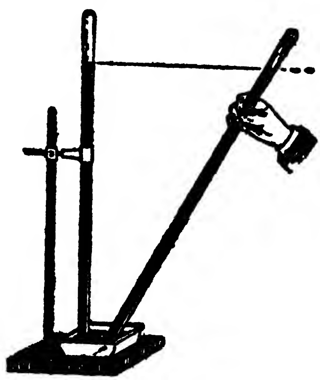


Fig. 67

above the mercury in the tube is a vacuum* called the Torricellian vacuum. Thus the pressure on the surface of the mercury column is zero; but the pressure inside the tube at the level of the mercury in the trough is equal to the product of the density of mercury and height of the column. This must also be the pressure on the mercury in the trough, for the mercury in the tube as well as in the trough is in free communication. But the pressure on the mercury in the trough is due to that of the atmosphere. Therefore the atmospheric pressure is equal to that of a column of mercury in the tube above that in the trough. This height is called *Barometric height.* Normally this is taken as 30 inches or 760 mms. There is very slight difference between the two. It is always the vertical height in the tube above that in the trough, which is the measure of the barometric height; for if the tube be inclined to the vertical as shown in figure 67, we notice that while the vertical height remains the same, the length of the tube filled with mercury increases. Of all the liquids, mercury is preferred on account of :—

(1) *Its great density.* It is 13·6 times as heavy as water, therefore a mercury barometer is not very long. A water barometer would be 13·6 times as long as a mercury barometer.

(2) *Its low vapour-pressure.* Mercury vapour exerts inappreciable pressure in the Torricellian vacuum. The vapour of any other liquid would exert considerable pressure.

(3) *Its purity.* Mercury can be had pure; while it is difficult to get any other liquid in the purer state.

(4) *Its lustre.* On account of this, it can be easily seen in the tube.

Fortin's Barometer. For finding the Barometric pressure practically with any accuracy, Fortin's Barometer,

* This space contains a little mercury vapour; but the pressure exerted by it is very small, and hence for ordinary purposes it is neglected.

as shown in figure 68, is used. The mercury cistern in this is made of leather and its bottom can be raised or lowered slightly by means of a screw. The zero of the scale coincides with the point of a small ivory index. The mercury surface is adjusted by means of the screw, until it just touches the index-point. Usually the scale is graduated only between 27 and 32 inches; and the reading of the barometric height is taken by the help of a vernier attached to it.

FIG. 68

The Aneroid Barometer. This form of barometer does not contain any mercury. It consists of an air-tight small box partially exhausted. The upper lid of this box consists of a thin metal diaphragm, which will bulge inside, more or less, according as the pressure of the atmosphere is greater or less than the normal pressure. The motion of the diaphragm is magnified by means of mechanical means and that indicates the pressure of the atmosphere. It is calibrated by comparison with a mercury barometer.

This instrument has the advantage of being very easily portable; for it is generally made of small size and convenient shape and on account of the absence of any liquid, there is no danger of its being spilt. The disadvantage is that it is impossible to ensure the same accuracy as in a mercury barometer so much so that it is essential to verify the calibration every now and then.

75. Measurements of heights by a Barometer. As the pressure of the atmosphere is due to the weight of air above the mercury in the trough, Pascal predicted that the mercury would fall if a barometer were taken to a mountain-top. In 1648, Clermont verified this fact and found the height of Puy-de-Dome. To find out by this method, the difference in heights between two points, we take the barometric

readings and from this the weight of the column of air between these two points is calculated. Dividing this by the density of air, we get the height,

$$H = \frac{h_2 - h_1}{d} \times 13{\cdot}6,$$

where H=height between the two points; h_1 and h_2 the barometric readings and d the density of air—13 6 being taken as the density of mercury.

In this consideration, we have assumed the air to be of uniform density throughout; but this is far from being true. As we go up, the density goes on decreasing. Further, we have made no allowance whatsoever for the pressure of the aqueous vapours present in the atmosphere. Due to these, the above formula does not give very satisfactory results, especially when the distance between two points is very great. *To get better results, "Multiply the difference between the logarithms of the two barometric readings by two millions. The result would be the height required in centimetres."* The mathematical treatment of this is outside the scope of this book.

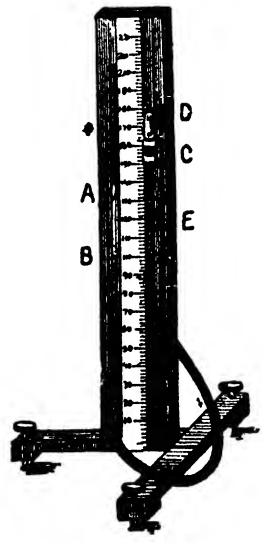


FIG. 69

76. Boyle's Law. The pressure exerted by a given mass of gas at constant temperature is inversely proportional to its volume. The law can be easily verified with the help of the apparatus shown in figure 69. The gas to be experimented upon is enclosed in the closed tube AB, its volume is proportional to the length of the tube occupied by the air and this can be read directly on the vertical scale. The pressure is found by reading the height of mercury surface in the open tube D above B and then adding to this the barometric height

By raising or lowering the tube, it would be found that the product of the volume and pressure of the given mass is constant, provided the temperature remains the same throughout. The instrument may be used for verifying Boyle's law, for pressures less than the atmospheric, by lowering D below B. In this case, the pressure is given by subtracting the difference in height between B and D from the barometric height.

Thus from Boyle's law, we have

$$PV=K=P'V'.$$

A perfect gas is defined as one, which obeys Boyle's law absolutely. No gas is perfect in this sense; for all gases show considerable divergence from this law, near their point of liquefaction. Air, oxygen, hydrogen and nitrogen are however, considered as perfect gases for they obey Boyles' law over a greater range. It should also be noted from the above considerations that the density of a gas is proportional to its pressure.

To show that elasticity of a gas is equal to its pressure:—Let a given mass of gas at pressure P and volume V be subjected to an increased pressure $P+p$ and let its volume decrease by a small amount v. Then by Boyle's Law, we must have

$$PV=(P+p)(V-v)$$

$$PV=PV+pV-Pv-pv$$

i. e. $pV=Pv$, neglecting pv the product of two small quantities,

$$\text{or } P=\frac{p}{\frac{v}{V}}=\frac{\text{stress}}{\text{strain}},$$

i. e. Elasticity at constant temperature, in the case of gases obeying Boyle's law is equal to the pressure.

Boyle's law deduced mathematically. Suppose a volume V of a gas under a pressure P is subjected to a pressure P' and the volume is reduced to V'. Then increase of pressure is equal to $P'-P$ and decrease in volume is equal to $V-V'$. Elasticity is given by $\dfrac{P'-P}{\frac{V-V'}{V}}$, *i. e.* the ratio of stress to

strain and this is equal to P';

$$\therefore \frac{P'-P}{V-V'}=\frac{P'}{V} \text{ or } PV=P'V' ;$$

SUMMARY

1 The Barometer is an instrument used to measure atmospheric pressure. Its various forms are :—(*a*) **Fortin's** and (*b*) **Aneroid.**

2. The empty space above the mercury column in a barometric tube is called the **Torricellian** vacuum.

3 To find the height between two points in centimetres accurately, multiply the difference between the logarithms of the two barometric readings by two millions

4 **Boyle's law.** The volume of a given mass of a gas varies inversely as the pressure, provided the temperature remains constant, *i.e* $PV=K$.

EXAMPLES

1. The air in the closed limb of a Boyle's law tube occupies 8 c.c at a pressure of 75 cms of mercury. What would be the pressure, when it occupies a volume of 10 c.c.

$$PV=P'V'$$

$$\text{or} \quad 75\times8=P'\times10 \text{ or } P'=60.$$

2. A cylinder filled with air at atmospheric pressure (76 cms) is plunged in water with its mouth downwards. At what depth will it be half full of water?

$$PV=P'V'$$

$76\ V=P'\ ^1/_2 V$ (for the air is made to occupy half the volume)

$\therefore\ P'=152$ cms of mercury column.

The atmosphere exerts a pressure equal to 76 cms ; therefore the pressure due to water should be equal to 76 of mercury column or $76\times13{\cdot}6$ cms. of water-column

3. A barometer reads 30 inches and the Torricellian vacuum is 5 inches. If a bubble of air, which at normal pressure would occupy 5 inches of tube be introduced, what would be the reading?

$$PV=P'V'$$

$30\times5=x(5+x)$; for suppose the mercury column goes down by x inches, then the volume of the gas would be $5+x$ and the pressure due to it equal to x.

$$\therefore\ 150=x^2+5x$$

$$\text{or} \quad x^2+5x-150=0$$

$$\text{or} \quad x=10$$

Therefore the column would go down by 10 inches or the mercury will stand at 20 inches.

4. A barometric tube contains air in the Torricellian vacuum. The mercury stands at 72, when the space above is 10 cms.: and it stands at 70, when the space above is reduced to 6 cms, by lowering the tube in the cistern. Find the true pressure.

By Boyle's law PV is constant

$$10(P-72)=6(P-70)$$

$$4P=300$$

$$P=75.$$

5. The tube of a barometer has a cross-section of 1 sq. cm. and when the mercury stands at 77 cms. the length of the vacuous space above is 8 cms How far will the mercurial column be depressed, if one cubic centimetre of air be passed into the tube.

6. Prove that a pressure of one megadyne (10^6 dynes) per sq. centimetre corresponds almost exactly to a barometric height of 75 cms.

7. If a mercury barometer falls 2 inches, find the fall of the water barometer.

8. A barometer tube contains air above the mercury column. The mercury stands at 25 inches, when the space above is 6 inches; and at 24 inches when the space above is 5 inches. Find the true atmospheric pressure.

9. In a barometer containing air, mercury stands at 70 cms. when the space above is 20 cms.; and at 65 cms. when the space above is reduced to 10 cms., by lowering the tube in the cistern. Find the true pressure?

10. A good barometer reads 75 cms.; on admitting 1c c. of air, the reading is 70 cms. Find the volume of the space above the mercury at the end. (P. U. 1931)

CHAPTER XVII

HYDROSTATIC MACHINES

77. Siphon. It consists of a bent tube ACB open at both ends as shown in figure 70. It is used for pouring a liquid from one vessel at a higher level to another at a lower one, without disturbing the vessels themselves. To put the instrument in operation, air is exhausted out of it; either by sucking with the mouth from the lower end or by filling it with the liquid, contained in the vessel which is to be emptied. A steady flow of liquid from A to B will be maintained so long as the end A is inside the liquid, and the level of the liquid in A is higher than that in B.

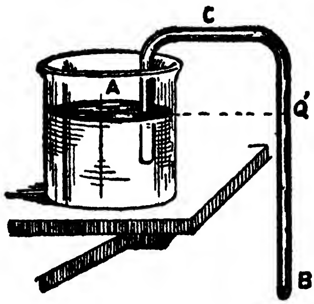


FIG. 70

To explain the action of the siphon, let us consider the pressures at A and Q' in the same horizontal line. The atmospheric pressure acting at A and B tends to raise the liquid in the two limbs of the siphon to the same height h, the barometric height for the given liquid. Thus the net pressure at A urging the liquid to move in the upward direction is equal to h; similarly the net pressure at Q' urging the liquid in the opposite direction is equal to $h-y$, where y is the level of Q' above the level of B. Now h being greater than $h-y$ the pressure on the side A overbalances the pressure at Q'. Thus the liquid would continue to

flow from A to B so long as A is at a higher level than B. The height AC must not be greater than the barometric height of the liquid to be emptied, for the column AC is to be supported by the atmospheric pressure.

In order to maintain a continuous flow of a liquid from one vessel to another, the following conditions must be fulfilled:—

(*i*) The end A must dip in the liquid.
(*ii*) The height AC must be less than the height CB.
(*iii*) The height AC must be less than the barometric height for the given liquid.

78. The Water Pump. It is an instrument for raising water from a lower level to a higher one. It is shown diagrammatically in fig. 71, one end of the tube T dips into the liquid to be raised, while the other end is soldered to a bigger tube. At the junction is a valve, which opens if the pressure be applied from below and is closed if it be applied from above. The bigger tube is fitted with a water-tight piston, having a valve of similar nature as above. There is also a spout for the water to go out from the bigger tube. To understand the action of the pump, suppose the piston in the bigger tube is moved upwards from the junction of the two tubes. The air in the smaller tube shall force open the valve and fill the vacuum created by the motion of the

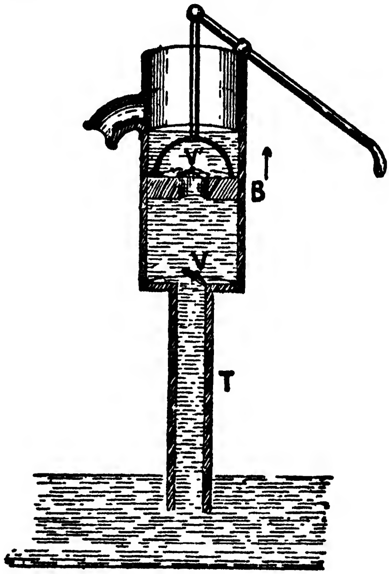


Fig. 71

piston *B* upwards; but in so doing its pressure would fall because by Boyle's law, $P \times V$ is constant. Thus if V increases P must decrease. Hence the atmospheric pressure acting on the surface of water will force some of it into the smaller tube. Next suppose, the piston is moved downwards. The valve at the junction of the two tubes will close, but the air between that valve and the piston shall force open the valve in the piston and escape. On moving the piston upwards again, some more liquid will be forced into the tube. On repeating this series of movements, water would rise up in the bigger tube, provided the height of the smaller tube above the surface of the liquid is not greater than its barometric height. Further suppose, that the bigger tube is full of water and the piston is at its top. On moving it downwards, the water comes up the piston through the valve. At the next stroke, this is bodily lifted up and escapes through the spout. The piston is generally worked by a lever.

78. (*a*) **The Force-Pump.** As shown in fig 72, it is like the water pump with this difference that there is no valve in the piston itself; but there is a side-tube and a valve at the junction of the two. This valve opens when force is applied from the side of the bigger tube. The action and working is similar to the water pump; but when the piston is moved downwards, the pressure of the piston forces the water up in the side tube This device is used when it is required to carry water to heights greater than the barometric height for water.

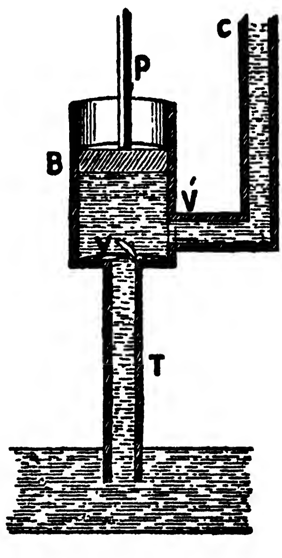


FIG. 72

79. Air Pump. This consists of a cylinder A called the barrel. In this, works a piston P having a valve F opening upwards. At the bottom of the barrel is another valve also opening upwards and closing a pipe OD, connected at the other end to the receiver E to be exhausted. To understand the action, suppose the piston is moved upwards in the barrel, air from the receiver would go into the barrel and thus its pressure would be decreased. On the

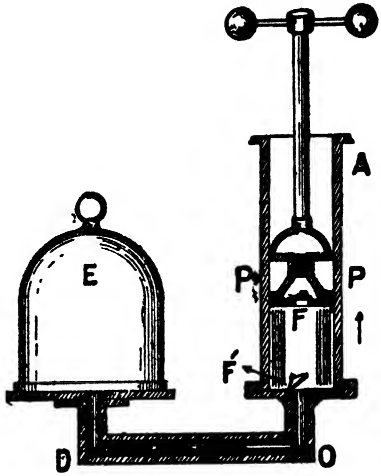


FIG. 73

downward motion of the piston, the valve at the junction of the barrel and the tube is closed; and the air in the barrel escapes through the valve F in the piston itself. Thus at each stroke, the air in the barrel is withdrawn. By repeated strokes, the receiver may be exhausted of air. In practice however, the pressure cannot be reduced beyond a certain limit, for the pressure of the air becomes so low that it becomes incapable of working the valve.

To determine the pressure of the air in the Receiver after any number of strokes :—

Suppose V be the volume of the receiver and the connecting tube,

v be the volume of the barrel,

P be the initial pressure of the gas in the receiver, and P_n be the pressure after n strokes.

After the first stroke, we have by Boyle's law,

$$PV = P_1(V+v) \text{ or } \frac{P_1}{P} = \frac{V}{V+v} \quad \ldots\ldots(i)$$

where P_1=Pressure after one stroke.

After second stroke, we have by Boyle's law

$$P_1V=P_2(V+v) \text{ or } \frac{P_2}{P_1}=\frac{V}{V+v} \qquad \ldots\ldots (ii)$$

where P_2=Pressure after second stroke.

Multiplying equation (i) and (ii) we have $\frac{P_2}{P}=\left(\frac{V}{V+v}\right)^2$.

Similarly after n strokes the pressure P_n will be given by

$$\frac{P_n}{P}=\left(\frac{V}{V+v}\right)^n.$$

Geissler's Air Pump. This pump, in its simplest form, consists of two big glass reservoirs of considerable capacity. One of these A (fig. 74) is fixed, while the other B can be raised or lowered at will. The two are connected by a strong India-rubber tubing. The reservoir B is open to the atmosphere, while A is fitted with an air-tight stopcock above and communicates with the receiver to be exhausted through a pipe, which can be closed by a tap D. The reservoir B in its lowest position is filled with mercury. Stopcock D is closed and C opened. B is raised slowly to such a height that mercury-level stands just upto the stopcock C, which is then closed and the reservoir B moved downwards till the mercury falls below D in the tube connected to A. D is then opened, the air from the receiver fills A and the mercury column comes downwards. D is then closed and B is again raised till the air in A is again compressed to nearly atmospheric pressure. At this time, C is opened and B is

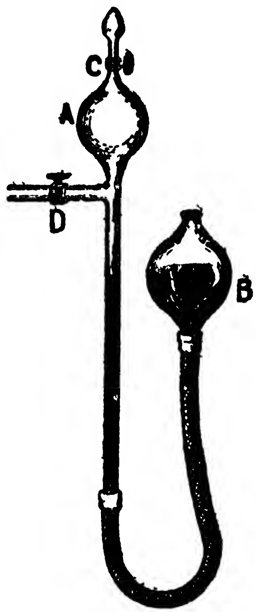


Fig. 74

still further raised so that mercury in A stands just upto C. C is closed and the process repeated. In this way high degree of exhaustion is attainable.

In this instrument, closing and opening of the taps every time is a tedious affair. Toppler's pump constructed after Geissler's model, obviates the difficulty of taps by slight modification.

Pressure Gauges. It is an instrument for measuring the pressure of a gas or vapour. The most usual form of this is a siphon manometer. It consists of a bent glass tube (figure 75), containing usually mercury or eiosine water. The end D is open to the atmosphere, while the end C is connected to the vessel of which the pressure is to be measured. If the pressure in C is greater than the atmospheric, the liquid in D would rise; otherwise, the reverse would take place. The difference in pressure on the two sides (C and D) is given by the difference in levels of the liquid in the two limbs of the guage.

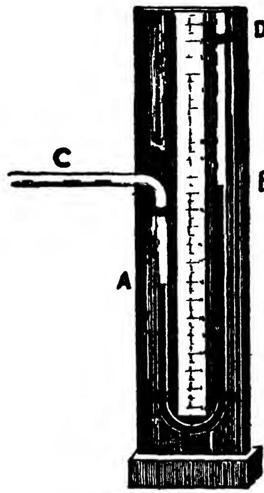


FIG. 75

SUMMARY

1. The pressure in the receiver of an air pump after n strokes is given by $\frac{P_n}{P}=\left(\frac{V}{V+v}\right)^n$, where v=the volume of barrel, V=volume of receiver, n is the number of strokes and P the initial pressure.

2. For high vacuum. **mercury air pump** is the instrument that can be used successfully.

EXAMPLES

1. In an air pump, the volume of the receiver is 150 c.c. and that of the barrel is 30 c c. Find in what ratio the density is reduced after 4 strokes.

$$\frac{d_n}{d}=\frac{P_n}{P}=\left(\frac{V}{V+v}\right)^n=\left(\frac{150}{150+30}\right)^4=\left(\frac{5}{6}\right)^4$$
$$=625:1296$$

2. To what depth must a diving-bell 6 feet high be immersed, so that the water may rise 4 feet within it, height of water barometer being 34 ft.

$6\times 34=2x$ for volume of air is reduced to 2 feet only.

$$\therefore \quad x=102$$

The depth below must be $102-34=68$ feet.

3. A bubble of air 1 mm. in diameter starts from the bottom of a lake 1033·6 centimetres deep. Find its size on reaching the surface.

The volume of the bubble $=\pi r^2$

$=\pi(1/2)^2$ cubic millimetres at the bottom of the lake;

and the pressure there $=76+\frac{1033·6}{13·06}$ cms of mercury column.

This is equal to 152 cms

By Boyle's law PV is constant. On the surface, the pressure is only equal to 76 cms of mercury column Let the radius of the bubble be denoted by r, then we have

$\pi\times 1/4\times 152=\pi r^2\times 76$

$$\text{or } r=\sqrt{1/2} \text{ and the diameter}=\frac{2\sqrt{2}}{2}=\sqrt{2}$$
$$=1·4142 \text{ mm.}$$

4. What is the greatest height of the shorter arm of a siphon, which can permit of the flow of (*i*) mercury and (*ii*) alcohol.

5. One limb of a U-tube is connected to the gas tap while the other is open. The difference of water-levels, in the two limbs is equal to 33·4 cms. Find the pressure of the gas in terms of the water-column—density of mercury being 13·6 and barometric pressure 76 cms.

CHAPTER XVIII

AVIATION

81. Aviation. Flying in air has been attempted in two ways:—(*i*) by means of machines 'Heavier than Air', commonly called aeroplanes and (*ii*) by means of machines "Lighter than Air", known as balloons. From earliest times, man has seen birds fly and has wondered how they accomplish this feat. Early attempts in this direction were made with balloons but with partial success.

(*a*) The balloon floats in air for the same reason as a boat floats in water. The weight of the volume of air displaced by a balloon is greater than the weight of the same volume of hydrogen gas with which balloons are usually filled.

(*b*) The aeroplane floats in air for the same reason as a boy's kite or a bird; *i.e.* by reason of the upward pressure of the current of air passing under it.

(*c*) A balloon affords a means of rising high in the air, but the ambition of human race from the beginning has been to fly in the air and not simply to rise. In order to achieve the latter object, *i.e.* of flying in air, a machine propeller is fitted to the big balloon and a rudder attached to direct its course. With a view to counterbalance the weight of this additional machinery, the balloon is made much bigger. Such a machine is called the *Dirigible Balloon or the Airship.* This is a combination of a balloon and an aeroplane. In order to reduce the resistance of the air to motion, the balloon is given the form of a cigar and is propelled by gasoline engines, which for a given weight give the maximum.

of power. Zeppelin and Wellman are the most important forms of airships.

Of late years, Airships have developed very rapidly for they can lift without being in forward motion and can hover over a given point. They are now used for comparatively heavy transport. The largest airships have a gross lift of 100 to 110 tons and a speed upwards of 60 miles. One such British airship R 101, provided sleeping accommodation for about 50 passengers. After a successful tour to the U.S.A., it met with a mishap in France on its way to India, which resulted unfortunately in the death of about 40 souls including that of Lord Thomson, the Air Minister.

To investigate fully the conditions under which a kite remains in equilibrium, let us refer to fig. 76. *AB* is the kite, which is held in position by the string *CE*. The forces acting on it are:—

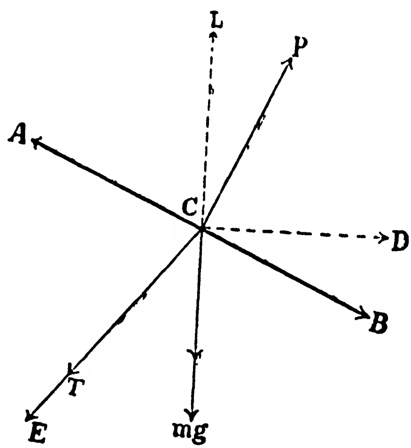


Fig. 76

1. Its weight *mg* acting vertically downwards.

2. The pull *T* of the string *CE*, acting in the direction *CE*.

3. The pressure *P* of the wind.

If these three forces can be represented by the three sides of a triangle taken in order, then the kite will remain in equilibrium; if however, it is not possible then the kite cannot be in equilibrium. In the latter case, the kite will move. To find out the direction of motion, let us suppose that the pressure *P* of the wind is resolved into two components *L* and *D*,

in the vertical and horizontal directions respectively. The vertical component of the pressure is called the 'Lift' and the horizontal component is called the 'Drift.' When the lift exceeds the weight of the kite and the downward component of the pull, it will rise upwards; if the lift is less than the weight and the downward component of the pull, the kite will fall down.

The lift increases as (*i*) the pressure of the wind increases and (*ii*) the inclination of the kite decreases. Thus to make the kite rise upwards, jerk (*tunka*) is given to it by the string; and this increases the pressure of the wind and decreases the inclination of the kite.

It makes no difference whether the wind, which passes under the planes supporting the aeroplane, is due to natural causes or artificial means, such as the speed of the apparatus through air by mechanical means. The underside of an aeroplane is concave and the upper side convex. The supporting plane, when the machine is flying through the air is usually inclined to the horizontal at an angle of 8 degrees. This slight inclination along with its concavity, allows the air through which it is passing to enter the concave surface and to exert upward pressure. This is, in short, the cause of the aeroplane floating.

It may be observed however, that while the upward pressure varies as the square of the velocity; the power required to produce the velocity varies as its cube.

There are two forms of aeroplanes, known as the 'Biplane' and the 'Monoplane.'

(*i*) The Biplane consists as shown in fig 77 of two planes, arranged vertically one above the other; together with the elevating plane, a tail plane *T*, steering arrangement and the engine propeller and accessories. From the middle of the lower plane, stretches out a construction (known as fusellage) to the rear. Its function is to carry the tale-plane and the rudder.

The elevator or the elevating plane, projects in

front of the two main planes. It is supported from the central platform of the lower plane by outriggers and its movements up or down are controlled by wires attached to levers. The elevator and the tail plane assist to support the machine as a whole. The engine with its accessories is fixed in the middle of the lower plane. The propeller, which is generally worked directly by the engine is fixed at the rear of the lower plane. Sometimes the Biplane is provided with hanging flaps attached to the outer ends of the upper plane; these are called (*ailerons*).

(*ii*) The Monoplane is very much like a bird in

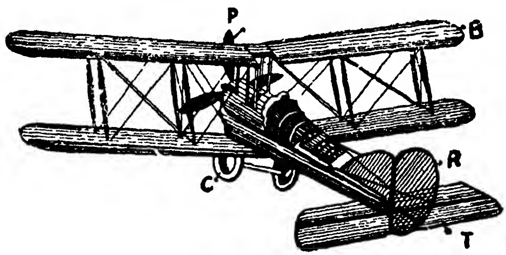


FIG. 77

form. The propeller is fixed in front of the machine, where the bird's head would be. At the other end is the combination tail, forming both the rudder and the elevator.

Rise and fall of aeroplane.—The aeroplane is made to move upward or downward by the aid of the elevator, which consists of a cambered surface stretched over a metal or wooden frame. When the elevator is tilted upwards, the pressure of the air on the underside, when it is carried in front and on the upperside, when it is carried in rear, turns the front of the machine upwards. But when the elevator is turned downwards, the pressure of the air, on its upper side, when it is carried in front and on the lower side, when it is carried behind, causes the front of the machine to turn downwards.

Turning to right and left.—The aeroplane is turned to right or left by the aid of a vertical rudder, which is always carried on the tail. Turning the rudder to the right turns the head of the machine to the right and turning it to the left, turns the head of the machine to the left.

The propeller.—The propeller is similar in shape to the blades of an electric fan. It is revolved at a very rapid rate of about 1000 revolutions per minute or over. A propeller may have 2 or 4 blades. Just as a screw cuts its way through the wood, so do the screw-blades forming the propeller cut their way through the air or water, and force the aeroplane forward.

The landing apparatus or chasis.—It consists of two or four bicycle wheels with comparatively large pneumatic tyres. Two wheels are generally fixed on each side of the aeroplane. The wheels can move in any direction and are shock absorbers. It is raised above the ground, when the velocity of the machine is so great as to exert upward pressure, greater than the weight of the machine.

EXAMINATION QUESTIONS IV

1. What is a hydrometer? Explain the construction and principle of a constant-weight type hydrometer.

2. What is Boyle's law? How is it proved experimentally? Mercury used in the construction of a barometer has air dissolved in it? What will be the consequence of this? How will you measure correct Atmospheric pressure with such a barometer?

3. Explain the action of a siphon. What is the greatest height, over which it is possible to carry turpentine?

4. Explain with the help of a neat diagram how a boy's kite flies? What is the difference between a *dirigible* and an *aeroplane*?

5. Describe a mercury air pump. Why does an ordinary

pump fail to produce high vacuum?

6. A bottle filled with air at atmospheric pressure is plunged mouth downwards in sea-water. At what depth will it be one-fourth full of water?

Density of mercury = 13·56

„ „ sea-water = 1·13.

HEAT

CHAPTER I

INTRODUCTION

82. Temperature and Heat. You are all familiar with the ordinary sensations of *heat* and *cold.* When you touch any substance with your hand, you can at once say, whether *it is* hot or cold; thus our unaided senses are sufficient to distinguish between conditions termed as hot or cold. It must however, be clearly understood that the knowledge, we get by our senses, of the hotness of a body is only relative and cannot be relied upon absolutely.

Experiment.—Take three beakers A, B and C containing cold, luke-warm and hot water respectively. Immerse your left-hand in cold water and your right-hand in hot water. Now immerse both hands into B containing luke-warm water. You will notice that the left-hand feels warm, while the right-hand feels cool.

Although in particular cases, our senses may sometimes deceive us, yet our ideas of hotness and coldness are derived primarily from our senses. *This hotness or coldness of a body in scientific language is spoken of as its temperature.*

Further we know that when two bodies at different temperatures are placed in thermal communication, heat flows from a body at a higher temperature to one at a lower temperature, *i.e.* difference of temperature is a necessary condition for the flow of heat. *Thus temperature is defined as the thermal condition of a body which determines, whether it will give heat to or receive heat from another, when placed in contact with it.*

Now the question arises, what is it, which brings

about these differences of temperature in bodies? *The agent, which brings about these differences of temperature is termed Heat.*

Thus, when a body is placed in the Sun or in a furnace, it absorbs heat and its temperature rises. What is the nature of this agent, known as Heat? It will be discussed at some length in Chapter X, but it will not be out of place to say that it is a form of energy. Heat and temperature are very closely related to one another, for generally whenever a body absorbs heat, its temperature rises and whenever it loses heat, its temperature falls. Though heat and temperature are seen to be closely connected, it is very important to distinguish clearly between the two.

Distinctions between Heat and Temperature:—

1. *Temperature refers to a particular thermal condition and is quite distinct from the quantity of heat, which a body possesses.* Thus a red-hot needle is at a very high temperature as compared to sea-water; although the total quantity of heat is enormous in the latter case and that possessed by the needle is negligibly small.

2. *Temperature alone determines the direction of flow of heat.* Thus in the above example, if the red-hot needle be thrown into the sea, heat will flow from the needle to the sea-water.

To make the distinction between heat and temperature still more clear, we may say that heat corresponds to the quantity of water in a vessel and temperature to the level of the liquid in it. Just as water flows from a higher to a lower level, similarly heat flows from a body at a higher temperature to one at a lower temperature. Or we may say that heat corresponds to the quantity of electricity and temperature to potential. And just as electricity flows from a body at a higher potential to one at a lower potential, similarly does heat flow from a body at a higher temperature to one at a lower temperature.

83. Effects of Heat. The effects produced by heat on different substances are very diverse and depend upon the circumstances, under which heating or

cooling takes place. Some of the more important effects are the following:—

(*i*) *Change of temperature.* Whenever the state of aggregation of a body does not change, its temperature increases with gain and falls with loss of heat. It should be clearly noted however, that there is no rise or fall of temperature with the gain or loss of heat, when the body is changing its state. Thus when heat is applied to melting ice, there is no rise of temperature, till the whole of it is melted.

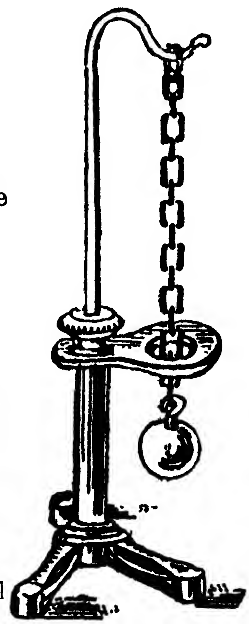


FIG. 1

(*ii*) *Expansion.*—It is found that with a few exceptions * all bodies expand with the rise of temperature and contract with the fall of temperature. The amount of expansion varies with the nature of the substance, the rise of temperature and its initial volume. In general, gases expand more than solids for the same rise of temperature.

(*a*) **Expansion of solids.** Take Gravesand's ring as shown in fig. 1. It consists of a ring and a solid sphere, which just passes through the ring, when both are at the same temperature. Heat the ball over a Bunsen's flame and place it on the ring. Observe that it does not pass through the ring as it has expanded. Allow it to cool; it contracts and falls through.

(*b*) **Expansion of liquids.** Take a small flask fitted with a rubber stopper through which passes one end of a tube of fine bore, about 6 inches in length. Fill the flask completely with coloured water and insert the stopper. Some of the liquid will be forced up into the tube. Now heat the flask gently by putting it into a beaker of hot water. Notice that the level of the liquid rises in the tube, thus showing that it

*Water very near its freezing point and stretched India-rubber tubing contract, when heated.

expands when heated. On cooling, the liquid-column in the tube falls.

(c) **Expansion of gases.** Take a flask similar to the one used to demonstrate the expansion of liquids. Introduce a short column of mercury into the stem, so as to separate the air in the bulb from the outer air. Now warm the bulb slightly. Notice that the mercury column is pushed upwards, showing that the gas expands when heated. On cooling, the mercury-thread in the tube falls

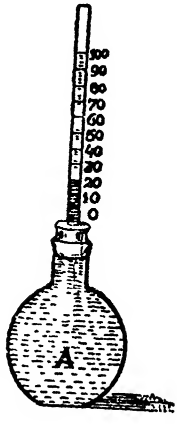


FIG. 2

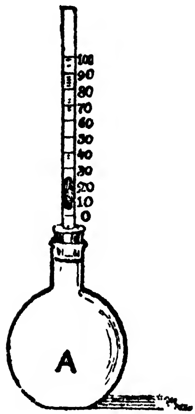


FIG. 3

(*iii*) *Change of State.*—Whenever heat is given to a substance, then under suitable circumstances, it may change from solid to liquid or from liquid to gaseous state; and if the heat were taken away the reverse changes may take place. Thus when heat is applied to ice, it melts into water, the temperature remaining constant during the process. This heat, it may clearly be understood, does not increase the molecular kinetic energy; but is spent up in bringing about a complete molecular re-arrangement. It does not increase its temperature and hence is called *latent* or hidden. If more heat be supplied, the temperature of the water so formed rises till it begins to boil; when again the temperature remains stationary and the liquid is converted into vapour. During the process of conversion, heat is utilized to augment the potential molecular energy. The first process, *i.e.* conversion from the solid to liquid state is known as *fusion* or *melting*; and the latter, *i.e.* conversion from the liquid to vapour state is known as *vaporisation.*

(*iv*) *Change of Physical Properties.*—Most of the physical properties, such as elasticity, viscosity, surface

tension, conductivity, optical properties etc. are changed, when the temperature changes. Sometimes differences of temperature may give rise to an electric current as in a thermopile.

(*v*) *Chemical Effects.*—Heat promotes chemical action as is shown by the combustion of coal. Coal and oxygen may lie together for years, yet no action will take place till the temperature is raised to ignition-point.

SUMMARY

1. **Heat** is a form of energy, which brings about differences of temperature.

2. **Temperature** is the thermal condition of a body, which determines the direction of flow of heat, when it is placed in contact with another, kept at the standard temperature.

3. **Heat produces the following effects:—**

(*i*) Change of Temperature.
(*ii*) Expansion.
(*iii*) Change of State.
(*iv*) Change of Physical Properties.
(*v*) Chemical Effects.

CHAPTER II

THERMOMETRY

84. Thermometry. Our sense of touch no doubt gives us some idea of the temperature of a body, yet as has been pointed out in the first chapter, this sensation is not always a safe guide in estimating the degree of hotness or coldness of bodies.

Experiment.—On a *rather cool* day, touch a piece of metal and a piece of wool, which have been exposed to the atmosphere. The piece of metal being a good conductor absorbs heat more quickly than the piece of wool and hence the former feels cooler; although both are at the same temperature.

Thus we see that our sense of touch cannot be relied upon for the correct measurement of temperatures, as this is neither delicate nor reliable. Moreover, the measurement is influenced by the previous temperature of the hand.

Thermometer. Any instrument, which is a trustworthy mode of comparing or measuring temperatures, is called a *thermometer.*

Principle of a thermometer. We have seen in the Introductory Chapter that when heat is communicated to a body, not only does its temperature rise but other changes also take place simultaneously. Any *property* of matter, which *varies* continuously with the temperature might be used to measure *temperatures.* Thus linear or cubical expansion, electric resistance of a conductor or the electromotive forces generated at the junction of two unlike metals, may be used to measure temperature, which does not admit of direct measurement. Thermometers, based on each of the above changes, have in fact been constructed and are used for special purposes. They will

be considered at their proper place in this book. But the effect, which is most commonly used for the purpose of measuring temperatures is the expansion of liquids. Solids expand little, while gases expand too much and liquids on account of moderate expansion are best adapted for the purpose.

N. B.—A thermometer actually denotes the temperature of the substance, used in its construction; but it is so arranged that the substance acquires the temperature of the medium in which it is placed.

Experiment—Take a glass tube of fine bore with a bulb blown at one end as shown in fig. 4, let the bulb and part of the tube be filled with a liquid. Place this in a beaker of water and heat it. Notice that the level of the liquid in the tube rises continuously with the rise of temperature. Now if the water cools down to its original temperature, the level of the liquid in the tube will be the same as in the beginning.

If we were to make a series of marks on the stem, then each one of them would denote a definite temperature and the instrument would no doubt serve as a thermometer; but such a thermometer shall have the following disadvantages.

(1) Upper end of the stem being open, foreign substances might enter the tube and thus cause trouble.

(2) Some of the liquid in the stem and the bulb will leave them by evaporation.

Fig. 4

(3) The readings being arbitrarily marked, would not be intelligible to others.

To obviate the above disadvantages, the upper end of the stem is closed, this remedies the defects numbers (1) and (2) above. To remedy defect number (3), two marks known as the fixed points are made on the stem. These indicate two well-known and standard temperatures, *i. e. the temperatures of melting ice and boiling water* under normal pressure. The distance between these two marks is divided into a definite number of equal parts known as degrees, provided the bore of the stem

is uniform. The liquid used is generally mercury. The reasons for selecting it will be described at its proper place. Such a thermometer is called a Mercury-in-glass Thermometer.

85. Mercury Thermometer. It consists of a narrow, thin-walled, cylindrical glass bulb and stem of a thick-walled capillary tube of uniform bore closed at the upper end. The stem terminates at its upper end in a small bulb, which allows mercury space for expansion in case the thermometer is heated beyond the range for which it is intended. The graduations are etched on the stem and are filled with black paint. A layer of white enamel is generally embedded at the back to serve as a white background in order to see the mercury easily.

To construct a thermometer, a length of thick capillary tubing of *uniform bore** is selected. It is then thoroughly cleaned by washing it with nitric acid and distilled water. It is then dried by passing hot air from the drying apparatus. A bulb is then blown at one end and small thistle funnel at the other.

FIG. 5

The next operation is to fill the bulb and a part of the stem with mercury. To achieve this, the funnel is filled with pure dry mercury, which does not go down into the stem as the bore is too narrow. To do this the bulb is gently heated, the gas imprisoned in the bulb and the stem expands; and a portion of it escapes. The bulb is allowed to cool, the pressure of the inside air decreases and a small quantity of mercury is forced into the bulb. This process of alternate heating and cooling is repeated several times until the bulb and a small portion of the stem are filled with mercury. The mercury is then boiled

* It is easily done by introducing a small thread of mercury and then measuring its length at various points. If it is the same at all points then the bore is uniform, otherwise not. In the latter case the tube must be discarded and another of uniform bore selected.

in order to drive off the air and the moisture inside it. With a view to seal the upper end of the thermometer, the instrument is placed in a bath, the temperature of which is slightly above the highest temperature, it is intended to measure. A small pointed flame is then directed near the upper end, to fuse the glass and thus close that end.

The next operation is the determination of fixed points. The convenient fixed points are those corresponding to the temperature of pure melting ice and the boiling point of water under normal atmospheric pressure (760 mms. of mercury at sea-level in latitude 45°). The marking of the fixed points should preferably be postponed for at least a week after the thermometer is sealed off.

The *lower fixed point* is determined by placing the bulb and part of the stem in melting ice, which has been formed by freezing distilled water, as shown in fig. 6. The point on the stem, where the mercury thread is stationary for a time, denotes the *Freezing Point* and is scratched.

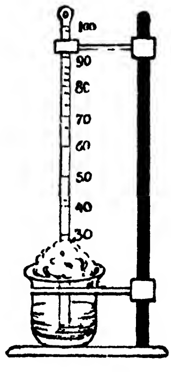


FIG. 6

The *upper fixed point* is determined by surrounding the bulb and stem by steam issuing from boiling water. For this purpose, the instrument known as hypsometer, as shown in fig. 7 is used. The apparatus is simply a copper boiler, having two concentric tubes so arranged that the steam ascends the inner tube and descends through the space between the two. Thus the steam in the inner tube is prevented from cooling and also the inner tube is maintained at the temperature of the steam, and errors due to loss of heat by radiation are avoided. The thermometer tube, the upper fixed point of which is to be marked, is thrust through the India-rubber cork. The whole of the stem upto the extremity of the mercury column and the bulb should be surrounded by steam. The bulb on

no account should touch the boiling water, for the same is greatly affected by the presence of impurities in it, while the temperature of steam is not very much altered for that reason. When the level of the mercury in the stem has become stationary, a fine scratch is marked to denote the upper level of the mercury meniscus and the atmospheric pressure is immediately noted. If it is normal, the scratch marks the upper fixed point. As the boiling point of water depends upon pressure, correction shall have to be applied if the atmospheric pressure is not normal. The temperature of boiling water is given with sufficient accuracy by the formula $T=100-{\cdot}037\,(760-h)$, where h denotes the barometric height in mms. of mercury column.

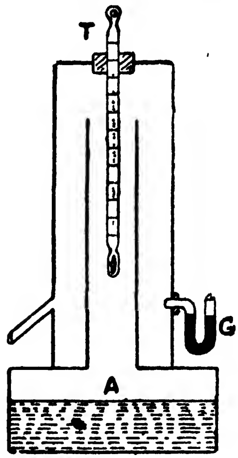


FIG. 7

Thus if l denotes the distance between the lower and the upper fixed points as marked, then the true upper fixed point shall be $\dfrac{l\times 100}{100-{\cdot}037\,(760-h)}$* above the lower fixed point. A new scratch is made at the point given by the above formula and that in fact denotes the true upper fixed point.

86. Graduation of a thermometer. The fixed points having been marked on the stem, a thermometer is graduated by dividing the distance between the two fixed points into suitable and recognized scales of temperatures and continuing this marking of divisions throughout the whole length.† There are

* Provided temperatures are measured on the c.g. Scale.

† To etch the divisions on the stem, the whole of the thermometer is covered over with wax. The divisions are made by means of a fine needle mark and the whole instrument is put into hydroflouric acid the acid does not affect the parts of glass covered over by wax but deep marks are etched on the exposed spaces. To make them easily visible, lamp black is embedded in them.

three scales of temperatures, which are more or less, in general vogue.

(1) **Centigrade.** The lower fixed point is marked zero, the upper 100 and the space between the two fixed points is divided into 100 equal parts known as degrees. The scale is extensively used for all scientific purposes.

(2) **Reaumur.** The lower fixed point is marked zero, the upper 80 and the space between the two fixed points is divided into 80 equal parts. This scale is used on the Continent of Europe, but is now getting out of use.

(3) **Fahrenheit.** The lower fixed point is not marked 0 but as 32, the upper fixed point is marked 212 and the space between the two fixed points is divided into 180 equal parts. This scale is used in Great Britain for domestic purposes.

Comparison of scales of temperature. The relation between the three scales is given by the following table :—

Name of scale.	Freezing point marked.	Boiling point marked.	No. of degrees between two points.
Centigrade (C)	0°	100°	100°
Reaumur (R)	0°	80°	80°
Fahrenheit (F)	32°	212°	180°

To convert temperatures from any one scale into another, we see that a difference of 100° on the Centigrade scale corresponds to one of 80° on the Reaumur

and 180° on the Fahrenheit. Thus a Reaumur degree is $\frac{100}{80}=\frac{5^\circ}{4}$ Centigrade and a Fahrenheit is $\frac{100}{180}=\frac{5}{9}$ of a centigrade degree. Thus a difference of temperature which would be represented by 5° on C will be represented by 4°R and 9°F. But the melting point of ice on both the C and R scales is represented by 0°, while on the Fahrenheit, it is represented by 32°. Thus a temperature F on the Fahrenheit is $F-32$ above the freezing point; and to convert it into centigrade, we must multiply $F-32$ by $\frac{5}{9}$ and to convert it into Reaumur, by $\frac{4}{9}$. If C, R and F were to denote the readings of the same temperature on the three scales we must have

$$\frac{C}{100}=\frac{R}{80}=\frac{F-32}{180} \text{ or } \frac{C}{5}=\frac{R}{4}=\frac{F-32}{9}.$$

Choice of the thermometric substance. The choice of a liquid for a thermometer is governed in the first instance by the use to which it is to be put and in the second, by the consideration how far it satisfies the properties required of an ideal thermometric substance, which may be enumerated as follows:—

1. It should be easily and cheaply procurable in pure state.
2. It should neither wet the glass nor stick to the walls of the capillary tube.
3. It should be distinctly visible in a capillary tube.
4. It should remain liquid over a long range of temperature.
5. It should have a large and uniform co-efficient of expansion.
6. It should transmit heat quickly either by conduction or by convection.
7. It should have low thermal capacity.
8. It should have low specific gravity.

The only substance possessing all the above properties is a gas, which obeys Boyle's law. The air thermometer to be described in the next chapter is thus the most perfect thermometer. Next to that comes *mer-*

cury between the temperatures of 0°C. and 100°C.*

88. Other Forms of Thermometers.

1. Alcohol thermometer. Sometimes alcohol is used instead of mercury in the construction of thermometers. *Its advantages over mercury are the following:—*

(*i*) It solidifies at $-130°$C. while mercury solidifies at $-39°$C. Thus it can be used for the measurement of very low temperatures.

(*ii*) It is cheap.

(*iii*) It expands much more, *i.e.* its expansion is nearly 10 times as much as that of mercury and is thus very sensitive.

(*iv*) Its thermal capacity is less than that of mercury.

(*v*) It moves smoothly in a capillary tube and does not stick to the walls.

It has however, the following *disadvantages* too—

(*i*) It boils at 78°C. and thus cannot be used for measuring high temperatures. For this reason, it is graduated by comparison with a standard thermometer.

(*ii*) Its expansion is not uniform.

(*iii*) It is colourless and thus has to be coloured to make it visible in a capillary tube.

2. Maximum and Minimum Thermometers.

These thermometers are used generally to record the highest and lowest temperatures during a given period of time. These temperatures are used for agricultural or meteorological purposes.

(*a*) **Six's Maximum and Minimum Thermometer.**

It is at once both a maximum and a minimum thermometer and it is perhaps the oldest of its kind. Its general shape is shown in fig. 8. The bulb A and part of the stem upto B are filled with alcohol. The U-shaped column of mercury BC is used to move the indices, which denote the lowest and the highest temperatures respectively. The small bulb E along with the stem upto C again contains alcohol, which simply protects the index on *maximum* temperature side. As the alcohol in bulb A expands or contracts with variations of tempe-

* The student should himself write down the advantages and disadvantages of mercury as a thermometric substance.

rature, the extreme points *C* and *B* respectively of mercury column move upwards along with their respective indices, which are prevented from returning by means of a spring. The position of the index on the *minimum* temperature side evidently indicates the lowest temperature, while that of the index on *maximum* temperature side denotes the highest temperature. It is important to note that the limb *B*, the index in which indicates the *lowest* temperature, is graduated from *top to bottom*, while the limb *C*, the index in which denotes the *highest* temperature, is graduated from *bottom to top*. To set the instrument ready for any observation, the two indices are brought in contact with the mercurial column by means of a small magnet. The construction of an index is shown separately. It is a dumb-bell shaped piece of soft iron, carrying a small side spring which prevents it from smooth movements in a capillary tubing; but allows it free movement, when the upward thrust of mercury or the force of magnetic attraction acts upon it.

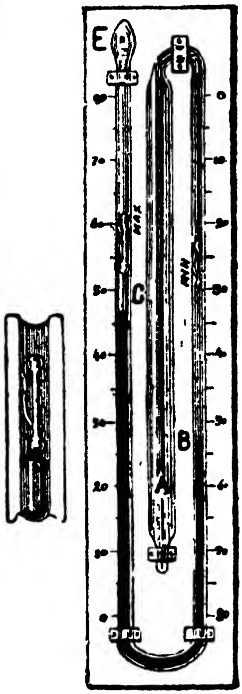


FIG. 8

(*b*) **Rutherford's Maximum and Minimum Thermometers.**

These are two separate thermometers, one to register

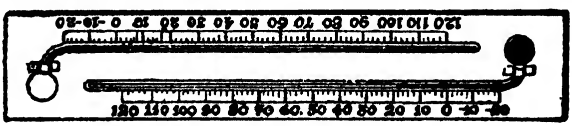


FIG. 9

the maximum temperature and the other to register the minimum temperature, set on one horizontal wooden board, as shown in fig. 9.

(1) The maximum thermometer is an ordinary mercury thermometer with a steel index, which is pushed forward as the mercury expands due to the rise of temperature. Thus the position of the index denotes the maximum temperature reached.

(2) The minimum thermometer is an alcohol thermometer, which also contains a similar light index. When the alcohol contracts due to the fall of temperature, its surface tension is sufficient to draw the index back with the meniscus; but on expansion it flows past the index without affecting it.

(*c*) **The Clinical Thermometer.** It is a kind of maximum thermometer. Just at the junction of the stem and the bulb is a narrow constriction as shown in fig. 10. To use the instrument, the mercury is transferred into the bulb by jerks and it is left to attain the maximum temperature. On expansion the mercury flows through the constriction, but on contraction, the column of mercury breaks at the constriction, and is thus prevented from flowing back into the bulb. It is generally used by physicians to find the temperature of a patient, for this purpose the thermometer is generally graduated between 95°F. and 110°F. by comparison with a standard thermometer.

89.* High Temperature Thermometers. A mercury thermometer is unsuitable for measuring very high or very low temperatures and a gas thermometer is very tedious to manage; therefore for measuring such temperatures, use is made of certain electric properties of a substance, which vary with the temperature.

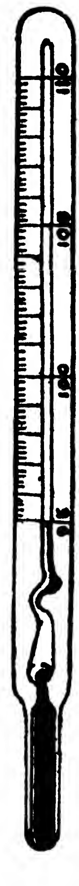


FIG. 10

(*d*) **Platinum Resistance Thermometer.** It has

* This section may be omitted at first reading.

been found that the resistance of a conductor increases continuously with the temperature. Thus if we were to determine the resistance of a given wire at two known standard temperatures and its resistance at another unknown temperature T were also determined then we can get the value of that temperature by the formula,

$$T = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

Because if a = the co-efficient of increase of resistance per degree centigrade, we have

$$\frac{R_{100} - R_0}{R_0 \times 100} = a$$

and also

$$\frac{R_t - R_0}{R_0 \times T} = a$$

Equating the left-hand side expressions, we have

$$T = \frac{R_t - R_0}{R_{100} - R_0} \times 100$$

(2) Thermopile. The fact that an *E.M.F.*, proportional to the difference of temperatures at the two junctions of a couple of two different metals, is generated, is made use of, for measuring temperatures. One junction of such a couple is generally maintained at the standard temperature of melting ice, while the other is allowed to be in contact with the body, the temperature of which is required. The *E.M.F.* generated is measured and from that the temperature is ascertained.

90. Sources of Error in a Mercurial Thermometer.

(*i*) *Change of zero.* If a thermometer tube, after being hermetically sealed, has not been kept for a long time before graduation; then as glass takes a considerable time to come to its original volume, when once heated to a high temperature, the volume of the bulb of the thermometer will continue to decrease due to contraction and consequently the column of mercury would stand higher than the graduation marked 0°, when the instrument is immersed in ice. To avoid this, the thermometer tube, before it is graduated, must be

kept for a long time after being sealed. To correct it however, all we have to do is to keep the thermometer in melting ice, note down its temperature and subtract the the reading from all further observations.

(*ii*) *Recent heating.* When a thermometer is used to measure high temperatures and is immediately afterwards used for measuring low temperatures, the bulb having had little time to contract, the thermometer, if it is immersed in melting ice will denote a temperature below zero. This error is avoided if different thermometers are used for measuring high and low temperatures.

(*iii*) *Exposed column.* If the whole of the bulb and part of the stem, upto which the mercury column stands, are not immersed in the substance the temperature of which is required ; then the temperature of the mercury column, which will be exposed to the atmosphere will not be the same as that of the substance and thus error might be introduced. To avoid this, the whole of the bulb and the stem up to which the mercury column stands, must be immersed in the substance

(*iv*) *Position and Pressure.* If the thermometer is read in positions and under pressures different from those under which it was graduated, then its readings will differ; hence it must be read in the position in which it was graduated and it should not be subjected to variations of pressure.

91. The Differential Air Thermoscope. It consists as shown in fig. 11 of a bent glass tube terminating in two bulbs *A* and *B*. The bent tube contains coloured sulphuric acid.

If the two bulbs are at the same temperature, the level of the liquid remains the same in the two limbs. If however, the temperature of one of the bulbs rises, the air in it expands and the level of the liquid on that side is depressed while that on the other, is raised. This instrument is

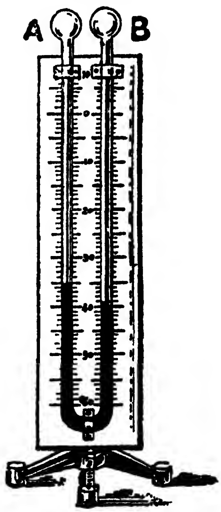


Fig. 11

used to indicate the difference of temperatures between the two bulbs. To make it sensitive, the bulbs are blackened.

SUMMARY

1. **Temperature** means the hotness of a body. Although its idea is derived primarily from our senses, yet for accurate determination, some physical property, which varies continuously with the temperature, is measured.

2. Thermometer is any instrument designed to measure temperatures.

3. The **expansion of mercury** is utilized in the construction of thermometers.

4. The **Fixed points** correspond to the two arbitrary temperatures, one that of melting ice and the other that of boiling water under normal pressure.

5. Relation between the three scales is given by $\frac{C}{100} = \frac{R}{80} = \frac{F-32}{180}$, or $\frac{C}{5} = \frac{R}{4} = \frac{F-32}{9}$.

6. A **maximum thermometer** denotes the highest temperature and a **minimum thermometer** denotes the lowest temperature, during a given time.

7. A **Platinum Resistance Thermometer** measures temperatures by changes in the resistance of a wire.

8. A **thermo-couple** measures temperatures by the measurement of *E.M.F.* generated.

9. The **Thermoscope** indicates difference of temperatures.

EXAMPLES

1. Find the temperature corresponding to 104°F. on the centigrade scale.

2. Find what temperature is represented by the same number on the *F.* and *C.* scales.

3. Express the temperature of human blood, *i. e.* 98·4°F. on the other two scales.

4. A thermometer with an arbitrary scale of equal parts reads 14·6 in melting ice and 237·6 in water, boiling under standard pressure. Find the centigrade temperatures indicated by the readings 97·1 and 214·0 on this thermometer. (Lond. F. of M. 1897).

5. If when the temperature is 0°C., a mercury thermometer reads 0·7°C. and when it is 100°C., it reads 101°C.; and if the bore be uniform and degree divisions of equal length. What is the temperature when the thermometer reads 20°C.

CHAPTER III

EXPANSION

92. Expansion of Solids. The fact, that almost all bodies whether solids or fluids expand when heated, has already been illustrated in the Introductory Chapter. Solids have the property of retaining their shape without the support of a containing vessel. Thus it is possible in their case to measure the increase in length, breadth or thickness, due to a given rise of temperature.

The increase in length of a metal bar is found by experiment to *vary directly with the original length* and *the rise of temperature*; but it is found to depend on the nature of the substance of which the bar is made. Thus, if we were to measure the increase in length per cm. length and per degree centigrade rise of temperature, the increase would vary with the nature of the substance and is known as the *co-efficient of linear expansion of the substance of which the bar is composed. It is defined as the increase in length which a bar 1 cm. long would experience, if heated through 1°C.*

If the co-efficient of linear expansion is known, the length of a rod at any temperature can be easily calculated,

Thus let a=the co-efficient of linear expansion,
l=the original length of the bar,
and t=the rise of temperature.

Assuming that the expansion is uniform throughout the range of temperatures, we have

Increase in a bar of unit length $=at$
" " " length $l = lat$
∴ Final length of the bar $= l + lat$
or $L_t = l(1 + at)$.

Co-efficient of Superficial expansion. It is the incr-

ease in area per unit area, per unit degree centigrade rise of temperature.

Suppose we have a square plate, each edge of which is 1 cm. long, then using the same symbols as before, we have

$$\text{Final length of each edge} = 1 + at$$
$$\therefore \text{ the area of the plate} = (1 + at)^2$$
$$= 1 + 2at + a^2t^2.$$

Now a is always very small; hence the term containing the square of a may be neglected.

Hence the area of the plate $= 1 + 2at$

And the increase in area $= 1 + 2at - 1 = 2at$.

Therefore the co-efficient of superficial expansion is double that of the co-efficient of linear expansion for the same substance.

Co-efficient of cubical expansion. It is the increase in volume per unit volume, per unit degree centigrade rise of temperature.

Let the volume of a given mass of substance at 0°C. be V_0, c the co-efficient of cubical expansion and t the rise of temperature. Let the volume at t°C. be denoted by V_t, then we have V_0c the increase in volume of volume V_0 for 1° rise of temperature and $V_0\,ct$ the increase in volume of the volume V_0 for t° rise of temperature.

$$\therefore \text{ the final volume } V_t = V_0 + V_0\,ct$$
$$\text{or } V_t = V_0\,(1 + ct)$$

This may be put in the form $c = \dfrac{V_t - V_0}{V_0\,t}$

Suppose we have a cube of unit edge, then using the same symbols as before, we have

$$\text{Final length of each edge} = 1 + at$$
$$\therefore \text{ the final volume of the cube} = (1 + at)^3$$
$$= 1 + 3at + 3a^2t^2 + a^3t^3.$$

Neglecting the terms containing the square and cube of a, we have the volume of the cube $= 1 + 3at$.

Hence the increase in volume $= 1 + 3at - 1 = 3at$.

Thus the co-efficient of cubical expansion is three times the co-efficient of linear expansion.

93. Measurement of the co-efficient of linear ex-

pansion. As the expansion of solids is very small, therefore special methods have to be devised to measure the small increase in length. Two methods are used for this purpose. According to one, by means of a system of levers, the expansion is magnified in a known proportion, as in Laplace and Lavoisier's method ; according to the other, the expansion is directly measured by delicate instruments such as a screw-gauge or a spherometer. Modern laboratory methods work on the latter principle.

(*a*) **The method of Laplace and Lavoisier.** The principle of the method is illustrated in fig. 12. The bar AB, whose expansion is required, is supported hori-

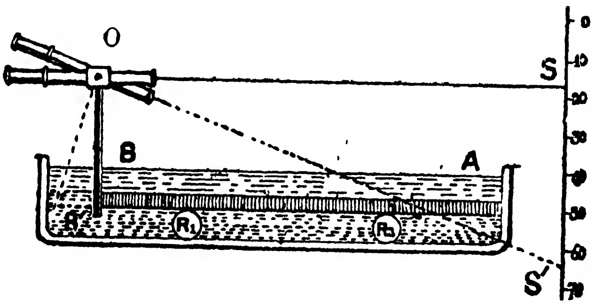


FIG. 12

zontally on rollers R_1, R_2 in a trough containing water, which can be heated. The end A of the bar is fixed, while the end B is free to move and pushes the end B of the lever OB having its fulcrum at O. A telescope which can be focussed at the screen SS' is rigidly fixed at the point O in a horizontal direction. A small motion of the end B causes the line of sight of the telescope to travel over a greater length SS' on the scale.

To begin with, the temperature of the bar AB is reduced to zero by surrounding it with melting ice and ice-cold water and the reading S is taken with the telescope. The temperature of the trough is then raised through $t°$ and when this temperature remains constant

for a considerable time, the reading S' is again taken with the telescope.

To calculate the expansion, we have by the similar triangles BOB' and SOS' $\left\{\begin{array}{l}\text{for } \angle BOB' = \angle SOS' \text{ and} \\ \text{both of them are } rt. \angle ed\ \Delta s.\end{array}\right.$

$$\therefore \quad \frac{BB'}{OB} = \frac{SS'}{OS}$$

$$\text{or} \quad BB' = SS'.\frac{OB}{OS}.$$

The distance SS' is already known from the readings and the other two distances admit of easy measurements.

Now BB'=expansion in length of AB

But expansion $BB' = AB \times a \times t$

$$\therefore \quad a = \frac{BB'}{AB.t} = \frac{SS'.\ OB}{AB.\ OS} \times \frac{1}{t}$$

One objection to this method is that it is rather tedious to determine the ratio $\frac{OB}{OS}$ and the second objection is that the end A of the rod also expands as the vessel expands.

(*b*) **The Laboratory or the spherometer method.** The principle of the method is illustrated in Fig. 13. The rod, whose co-efficient of expansion is required, is

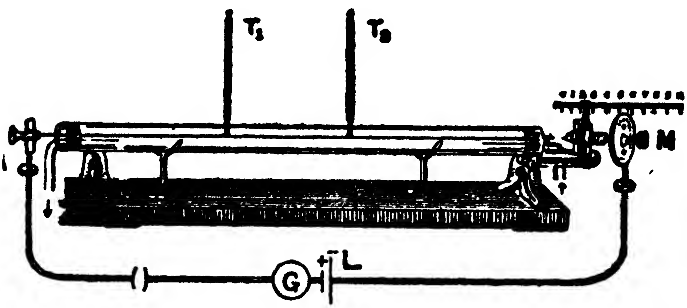


Fig. 13

taken about 80 cms. long and is supported in the horizontal position, as shown in the figure. One end of

this abuts against the fixed support which prevents it from expanding in that direction. The other end is free to expand and the expansion is measured by the sphero-meter screw. The rod has two thermometers T_1 and T_2 attached to it to record its temperature and the whole is surrounded by a big glass tube having inlet and outlet tubes for steam.

To calculate the expansion, the rod is measured at the ordinary atmospheric temperature and its length L recorded.

The reading R_1 of the screw, when it just touches the end of the rod, at the ordinary atmospheric temperature, is also recorded.

The screw M is then moved outwards and steam is passed into the jacket; when the temperature is stationary, the screw M is moved inwards to touch the end of the rod and the reading R_2 again recorded.

To ensure the proper contact of the screw with the rod, a cell L and a galvanometer G are connected as shown in the diagram. When the screw M just touches the rod, the circuit is complete and a current begins to flow. This deflects the galvanometer needle immediately, which indicates that contact between the rod and the screw is established.

The difference between these two readings of the screw gives the increase in length.

Hence $R_2 - R_1 = L \times a \times t$, where t denotes the rise of temperature.

Therefore $$a = \frac{R_2 - R_1}{L \times t}.$$

94. Change of Density with Temperature. Since the mass of a substance does not change with variations in its temperature, but the volume increases with rise of temperature; hence the density necessarily decreases. Thus if V_o and V_t denote the volumes of a substance at 0° and t° C. respectively and d_0 and d_t the densities of the same substance at the two temperatures; we have $V_o d_o = V_t d_t = M$

i. e. $\dfrac{d_o}{d_t}=\dfrac{V_t}{V_o}=\dfrac{V_o(1+ct)}{V_o}$,

where c denotes the co-efficient of cubical expansion of the substance.

or $d_t=\dfrac{d_o}{1+ct}$.

This is true for solids, liquids and gases. But for solids and liquids for which c is small, we may have the approximate formula $d_t=d_o(1-ct)$, provided the rise of temperature is small, because $d_t=\dfrac{d_o}{1+ct}$ $=d_o(1+ct)^{-1}=d_o(1-ct)$ approximately.

Also from the above formula we have $\dfrac{d_o}{d_t}=1+ct$;

subtracting one from both sides we get $\dfrac{d_o-d_t}{d_t}=ct$.

Dividing by t, we get $c=\dfrac{d_o-d_t}{d_t\times t}$.

95. Practical consequences of expansion or contraction. In many cases, in which metals are used, their expansion with rise of temperature becomes a source of trouble. Thus in building iron bridges, space is to be left for their expansion, similarly in constructing a railway line, a small space is left between two consecutive rails. The fish plates, which join the two rails are so slotted as to allow for their free expansion. The joints of water pipes and gas mains are made like those of a telescope. Space has to be left for the expansion of furnace bars in order to prevent them from bending. In order to work railway points at a distance from the cabin, small cross-pieces as shown in fig. 14, movable about their middle points are used at intervals, for when expansion takes place *CL* the cabin lever and *P*

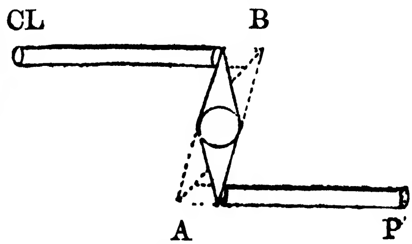
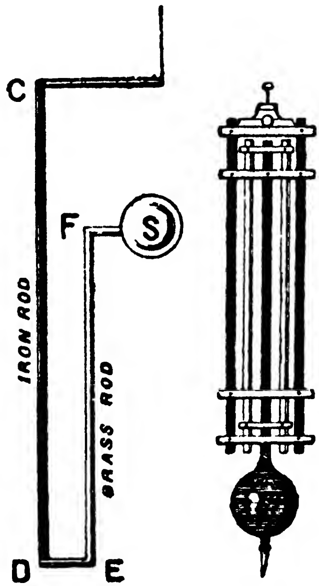


FIG. 14

the *rail point* end remain fixed, while the cross-piece turns round as shown in fig. 14 by dotted lines; but if CL be moved backward by a pointsman, the point P will move the same distance backward. The standard yard is the distance between two marks at 62°F., any variations of temperature will necessitate corresponding correction. The fact, that metals contract when their temperature is lowered, is made use of in fitting iron tyres to the wheels of carriages and carts.

Compensated Pendulum. The time of vibration of a simple pendulum is given by $t=2\pi\sqrt{l/g}$; hence it is essential that the length l should remain constant, otherwise the clock under its control will go slow if



(a) (b)

Fig. 15 Fig 16

l increases due to the rise of temperature and will go fast when it decreases due to the fall of temperature. To

prevent this, the following different devices are used.

(*i*) **Graham's Mercury Pendulum.** An iron cylinder is screwed on to the end of an iron rod as shown in fig. 15. The cylinder contains mercury whose quantity is so adjusted that its expansion upwards for any rise of temperature will cause its centre of gravity to rise by just the amount through which it (*i.e.* *c.g*) is lowered by the expansion of the rod downwards due to the same rise of temperature.

(*ii*) **Harrison's gridiron Pendulum.** The principle of the instrument is explained by reference to fig. 16(*a*). *CD* and *EF* represent two rods of iron and brass respectively, *EF* is $\frac{2}{3}$ of *CD*. As the co-efficient of linear expansion of brass is $\frac{3}{2}$ times that of iron, it is evident that the expansion of iron rod will be equal to that of the brass rod to whatever temperature the two bars may be heated. The rod *CD* is fixed at *C* and expands downwards, while the rod *EF* expands an equal distance upwards, therefore the distance of *F* from *C*, *i.e.* the effective length of the pendulum remains unaltered, whatever the variations of temperature.

Such a form of pendulum would be rather awkward to use, therefore the pendulum is given the form as shown in fig. 16 (*b*). The iron bars are shown black and the brass bars white. The expansion of iron rods tends to lower the pendulum bob, while that of brass rods tends to raise it. The total length of iron rods is $1\frac{1}{2}$ times that of brass rods, therefore the expansion of two sets of bars will be equal and opposite and thus the distance between the centre of suspension and the centre of the bob will remain unaltered.

(*iii*) In chronometers and watches, the balance-wheel controls the time. The period of vibration increases with increase of dimensions, for a larger wheel will oscillate more slowly than a smaller one under the same force; moreover the elasticity of the hair spring decreases with rise of temperature. Due to both these causes, a rise of temperature makes the watch go

slowly. To remedy this, the circumference of the wheel is made of two metals, the outer one being more expansible. The circumference is further divided into 3 parts, each part being supported by one spoke. The effect of rise of temperature on such a wheel is that the point of the segment near the end of the spoke will be pushed out from the centre as the spoke expands; while the one far away from the spoke will curl towards the centre of the wheel, on account of the greater expansion of the outer metal and the attached screw weights. By proper adjustment exact compensation may be obtained.

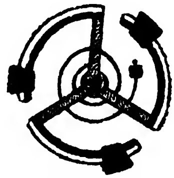


FIG. 17

96. Metallic Thermometer. It consists of a long spiral made up of 3 strips of silver, gold and platinum, soldered so as to form a single ribbon. Silver being most expansible of the three is placed inside, then gold and then platinum on the outside. At the lower end, the spiral carries a pointer, which moves over a dial graduated in degrees. As the spiral winds or unwinds with fall or rise of temperature, the pointer moves over the dial and denotes the temperature. The instrument is only of historic importance now and is of no practical utility.

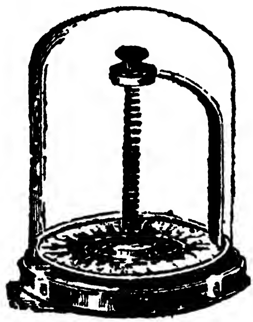


FIG. 18

97. Expansion of Liquids. As liquids do not possess any shape of their own, but assume the shape of the vessel in which they are poured, their linear or superficial expansion cannot be constant and thus their measurement serves no purpose. The cubical expansion however, is constant and is the one which is measured. But the problem becomes a bit difficult; because when heat is applied, the containing vessel also expands and its internal volume increases. The liquid must first com-

pensate for this increase in volume, before its expansion can be made visible. Consequently the expansion that is observed, is the difference between the real expansion of the liquid and the expansion of the vessel. Thus liquids have two kinds of expansions: (*i*) the real and (*ii*) the apparent.

Show that the real co-efficient of cubical expansion of a liquid is equal to its apparent co-efficient of cubical expansion plus the co-efficient of cubical expansion of the material of the vessel.

Let V_0 = volume of the liquid at 0°
and t = rise of temperature
V_r = real volume at $t°$ and C_r = real co-efficient of expansion.
Va = apparent ,, $t°$ and Ca = apparent co-efficient of expansion
and C = co-efficient of expansion of glass

Va the apparent volume at $t°$ must be equal to $V_0(1+Cat)$ for the apparent increase in volume is evidently equal to V_0Cat.

V_r the real volume at $t° = V_0(1+C_rt)$, for the real increase in volume $= V_0C_rt$.

Also V_r the real volume $= Va$ the apparent volume plus the increase in volume of the vessel, which is equal to $VaCt$. Thus $V_r = Va(1+Ct)$.

Writing these equations we get

$$V_a = V_0(1+C_at) \qquad (i)$$
$$V_r = V_0(1+C_rt) \qquad (ii)$$
$$V_r = V_a(1+Ct) \qquad (iii)$$

Substituting the value of Va from equation (*i*) in equation (*iii*), we get

$$V_r = V_0(1+C_at)(1+Ct) \qquad (iv)$$

As the left-hand expressions of equations (*ii*) and (*iv*) are equal, therefore the right-hand expressions must also be equal. Thus

$$V_0(1+C_rt) = V_0(1+C_at)(1+Ct)$$
$$\text{or} \quad C_rt = C_at + Ct + C_aCt^2.$$

Neglecting the product of Ca and C which are very

small quantities and dividing by t, we get

$$C_r = C_a + C.$$

The relation is of great importance, for knowing the apparent co-efficient of expansion of the liquid and the co-efficient of expansion of the material of the vessel, we can find out the real co-efficient of expansion of the liquid.

98. To determine the co-efficient of apparent expansion of a liquid by the Weight Thermometer. The Weight Thermometer fig. 19, usually consists of a cylindrical glass tube of about 5 c.c. capacity, ending in a fine capillary tube bent as shown in the figure.

In order to find C_a, the apparent co-efficient of expansion, the instrument is thoroughly cleaned, dried and weighed. Let it be w_1. It is then filled to the brim by alternate heating and cooling with the liquid, the co-efficient of expansion of which is required. It is then left for a short time in the atmosphere with its open end dipping well below the liquid level, until the whole instrument acquires the temperature $t_1°$ of the atmosphere. The instrument is then wiped dry and weighed again and let it be w_2.

FIG. 19

The instrument is kept in a steam bath maintained at temperature t_2°C. On account of the greater expansion of the liquid, some of it will be forced out The instrument is then taken out of the steam bath, dried and allowed to cool. The liquid in the instrument contracts, leaving some spac eempty at the top. Weigh this again and let it be denoted by w_3.

The mass of the liquid which remains after heating is $w_3 - w_1$ and the mass which overflows is $w_2 - w_3$. Thus $(w_3 - w_1)$ grammes when heated through $(t_2° - t_1°)$ will occupy the same volume as is occupied by the total mass, *i.e.* that which remains and that which has overflown, or $(w_3 - w_1) + (w_2 - w_3) = w_2 - w_1$. Now at the same temperature the volumes are proportional to the

corresponding weights, hence a volume proportional to $w_3 - w_1$ increases by a volume proportional to $w_2 - w_3$ when heated through $t_2^\circ - t_1^\circ$.

∴ the apparent co-efficient of expansion

$$C_a = \frac{w_2 - w_3}{w_3 - w_1} \times \frac{1}{t_2^\circ - t_1^\circ}$$

$$\text{i e} = \frac{\text{Mass of liquid expelled}}{\text{Mass of liquid left behind at the higher temp.} \times \text{rise of temperature}}.$$

Dulong and Petits' direct method of measuring the absolute co-efficient of expansion of a liquid.

The apparatus as shown in fig. 20 is a U-tube, of which the two limbs AB and CD are vertical, while the communicating arm BC is horizontal. Both the limbs are surrounded by wide glass tubes. A thermometer in each of these tubes denotes their temperatures. The liquid whose co-efficient is required, is poured into the U-tube and stands at the same level in each tube. Steam is passed from the *top*, in the tube surrounding the limb CD and ice-cold water is made to circulate from the *bottom*, in the tube surrounding the limb AB. As the temperature of the liquid in the limb CD rises, its density decreases and its level is seen to rise higher than that of the liquid in the limb AB. To prevent the flow of heat along CB, a piece of muslin is made to surround the tube at E and ice-cold

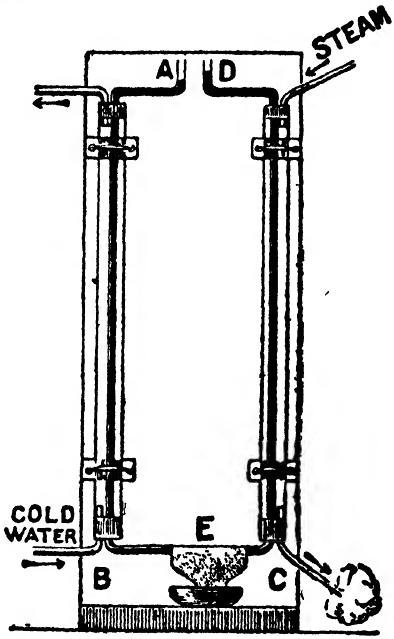


FIG. 20

water is constantly poured over it. When the temperatures of AB and CD become steady, their heights h_1 and h_2, and temperatures t_1 and t_2 respectively, are measured by a cathetometer.*

The pressure at points B and C in the same horizontal line must be the same, when the whole system is in equilibrium.

Therefore $h_1 d_1 g = h_2 d_2 g$,
where d_1 and d_2 are the densities of the liquid at temperatures t_1 and t_2 respectively.

or $h_1 d_1 = h_2 d_2$

or $\dfrac{d_1}{d_2} = \dfrac{h_2}{h_1}$.

But $\dfrac{d_1}{d_2} = 1 + C_r t$, (*vide* Article 94)

$\therefore \quad 1 + C_r t = \dfrac{h_2}{h_1}$, (subtracting 1 from both sides and dividing by t)

we get $\quad C_r = \dfrac{h_2 - h_1}{h_1 t}$.

Dulong and Petit observed by this method that the co-efficient of expansion of mercury increases with rise of temperature.

99. Expansion of water. The expansion of water presents some interesting peculiarities and is conveniently studied with the help of the instrument known as Hope's apparatus. It consists as shown in fig. 21 of a tall metal jar A, having two side-openings fitted with thermometers, T_1 and T_2. It is surrounded at its centre with a circular trough, which is filled with freezing

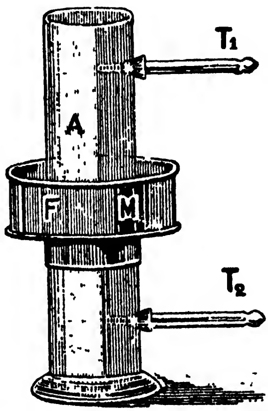


FIG. 21

*A cathetometer is a low-power telescope, which can slide up and down a vertical graduated scale.

mixture containing ice and salt. *A* is filled with water, which is cooled down to 10°C. As cooling proceeds, the temperature of the lower thermometer goes on falling, while that of the upper thermometer remains unaffected. But when the temperature of the lower thermometer reaches about 4°C. the temperature of the upper one begins to fall and continues to do so till it reaches 0°C If the action of the freezing mixture continues still further, a thin crust of ice may be formed on the upper surface of water, while the temperature of the water down below, still remains 4°C.

These facts are explained as follows:—The freezing mixture cools the water adjacent to it which contracts, becomes denser, goes down and causes the temperature of the lower thermometer to fall. This continues as long as a fall of temperature causes contraction. But as soon as the temperature reaches 4°C., any further lowering of the temperature causes the water to expand rather than contract and makes it go up and lower the temperature of the upper thermometer to 0°C. The lower thermometer however, remains stationary at 4°C. and shows that water must have maximum density at that temperature.

The fact that water has its maximum density at 4°C. is of great importance in Nature. During severe cold, the temperature of the whole water in lakes and seas goes on decreasing upto 4°C. but after that the temperature of the upper layers only falls and they may be frozen into ice, while down below there will be liquid. Thus the lives of aquatic animals are preserved. As water expands on freezing, therefore ice floats on the surface, and the whole mass of water is prevented from being frozen into ice.

100. Expansion of Gases. No notice has been taken of the pressure to which a solid or a liquid has been subjected during expansion, for the simple reason that the co-efficient of expansion of a solid or of a liquid is not appreciably affected by changes of pressure; but the volume of a gas depends not only on its tempera-

ture but also on the pressure to which it is subjected. Thus in the case of gases, the volume varies with two factors, *i e.* Pressure and Temperature; hence the *state* of a given mass of gas depends upon three variables—volume, pressure and temperature; and any one of them may be determined, when the other two are known. To determine the relation between these quantities, we resort to experiment If all the three quantities be allowed to vary at once, then the cause of the change cannot be ascribed to any particular factor; we therefore keep one variable constant and allow the other two to change and study the effects produced This necessitates three sets of experiments to be performed.

(*i*) Relation between pressure and volume, when temperature is constant.

(*ii*) Relation between volume and temperature, when pressure is constant.

(*iii*) Relation between pressure and temperature, when volume is constant.

101. Relation between volume and pressure of a gas at constant temperature. The relation between the pressure p and the volume v is studied with the help of the Boyle's law apparatus shown in fig. 22. A graduated tube AB has a closed end at A; and is connected, at the other end to a mercury reservoir consisting of a wide-mouthed tube D, by means of a long thick India-rubber tubing BC. The reservoir is attached to a sliding panel capable of vertical motion along a graduated scale E. This arrangement is very convenient to read the level of the mercury in the reservoir. To find the relation be-

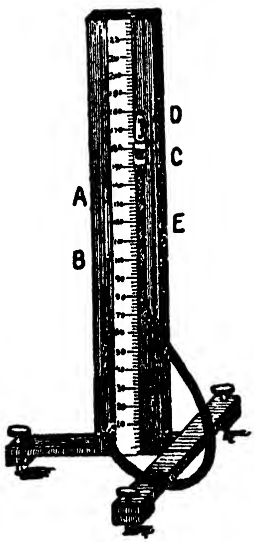


Fig. 22

tween p and v, the following operations are performed:— The tube is filled with dry air and the reservoir is raised or lowered till the level of mercury in the tube and the reservoir is the same; the readings are noted. Barometer is read to give the atmospheric pressure. After this the reservoir is raised every time through about 20 cms., the mercury surface in the tube as well as in the reservoir is read and the observations noted in the following way.—

Temperature of air.. . . . . =

Atmospheric pressure in cms. of mercury=

Mercury surface in the tube. 1	Mercury surface in reservoir. 2	Head of Mercury. 3	Total Pressure. (p) 4	Volume of air. (v) 5	$p.\ v.$ 6

The head of mercury is given by subtracting the reading in column no. 1 from that in column no. 2. The volume is given directly by the reading in column 5 and the total pressure is given by adding the atmospheric pressure P to the head of mercury. It will be seen that the value in the last column will be constant within experimental errors.

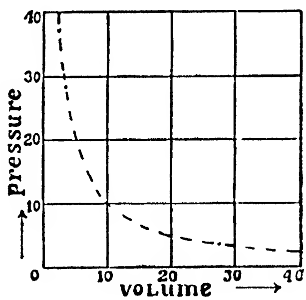


Fig. 23

Thus at constant temperature the product, of the volume of a given mass of gas and the pressure to which it is subjected, remains constant. This statement is known as Boyle's law.

It is written as $P.\ V=K.$

The curve shown in fig. 23 *represents graphically

*The curve shows the relation of 10 volume-units of the gas subject to 10 pressure-units. If the volume is reduced to 5 units, the pressure increases to 20 units, so that the product of P and $V=100$. Such a curve is known as rectangular hyperbola.

the relation between P and V, when the temperature is kept constant. Such a curve is known as an Isothermal. All gases do not obey Boyle's law rigidly. Even the so-called permanent gases, such as oxygen, hydrogen, helium etc. obey this law at ordinary temperatures, but present great divergence at low temperatures, near their boiling points. A gas, which obeys Boyle's law, is termed a *perfect gas*.

102. Relation between volume and temperature of a gas, at constant pressure. This relation is the one, we have considered in the expansion of solids and liquids, *i. e.* we have to determine the co-efficient of cubical expansion of gases under the influence of temperature. As the expansion of gases is much more than that of solids or liquids,† therefore the expansion of the vessel containing the gas may be neglected in comparison to the expansion of the gas. Thus the real and apparent co-efficients of expansion of a gas will differ by a negligibly small quantity; and in practice, it is the apparent co-efficient, which is measured. The two important experimental methods of determining the above relation are:—(*i*) Gay Lussac's and (*ii*) Regnault's.

(*i*) **Gay Lussac's method.** Gay Lussac's apparatus is shown in fig. 24. A long thermometric tube A with a spherical bulb is graduated and filled with dry air; a short column of

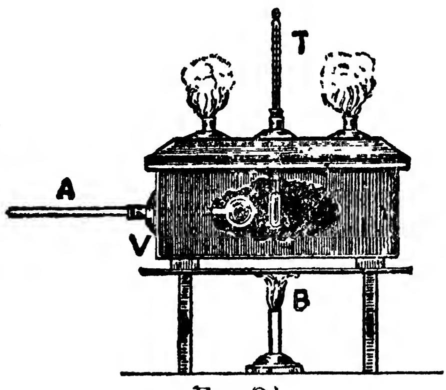


Fig. 24

† Expansion of air is nearly 20 times that of mercury, and 9 times that of water.

mercury is left in the stem of the tube to serve as index. It is placed for a very long time in a vessel V containing melting ice. The temperature 0° is denoted by the thermometer T and the volume V_0 is given by the position of mercury index. After this the vessel is heated and its temperature begins to rise slowly. The temperature t is noted when the liquid in the vessel V is boiling and the volume V_t is noted by reading the index position. From these observations we get C_v, the co-efficient of expansion at constant pressure, by the formula:

$$C_v = \frac{V_t - V_0}{V_0 t} \text{ or } V_t = V_0 (1 + C_v t)$$

Gay Lussac found $C_v = \cdot 00375$, which is much higher than the real value obtained by later observers. He experimented with different gases and found the co-efficient to be the same for all gases.

Note.—Gay Lussac's method is subject to two sources of error:—(*i*) The pressure inside and outside the tube may vary slightly without any corresponding movement of mercury index and (*ii*) Some interchange may take place between the enclosed gas and the gas outside the mercury index.

(*ii*) **Regnault's method.** His apparatus as shown in fig. 25 consists of the reservoir A, in which the gas is introduced, and the

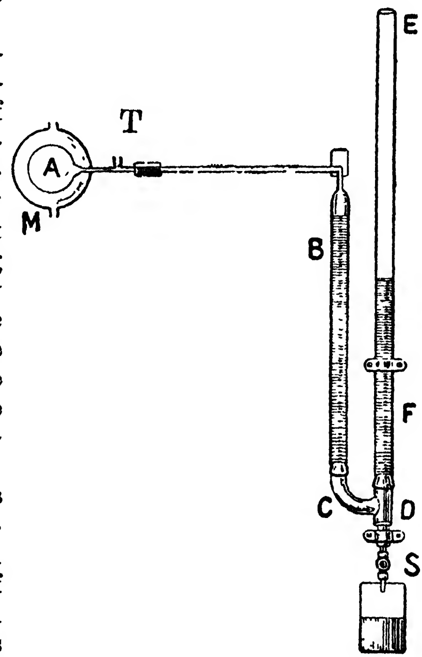


Fig. 25

manometer $BCDE$. The arm B of the manometer ends in a fine calibrated tube, which is bent at right angles and is connected to the reservoir A. Near A is a fine tube T by means of which the reservoir may be exhausted, dried and filled with the gas to be experimented upon. At the junction of the two arms of the manometer is a stop-cock S. To perform the experiment, the reservoir A is filled with dry gas and is surrounded by melting ice placed in a metallic sphere M, which completely surrounds the reservoir. More mercury is poured in the tube DE, till its level in the two arms of the manometer is in the same horizontal line. This adjustment is made by varying the quantity of gas in A by means of the tube T. After this adjustment the tube T is sealed, ice removed from M and steam allowed to play freely round A. The gas in A expands, depresses the mercury surface in BC and causes that in DE to rise. The stopcock S is opened and mercury allowed to flow till the level in the two arms is in the same horizontal line. The gas in A thus expands under atmospheric pressure.

Now if the increase in volume be denoted by v c.c.
and the volume of the gas at 0°C. " " " V "
and t=the temperature of steam,

we have $C_v = \dfrac{v}{V \times t}$

Regnault found by this method, $C_v = \cdot 0036705$

$$= \frac{1}{273}.$$

Note.—In Regnault's experiment, the mercury index is replaced by a mercury column and thus the sources of error of Gay Lussac's experiment are obviated; but the temperature of the gas contained in the part BC is not the same as that in A and therefore correction is essential.

Thus we arrive at the conclusion that *if a given volume of gas be heated through 1°C. it expands by* $\frac{1}{273}$

of its volume at $0°C.$, *provided the pressure remains constant.* The statement is known as **Gay Lussac's law** or more often as **Law of Charles.**

Caution.—It should be clearly noted that the increase in volume, per degree centigrade rise of temperature is $\frac{1}{273}$ of the volume of the gas *necessarily* at $0°C.$ It will be incorrect to substitute the value of volume at any other temperature. In solids and liquids this substitution does not matter much, for their expansion is very small; but in the case of gases, the expansion being very great, such a substitution will give totally wrong results.

$$\text{Thus} \quad V_t = V_0\left(1 + \frac{t}{273}\right)$$

Whenever it is required to find the volume V of a given mass of gas at any temperature T, when its volume v is given at a known temperature t, we first find out its volume at $0°C.$ and then apply the formula given above to know its volume at $T°$.

$$\text{Thus} \quad v = V_0\left(1 + \frac{t}{273}\right), \text{ where } V_0 = \text{volume at } 0°C.$$

$$\text{or} \quad V_0 = \frac{v.\,273}{273 + t}$$

But $V = V_0 \left(1 + \frac{T}{273}\right)$, (substituting the value of V_0)

$$\text{We get} \quad V = \frac{v.\,273}{(273 + t)} \times \frac{(273 + T)}{273}$$

$$\text{or} \quad V = \frac{(273 + T)}{(273 + t)}.v$$

103. Relation between Pressure and Temperature, when the volume is kept constant. The relation between the increase of pressure with the rise of temperature may be studied with the help of Regnault's apparatus shown in fig. 25, or with Jolly's

constant volume thermometer shown in fig. 26.

To find the increase in pressure per degree centigrade rise of temperature, when the volume is kept constant, the reservoir A is surrounded by melting ice to reduce its temperature to 0°C. and the pressure is so adjusted that the level of the mercury surface in the two arms at B and F is the same. This means that the pressure of the gas is equal to the atmospheric pressure. The reservoir A is heated by steam and the gas enclosed within A begins to expand, but is prevented from doing so either by pouring mercury in E fig 25. or by raising the tube F fig. 26. When the temperature of A becomes constant, the difference in the level of the mercury surface in B and in the open arm in either case gives the increase in pressure. If this be denoted by p, the initial barometric pressure at 0°C. be denoted by P_0 and the rise of temperature be equal to t°C, then C_p *i.e.* the co-efficient of increase of pressure at constant volume is given by

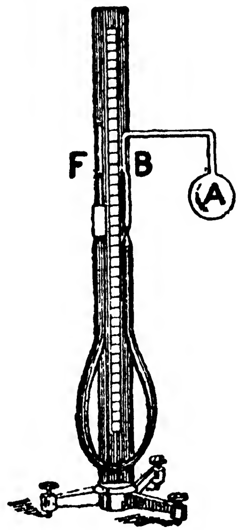


Fig. 26

$$C_p = \frac{p}{P_0 \times t} \text{ or } P_t \text{ the Pressure at } t^0 = P_0\,(1 + C_p.t)$$

Regnault found that the value of Cp for all gases was ·003665, whatever the initial pressure, and this is very approximately equal to 1/273. This *means that if the temperature of a given mass of gas at* **constant volume** *be raised through 1°C. its pressure will increase by* 1/273 *of its original pressure at 0°C.* **This is known as co-efficient of increase of Pressure.**

104. Relation between the two co-efficients.—It should be noted that the co-efficient of increase

of pressure is equal to the co-efficient of increase of volume of a gas, each being equal to 1/273. This result follows directly from Boyle's and Charles' laws. For consider a volume v of gas at 0°C. under a pressure p. Suppose the gas is heated through t^0 under constant pressure. Then by Charles' law, its volume v' will be equal to $v(1+C_vt)$, where C_v is the co-efficient of increase of volume. Imagine this volume v' at pressure p to be compressed at this very temperature t until its volume is reduced to the original volume v. Then by Boyle's law, the final pressure P is given by $Pv=pv'$.

Substituting the value of v'

we have $Pv=pv(1+C_vt)$

or $P=p(1+C_vt)$ but $P=p\,(1+C_p.t)$

$\therefore\ C_v=C_p.$

105. The Gas Thermometer. The expansion of gases finds a very important use in the construction of thermometers. It has been noted that gases expand uniformly and that their co-efficient of expansion is much more than that of solids or liquids, therefore a gas thermometer is more sensitive and accurate. A gas thermometer may take either of the two forms, *i. e.* the temperature may be measured either by noting the increase in volume at constant pressure or by the increase in pressure at constant volume.

Thus t is given either by $(i)\ t=\dfrac{V_t-V_0}{V_0C_v}$

or by $(ii)\ t=\dfrac{P_t-P_0}{P_0C_p}$

where C_v and C_p denote the co-efficients of the gas at constant pressure and volume respectively. In practice however, it is the latter form, which is very extensively used. The reason for this is that it is rather difficult to control the expansion in volume and at the same time to keep the temperature of the whole mass of gas constant. The apparatus used is Jolly's constant-volume air thermometer shown in fig. 26. The

formula employed is that given in equation (*ii*) above; *i.e.* $t=\frac{P_t-P_0}{P_0C_p}$.

Here P_0 is determined once for all at 0°C., $C_p=1/273$ and P_t is equal to H, the barometric pressure at the time of the experiment, + or − the difference between the level of the mercury surface in the two arms.

As the properties of air remain unchanged through a very long range of temperatures, air thermometer is used to determine very high and very low temperatures and is in fact the ***standard*** with which other thermometers are compared.

N.B.—The drawbacks of such a thermometer are:—(*i*) It is not easily portable, (*ii*) It cannot be graduated once for all, because graduations depend upon atmospheric pressure and (*iii*) Exact determination of Barometric pressure is required at each measurement.

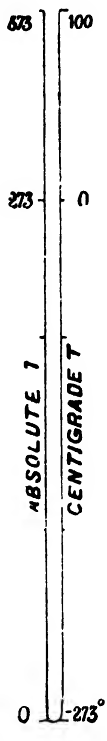


FIG. 27

106 Absolute temperature Suppose an air thermometer gives temperatures by increase of volume at constant pressure, and that at 0°C. the volume of a given mass of air is 273 units; evidently its volume at 100°C. will become 373 units. Now divide the distance between 0°C. and 100°C. into 100 equal parts then each division will correspond to 1°C. Carry this method of division down the scale below 0°C.; then at the bottom when the reading will be −273°C. fig. 27, the gas at this temperature should have no volume, if it were to obey Charles' law quite rigorously, for at this temperature the gas being thoroughly devoid of any volume and pressure shall have no heat. This temperature is known as the absolute zero. Temperatures measured from −273°C. as zero, are called absolute temperatures and the scale so constructed is

called the absolute scale. In practice however, it is impossible to attain such a condition, for all gases do liquefy and freeze before this temperature is reached.

The absolute temperature of a body is its temperature measured from the absolute zero. Thus 0°C. is 273° absolute temperature and 100°C. is (273+100) =373° absolute temperature and so on.

If temperatures be measured on the absolute scale, then it can be shown that the volume of a given mass of gas at constant pressure is proportional to its absolute temperature and similarly its pressure at constant volume is proportional to the absolute temperature. Thus if V_2 and V_1 denote volumes of a given mass of gas at $t_2°$ and $t_1°$ Centigrade and V_0 denotes the volume at 0°C. then we have $V_2 = V_0(1+\frac{t_2}{273})$

$$\text{and } V_1 = V_0(1+\frac{t_1}{273})$$

$\therefore \quad \frac{V_2}{V_1} = \frac{273+t_2}{273+t_1} = \frac{T_2}{T_1}$, where T_2 and T_1 are the absolute temperatures.

$$\text{Hence } \frac{V_2}{T_2} = \frac{V_1}{T_1}.$$

Similarly if P_2 and P_1 were to denote the corresponding pressures at $t_2°$ and $t_1°$ respectively, when the volume is kept constant, and P_0 were the pressure at 0°C.

$$\text{Then we have } P_2 = P_0(1+\frac{t_2}{273})$$

$$\text{and } P_1 = P_0(1+\frac{t_1}{273})$$

$$\therefore \quad \frac{P_2}{P_1} = \frac{273+t_2}{273+t_1} \text{ or } \frac{P_2}{P_1} = \frac{T_2}{T_1};$$

$$\text{Hence } \frac{P_2}{T_2} = \frac{P_1}{T_1}.$$

Relation between Pressure, Volume and Temperature of a gas.

Let P=Pressure,

V=Volume,

and T=Absolute temperature.

From Boyle's law, we have $V \propto \frac{1}{P}$, when T is constant; and from Charles' law we have $V \propto T$, when P is constant.

$\therefore$ when both P and T vary

$$\text{we have } V \propto \frac{T}{P}$$

$$\text{or } V = R.\frac{T}{P}, \text{ where } R \text{ is a constant.}$$

$$\therefore \quad \frac{PV}{T} = R = \frac{P'V'}{T'},$$

where P', V' and T' denote another set of values of pressure, volume and absolute temperature respectively, for the same mass of gas. The above is a very important result, which can be used to find the volume of a given mass of gas at any pressure and temperature when its volume at a known pressure and temperature is given.

Table of Co-efficients of Expansion.

1. *Linear co-efficients of solids.*

Aluminium	0·000022
Brass	0·000019
Copper	0·000017
German silver	0·000018
Glass	0·0000084
Gold	0·000014
Iron (cast)	0·000011
Lead	0·000028
Nickle	0·000013
Platinum	0·0000089
Quartz	0·0000027
Silver	0·000019
Tin	0·000023

Zinc	0·000029

2. Real co-efficients of liquids

Aniline	0·00085
Alcohol (methyl)	0·00122
Alcohol (ethyl)	0·00110
Benzene	0·00124
Brine	0·00043
Carbon Bisulphide	0·00121
Chloroform	0·00126
Ether	0·00163
Glycerine	0·00053
Mercury	0·000182
Nitric acid	0·00120
Sulphuric acid	0·00057
Turpentine	0·00095
Water	0·00050 0·00030

SUMMARY

Co-efficient of linear expansion of a substance is the increase in length per unit length, per unit degree centigrade rise of temperature.

Co-efficient of Cubical expansion is the increase in volume per unit volume, per unit degree centigrade rise of temperature.

Co-efficient of cubical expansion of a substance is three times its co-efficient of linear expansion.

Compensated pendulum and wheels are so constructed as to keep their time the same when variations in temperature take place.

Space must be allowed for the expansion of girders and other metal parts in the construction of bridges.

Co-efficient of apparent expansion of a liquid is the apparent increase in volume per c.c., per degree centigrade rise of temperature.

Co-efficient of real expansion of a liquid is the real increase in volume per c.c , per degree centigrade rise of temperature. The real co-efficient is equal to the apparent co-efficient and the cubical co-efficient of the substance of the vessel.

Weight thermometer method gives the apparent co-efficient of expansion.

Balancing-column method gives directly the real co-efficient of expansion.

Water has its maximum density at 4°C.

Boyle's law. The product of the pressure and volume of given mass of gas at constant temperature remains constant. A graph showing such relation is known as an isothermal.

Charles' law. The volume of a given mass of gas increases by $\frac{1}{273}$ of its volume at 0°C. for 1°C. rise of temperature, provided the pressure remains constant. The pressure of a given mass of gas increases by $\frac{1}{273}$ of its pressure at 0°C. for 1°C. rise of temperature, provided its volume remains constant.

Perfect gas is one, which rigidly obeys Boyle's law. Temperatures measured from -273°C. as zero are known as absolute temperatures.

The increase in pressure of a gas at constant volume is used to measure temperatures by Air Thermometer. It is very sensitive and reliable.

EXAMPLES

1. The length of a copper rod at 50°C. is 2·00166 metres and at 200°C. it is 2·00684 metres. Find its length at 0°C. and the co-efficient of expansion of copper. (C.U. 1915).

Let L_0 be the length at 0° and c the co-efficient of copper. Then we have

$$200 \cdot 166 = L_0(1+50c) \quad \ldots \ldots (i)$$

$$\text{and } 200 \cdot 684 = L_0(1+200c) \quad \ldots \ldots (ii)$$

Dividing (i) by (ii), we get $\frac{200 \cdot 166}{200 \cdot 684} = \frac{1+50c}{1+200c}$

whence $c = 0 \cdot 0000166$

Substituting this value of c in equation (i), we get

$$200 \cdot 166 = L_0(1+50 \times \cdot 0000166)$$

whence $L_0 = 199 \cdot 9998$ cms.

2. The density of mercury at 0°C. is 13·60 and at 100°C. it is 13·35. Calculate the co-efficient of absolute expansion of mercury.

We have $c = \frac{d_o - d_t}{d_t \times t}$

$$\therefore \quad c = \frac{13 \cdot 60 - 13 \cdot 35}{13 \cdot 35 \times 100}$$

$$= \cdot 000185$$

3. A long glass tube of uniform capillary bore contains a thread of mercury, which at 0° is 1 metre long. At 100°C. it is 1·65 cms. longer. If the co-efficient of expansion of mercury be ·000182, find the linear co-efficient of expansion of glass.

Suppose the area of cross-section of the tube = 1 sq. cm. Then the original volume of mercury = 100 c. cs and the increase ought to be equal to $100 \times 100 \times {}^{\cdot}000182 = 1\,82$ c. cs; but the apparent increase is equal to 1 65 c. cs. ∴ glass has increased in volume by $1{\cdot}82 - 1{\cdot}65 = 17$ c. c.

∴ its co-efficient of cubical expansion $= \dfrac{17}{100 \times 100} = {}^{\cdot}000017$

and its linear co-efficient, which is $^1/_3$ of cubical co-efficient

$= \dfrac{{}^{\cdot}000017}{3} = 00000567.$

4. If the co-efficients of cubical expansion of glass and mercury are 000025 and 00018 respectively, what fraction of the whole volume of a glass vessel should be filled with mercury in order that the volume of the empty part should remain constant, when the glass and the mercury are heated to the same temperature.

It means the increase in volume of glass and mercury for any rise of temperature should be the same, *i e* if the total volume at 0° of glass be V_0 and a portion xV_0 be mercury, where x is a proper fraction.

Then $V_0 \times t \times {}^{\cdot}000025 = xV_0 \times t \times {}^{\cdot}00018$

whence $x = \dfrac{{}^{\cdot}000025}{{}^{\cdot}00018} = \dfrac{5}{36} = {}^{\cdot}139$

i.e. the glass should be filled upto $\dfrac{5}{36}$ of its volume with mercury.

5. A mass of gas is heated from 15° C. to 80°C. Calculate its final volume, if it initially occupied 225 c.cs., pressure remaining constant.

We have $\dfrac{V}{T} = \dfrac{V'}{T'}$

i.e $\dfrac{125}{273+15} = \dfrac{V'}{273+80}$

or $V' = \dfrac{353}{288} \times \dfrac{125}{1} = 153{\cdot}2$ c.cs.

6. A steel vessel used for storing compressed carbon dioxide has a safety valve, which opens when the pressure is 135 lbs. weight per square inch above that of the atmosphere, which may be taken as 15 lbs. wt. per sq. inch. At 27°C. the pressure of the enclosed gas is 100 lbs. wt. per sq. inch. At what temperature will the valve open, if the vessel were heated.

$$\text{We have } \frac{P}{T} = \frac{P'}{T'}$$

$$\text{or } \frac{100}{300} = \frac{150}{T'}$$

or $T'=450$; which is equal to $450-273=177$°C.

7. Assuming that the maximum summer temperature is 45°C. and the minimum winter temperature is 2°C., what allowance shall be made for expansion between successive rails each 20 feet long ? Co-efficient of linear expansion of steel=·000012.

8. A cube of which the sides are 100 cms. each at 0°C is raised to 100°C. If each of the sides becomes 101 cms. find the co-efficients of linear and cubical expansions. (C.U. 1918)

9. Find the density of silver at 175°, its density at 10°C. being 10·30 gms. per c.c.

10. The co-efficient of apparent expansion of glycerine in glass is ·000503 and its co-efficient of real expansion is ·00053. Find the linear co-efficient of expansion of glass.

11. The co-efficient of absolute expansion of mercury is ·00018. the co-efficient of linear expansion of glass is ·000008. Mercury is placed in a graduated tube and occupies 100 divisions of the tube. Through how many degrees must the temperature be raised to cause the mercury to occupy 101 divisions ? (*London Matric.*)

12. At a pressure of 80 cms. a gas has a volume of 750 c.cs. At *N.T.P.* its volume is 800 c. cs. What was the original temperature ?

13. Find the value of R in the equation $pv=RT$ for oxygen and hydrogen. given that the mass of one c c. of oxygen at 0°C. and 15 lbs. wt. per sq. inch pressure is 09 pound and one cubic foot of hydrogen under the same conditions has a mass of ·006 pound.

14. A given volume of air at 740 mms. pressure is at

17°C. What is the temperature, when its pressure is 1850 mms.? (*London Inter.*)

15. 200 c.cs. of air at 15°C. is raised to 65°C. Find the new volume, the pressure remaining unchanged. (C.U. 1915)

EXAMINATION QUESTIONS V

1. Distinguish between heat and temperature. Does a thermometer measure heat or temperature?

2. Describe Six's Maximum and Minimum Thermometer.

3. Convert the following temperatures (*a*) 80°F. to C.; (*b*)—40° F. to C and (*c*) 10°C. to F.

4. What is the relation between the real and apparent co-efficients of expansion of a liquid? Give a method of finding the apparent co-efficient of expansion?

5. Describe Hope's experiment to show that water has its maximum density at 4°C.

6. Assuming the truth of Boyle's and Charles' laws, prove that $pv=RT$.

7. Describe the construction and principle of a constant volume air thermometer. How can it be used to measure the boiling point of carbon dioxide?

8. What do you understand by absolute scale of temperature? Why is it important in the investigation of gaseous laws?

CHAPTER IV

CALORIMETRY

107. Heat a measurable quantity. We have already distinguished between heat and temperature and have also considered in detail the methods of measuring temperature. Now it is necessary to devise methods of measuring heat. It has been said that temperature corresponds to level of liquid, while heat corresponds to quantity of liquid. Just as the quantity of liquid in a vessel diminishes as the level of the liquid in it falls, similarly the quantity of heat in a body diminishes as its temperature is lowered. Further as the diminution in quantity of the liquid for a given fall in liquid-level depends upon the area of cross-section of the vessel, similarly the diminution in the quantity of heat in a body for a given fall of temperature depends upon the mass of the body. Lastly as the weight of the liquid, which has escaped from the vessel depends also upon its *density*; so does the quantity of heat in a body depend upon its *nature*, *i e.* upon its specific heat, which we will define presently. Thus W (the weight of liquid)$=l$ (the fall in level) $\times a$ (the area of cross-section) $\times d$ (the density) and Q (the quantity of heat)$=t$ (the fall in temp) $\times m$(the mass of the substance) $\times s$ (the specific heat).

To show the last result experimentally, let us take a block of ice fig. 28 and a copper ball. Heat the ball to 100°C. by holding it in the tube of a hypsometer and transfer it quickly into the ice. Some of the ice will melt and will be converted into water. Collect it and weigh, when the temperature of the ball itself has been lowered to that of ice. Thus the ball in cooling from 100° C. to 0°C. has given

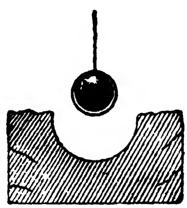


FIG. 28

up so much heat as has melted a certain amount of ice. Repeat the experiment with a ball, whose mass is double that of the first, we will find that the amount of ice melted is twice as much as in the former experiment. Similarly, if the first ball were heated initially to 200°C. instead of 100°C. we shall find again that twice as much ice melts as in the first case. *Thus the quantity of heat given out by a body is directly proportional to its mass and also to the fall of temperature.* Again if we were to take two balls of equal masses but one of copper and the other of lead and were to repeat the experiment, we shall find that the quantity of ice melted is much less in the case of lead than in the case of copper. Hence the quantity of heat given out depends upon the nature of the substance and this difference is said to be due to difference in their specific heats.

Hence the quantity of heat Q given out by a body in cooling from temperature t_2 to $t_1 = m$ (mass) $\times$ $(t_2 - t_1)$ $\times$ s (the specific heat).

108. Unit of heat. To measure a quantity of heat, we have to fix a unit. The unit chosen is that quantity of heat, which raises the temperature of a unit mass of water through unit degree.

Thus in the *C.G.S.* system, the unit of heat is the quantity of heat, which will raise the temperature of one gramme of water from 0°C. to 1°C. and it is known as a **Therm** or a **Calorie** or a **gram degree.**

While in the *F. P. S.* system, the unit of heat is the quantity of heat, which will raise the temperature of 1 ℔. of water from 0°F. to 1°F. and is known as a British Therm unit.

N. B. As the quantity of heat required to raise the temperature of a gramme of water from 0^0 to 1^0 is practically the same as that required to raise it from any temperature to the next higher one in the *C. G. S.* system, hence a *calorie* may be defined as the quantity of heat required to raise the temperature of one gramme of water through 1°C.

109. Specific heat. *Specific heat of a substance is*

the quantity of heat, measured in therms or calories, necessary to raise the temperature of one gramme of that substance through 1^0C.

It may also be defined as the ratio of the quantity of heat required to raise the temperature of a given substance through a certain range to the quantity of heat required to raise the temperature of an equal mass of water through the same range. Thus if Q units of heat are required to raise the temperature of a mass of water through t^0 and Q' units are required to raise the temperature of an equal mass of a given substance through the same range of temperature, then $\frac{Q'}{Q}=S$, the specific heat.

Thermal Capacity. *Thermal Capacity of a body is the quantity of heat necessary to raise the temperature of the given body through 1^0C.* It is evidently equal to the product of the mass of the substance and its specific heat. Thus the thermal capacity of a body $=m \times s$.

The total quantity of heat, gained or lost by a body during a given change of temperature, is equal to the product of the mass of the body into the specific heat of the substance of the body into the change of temperature considered, *i.e.* $Q=m \times s \times t$

110. Water-equivalent. Water-equivalent of a body is the *mass of water*, whose thermal capacity is equivalent to that of the body. Thus if w denotes the water-equivalent of a calorimeter of mass m and specific heat s, then $w=m \times s$; *i. e.* the water-equivalent of a body is the mass of water, which requires the same amount of heat to raise its temperature through 1^0C. as is required by the body for the same purpose; and is equal to the product of the mass of the body and its specific heat. In making calculations of the ultimate temperature attained, when heat is transferred from one body A to another B, it is assumed that no heat is wasted but that the whole of it is utilized in raising the temperature of the body to which it is transferred.

$$\left.\begin{array}{l}\text{Thus, Heat lost}\\ \text{by a body or set of bodies}\end{array}\right\}=\left\{\begin{array}{l}\text{Heat gained}\\ \text{by a body or set of bodies.}\end{array}\right.$$

Determination of water-equivalent of a calorimeter. Take a clean dry calorimeter with a stirrer, weigh it; then fill it about one-third with cold water and weigh it again. Hence find the mass of water taken. Heat water in a beaker approximately to a temperature, as much above that of the atmosphere, as the temperature of cold water is below it. Note the temperature of cold water and that of hot water in the beaker; pour hot water into the calorimeter quickly. Stir the mixture, note the final temperature and weigh again. Enter your results as follows:—

w_1 = wt. of empty calorimeter
w_2 = „ „ + cold water
t_1 = temperature of cold water and calorimeter
t_2 = temperature of hot water
t_3 = temperature of the mixture
w_3 = weight of cal. + mixt. of cold and hot waters

Heat lost = Heat gained.

Heat is lost by hot water; and is gained by cold water and calorimeter.

Thus heat lost by hot water $= (w_3 - w_2) \times (t_2 - t_3)$ and that gained by cold water and calorimeter

$$= \{(w_2 - w_1) + w\}(t_3 - t_1),$$

where w represents the water-equivalent of the calorimeter.

Equating, Heat lost = Heat gained, we get

$$w = \frac{(w_3 - w_2)(t_2 - t_3) - (w_2 - w_1)(t_3 - t_1)}{(t_3 - t_1)}.$$

Theoretically $w = w_1 \times s$, where s is the specific heat of the material of the calorimeter.

111. Determination of specific heat. Specific heat of a substance can be determined by three distinct methods: (*i*) *the method of mixture*, (*ii*) *the method of fusion of ice and* (*iii*) *the method of cooling*. Of these, the two latter will be described at a later stage at their

proper places, while now we proceed to describe the method of mixture.

(*a*) **To determine the specific heat of a solid by the method of mixture.** The solid, the specific heat of which is required, is put in the tube of a hypsometer and is heated to a known fixed temperature. During the time the solid is being heated, a calorimeter with stirrer and cover is weighed, first when it is empty and secondly when it is $^1/_3$ filled with cold water. The temperatures of cold water and hot solid are noted and the latter is quickly transferred into the former; the final temperature of the mixture is noted. The calorimeter is weighed once again in order to obtain the weight of the solid.

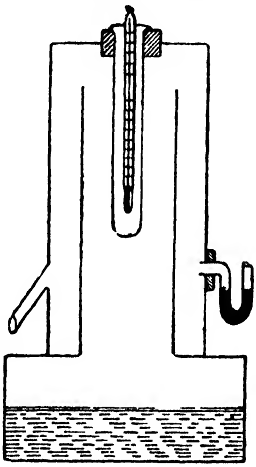


FIG. 29

The following observations are taken in order:—

1. The weight of the calorimeter $=w_1$
2. The weight of the calorimeter + cold water $=w_2$
3. Temperature of cold water $=t_1$
4. ,, ,, hot solid $=t_2$
5. ,, ,, mixture $=t_3$
6. weight ,, cal. + water + solid $=w_3$

Heat lost = Heat gained

Heat is lost by the solid the weight of which is equal to w_3-w_2 & Heat is gained by the cold water and the calorimeter.

$$\therefore \quad (w_3-w_2)S(t_2-t_3)=\{(w_2-w_1)+w_1s\}(t_3-t_1)$$

where S = specific heat of solid

and s = ,, ,, the material of the calorimeter.

The following precautions should be observed during the performance of the experiment—

1. The solid substance should be in small pieces.

2. The temperature of the solid should become constant.

3. The transference should be quick.

4. The calorimeter should be highly polished and screened from the burner.

5. In order to avoid error due to radiation, the temperature of cold water should be as much below the temperature of the atmosphere as the final temperature is to be above it.

6. A sensitive thermometer should be used to determine the final temperature of the mixture.

(*b*) **To determine the specific heat of a liquid.** The specific heat of a liquid is determined by the same method as described above, with the modifications that the *given liquid,* instead of water, is placed in the calorimeter and that the specific heat of the solid should be *known beforehand.* Then the equation to determine the specific heat of the liquid will be found to be

$$(w_3-w_2)s(t_2-t_3)=\{(w_2-w_1)S+w_1s\}(t_3-t_1).$$

In this equation s, the specific heat of the solid and s that of the calorimeter should be known and thus S the specific heat of the liquid becomes known.

(*c*) **Specific heat of gases.** Gases have two specific heats: (*i*) *at constant volume* and (*ii*) *at constant pressure.* The specific heat at constant pressure is always *greater* than that at constant volume; because to maintain a gas at constant pressure when it is being heated, it should be allowed to expand against the external pressure.

Thus work will have to be done by the gas and an equivalent amount of heat energy will be absorbed for the purpose, over and above the quantity of heat necessary to raise the temperature of unit mass of the gas through 1^0C. The specific heat of a gas at constant pressure is thus always *greater* than that at constant volume. Detailed methods of finding the two specific heats of a gas however, are outside the scope of this book. The specific heat of a gas at constant pressure was found by Regnault, by allowing the gas under constant pressure to be heated in a spiral, kept in an oil-bath and then bubbling it through a calorimeter;

while the specific heat of a gas at constant volume is obtained by Jolly's differential steam calorimeter.

This consists of two similar hollow spherical copper balls P_1 and P_2, provided with small pans to collect water. These are suspended from each arm of a balance and surrounded by a steam chamber S. The thermal capacities of the two pans are made exactly equal.

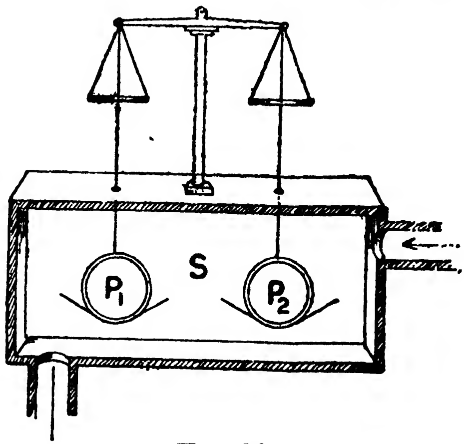


FIG. 30

Method Both the hollow balls are exhausted and exactly counterpoised. Now one of them is filled with the gas and counterpoised by adding weight w to the other pan. This gives the weight of the enclosed gas; θ_1 the temperature of the enclosed gas is also noted. Dry steam is now let into the chamber. Greater condensation takes place on the sphere which contains the gas, and this excess is found by the addition of weight w_2 to the other pan. The temperature θ_2 of steam is also noted. Then the specific heat of the gas at constant volume is given by $w_2 L = w_1 S(\theta_2 - \theta_1)$, where L is the latent heat of steam.

The advantages of a differential steam calorimeter are that common sources of error are automatically eliminated and the quantity of steam condensed due to the empty spheres is cancelled, with the result that calculations are simplified.

Note—Of all the solids and liquids as is evident from the tables of specific heats given below, water has the highest specific heat. Thus water absorbs a larger quantity of heat in being heated and gives out a larger quantity of heat

in being cooled through a certain range of temperature than an equal mass of any other solid or liquid. On account of this, water is used in hot water pipes and foot warmers.

TABLE OF SPECIFIC HEATS

SOLIDS

Aluminium	·213	Lead	·031
Brass	·094	Marble	·21
Copper	·095	Nickle	·109
Glass crown	·161	Platinum	·032
Glass flint	·117	Silver	·056
Gold	·032	Sulphur	·163
Iron	·114	Tin	·056
Ice	·504	Zinc	·096

LIQUIDS

Alcohol	·62	Petroleum	·511
Glycerine	·57	Turpentine	·46
Mercury	·033	Water	1·00
Olive oil	·471		

GASES (at constant pressure)

Air	·237	Oxygen	·217
Hydrogen	3·409	Steam	·47

To simplify calculations, specific heat of copper is generally taken as ·1 and that of ice as ·5.

SUMMARY

1. **Therm** or gram calorie is the quantity of heat required to raise the temperature of one gram of water through $1^{\circ}C$.

2. **Specific heat.** The quantity of heat measured in therms, which will raise the temperature of one gram of a substance through $1^{\circ}C$.

3. **Thermal capacity** of a body is the amount of heat, which will raise the temperature of the given body through $1^{\circ}C$. It is always equal to the product of the mass of the body and its specific heat.

4. **Water-equivalent** of a calorimeter is the amount of water, which will require the same amount of heat to raise its temperature through $1^{\circ}C.$, as will raise the temperature of the given calorimeter through $1^{\circ}C$.

EXAMPLES

1. A platinum ball weighing 80 grammes is heated to the temperature of a furnace and then dropped into a calori-

meter weighing 100 gms. and containing 390 grammes of water at 15°C. The final temperature is observed to be 20°C. Find the temperature of the furnace (C.U 1921)

Let the temperature of the furnace be T.

Heat lost=Heat gained.

$$80\times{}^{\cdot}032\times(T-20)=[390+100\times{}^{\cdot}1]5$$

$$T-20=\frac{400\times5}{2\cdot56}$$

$$T=781{\cdot}2+20=801{\cdot}2$$

2. 120 grams of a given liquid are contained in a copper vessel of mass 20 grams and are heated to a temperature of 100°C. and are then immersed in 300 grams of water at 13°C. in a copper vessel weighing 80 grams. The final temperature is observed to be 27·5°C. Find the specific heat of the liquid.

Let the specific heat of the liquid be S.

Heat lost=Heat gained

$$(120S+20\times{}^{\cdot}1)72\cdot5=(300+80\times{}^{\cdot}1)14{\cdot}5$$

$$\therefore \quad S={}^{\cdot}497$$

3. A copper vessel contains 100 gms. of water at 12°C., 56 gms. of water at 30°C. are added and the resulting temperature of the mixture is found to be 18°C. Find the water-equivalent of the vessel. (*London University*)

4. A piece of iron weighing 200 grams at 100°C. is immersed in 20 c. c. of water at 20°C. Find the resulting temperature, if the specific heat of iron be ·1124; the wt. of the calorimeter may be neglected

5. A mass of 700 grams of copper at 98°C. is put into 800 grams of water at 15°C, contained in a copper vessel weighing 200 grams and the final temperature is noticed to be 21°. Find the specific heat of copper. (S. and A. 1889)

6. 154 grams of a certain substance at 212°F. are placed in a vessel containing 182 grams of water at 15°C. and both come to a final temperature of 24°C. Calculate the specific heat of the body.

7. Find the temperature of a piece of iron weighing 10·4 lbs., which when immersed in 5·5 lbs. of water will raise its temperature from 14·8°C. to 26·4°C. (Sp. heat of iron =·119)

8. If 50 grams of lead shot (sp. heat=·031) at 97°C. are poured into 75 grams of a liquid at 31°C. contained in a calorimeter of water-equivalent 4·5 and the final temperature is 33°C. What is the sp. heat of the liquid?

CHAPTER V

CHANGE OF STATE

Liquefaction and Solidification

112. Liquefaction.—When heat is given to a substance, a rise of temperature may not take place. Indeed in some cases very large quantity of heat may be imparted without producing any rise of temperature. In such cases heat is utilized to change the state of aggregation of the substance.

Experiment.—Take sufficient quantity of ice in a calorimeter and place a thermometer in it. See that it denotes 0°C. Heat the calorimeter gently and continue to note the temperature. Notice that the thermometer indicates 0°C. till the whole of ice melts.

The change of solid into liquid state on heating is known as *liquefaction* or *Fusion* and the reverse process by which a liquid on cooling changes from liquid to solid state is called *freezing or solidification.*

Melting Point. The temperature at which a solid changes into the liquid form is known as the melting point of that substance and it is the same temperature at which the given liquid will solidify on cooling. When the temperature is above this point, the substance exists in the liquid state; and below it, in the solid state. When the temperature of the solid is rising liquefaction occurs here and when the temperature of the liquid is falling, solidification sets in at the same point. What happens at the melting point is, that the molecular motion increases so much as to overcome the force of cohesion, which in the case of a solid ordinarily preserves the molecules in their mean positions. The melting point temperature is different for different

substances, but for a given substance the melting point under normal conditions is a fixed temperature.

Laws of liquefaction. 1. Every crystalline substance begins to melt at a definite temperature under normal conditions.

2. During the process of conversion from solid to liquid state, the temperature remains constant till the whole of the solid is converted into the liquid form: and the rate of conversion depends upon the rate at which heat is supplied to the solid.

3. During conversion every substance requires per unit mass a certain amount of heat, *not* for any rise of temperature, called the *Latent heat of fusion* of that solid.

4. Those substances, which contract on fusion such as ice, have their melting-points lowered by an increase of pressure; while those substances, which expand by fusion such as wax, have their melting-points raised by an increase of pressure. Thus ice begins to melt at a temperature lower than 0°C. when subjected to high pressure and wax will melt at a temperature above 53°C. when subjected to high pressure.

In the case of amorphous bodies such as fats, wax, glass, iron etc there is no definite point, sharply marked, at which it can be said that the substance melts. It gradually passes through a viscous condition, where it possesses neither the properties of a solid nor those of a liquid distinctly.

In crystalline substances, melting point is the temperature at which the molecules arrange themselves in regular order. In amorphous substances there is no regularity of arrangement of molecules and hence there is no definite melting point.

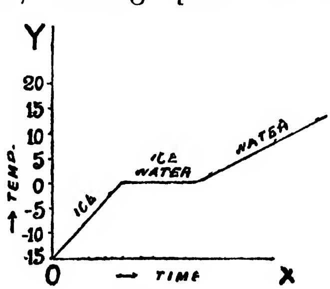


FIG. 31

If ice below 0°C. be taken, it will be noticed that on supplying heat to it, its

temperature will rise to 0^0C., then it will begin to melt and its temperature will remain constant till the whole of it gets melted and after that the temperature will begin to increase again. The graph will be of the shape shown in fig. 31.

Determination of Melting Point. When the melting point of a substance is neither too high nor too low, it is easily determined in the following way:— The solid is finely powdered and put into a thin capillary tube, which in turn is attached to the bulb of a thermometer by means of rubber bands. The whole is put into a beaker containing water, taking care that the open end of the capillary tube remains well above the surface of the liquid in the beaker. The water is gently heated and its temperature, when the solid in the capillary tube begins to melt, is noted and heating stopped. The water is kept well-stirred and allowed to cool and the temperature, when the liquid in the capillary tube begins to solidify, is noted. The mean of the two temperatures, *i.e.* that at which the melting as well as that at which solidification begins, gives the melting point of the given solid. For accurate work it is necessary to perform two experiments, one *preliminary*, to know the approximate melting point and the second, the *proper* experiment. In the latter case, temperature near the melting point is arranged to increase very slowly by regulating the supply of gas in the burner.

In the case of such substances as have comparatively higher melting points and do not pass through the plastic state, the method of cooling is resorted to. The substance is heated and allowed to melt. In the liquid state it is kept well-stirred, allowed to cool and its temperature noted after every minute till the liquid solidifies; a graph is drawn showing the relation between temperature and time. At the solidifying point, the temperature remains constant for some interval; that temperature gives the solidifying point or the melting point.

Whenever a solid is converted into the liquid form,

a change in volume takes place. In general a liquid occupies a space larger than its corresponding solid; but water, iron, bismuth and antimony are notable exceptions to the general rule. Water contracts on liquefaction and expands when it freezes, so is the case with iron. For this reason ice floats on the surface of water and iron floats on the surface of molten iron. Due to the above property, iron can be cast and sharp well-defined impressions produced; while silver and gold which contract on solidification, have to be stamped with a dye. The contraction of phosphorus, on the other hand, prevents its adhering to the mould in which it is cast.

The change in volume, which takes place when 1 gm. of ice at $-5°$C. is heated to 10°C., is shown beautifully by means of the graph in fig. 32.

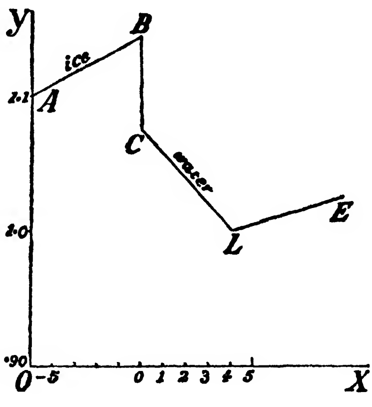


Fig. 32

The portion AB shows the expansion of ice between -5 and 0, BC shows the melting stage when the volume decreases without any rise of temperature, CL the contraction of water between 0°C. and 4°C. and LE the expansion of liquid with rise of temperature.

The fact that ice is lighter than water and floats on its surface is of great value in the economy of Nature; because if it were not so, then in the cold regions, frost in winter would have converted ponds, wells, lakes and seas into one block of solid ice by exposing fresh layers of water every time, and thus would have destroyed all aquatic life.

The expansion of water on freezing takes place with great force. Take a hollow cast-iron bottle and fill it with water to the brim, screw the plug and place

it in freezing mixture. Notice that the bottle bursts.

113. Regelation. Faraday showed that if two pieces of ice be taken, pressed for some time and then released, they will be found frozen together at the region of contact. The explanation of this is that the pressure applied lowers the melting point below 0^0C. Some of the ice at the points of contact melts and the temperature of surrounding points falls; because heat is required to melt ice and that is taken from the adjoining points. On releasing the pressure the water, being below its freezing-point, freezes again and thus joins the two pieces together *This phenomenon, in which ice melts when subjected to increased pressure and the liquid solidifies again when the pressure is released, is called regelation.*

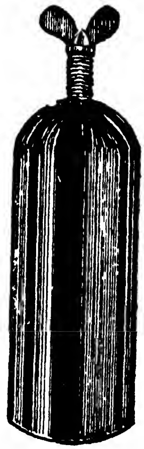


Fig. 33

The following beautiful experiment showing the phenomenon of regelation is due to **Bottomley**:—Take a big block of ice and support it on two tables so that it bridges the space between them Place a piece of long thin *copper* wire across the middle of the block between the supports and attach heavy weights to each end. It will be observed that the wire cuts its way slowly through the ice, without dividing the blocks into two pieces.

The explanation is that ice, immediately under the wire, is subjected to great pressure, it melts and heat necessary for the purpose is taken from its surroundings. The wire makes its way down and the water escapes to the upper surface; being released, it freezes again. During the process of solidification, it gives out heat, which is readily conducted by the copper wire downwards, and which further helps to melt the ice immediately below the wire. This explains why a

copper wire, which is a good conductor of heat, cuts its way through ice more easily than iron wire and why a piece of string will not cut its way at all.

The formation and motion of glacier are both explained by the phenomena of regelation. A glacier is a huge mass of ice, which moves very slowly from the place of its formation on high mountains above the snow-line to the valleys down below. The bottom layers of this huge mass liquefy under the great pressure of the superincumbent mass, flow down and solidify immediately. Thus in high mountains, we get large quantities of ice, being continually supplied from the snow above the snow-line.

114. Determination of latent heat of water. *The latent heat of fusion of ice generally spoken of as the latent heat of water is the quantity of heat required to convert one gram of ice into water at the same temperature. It is determined in the following ways.—*

(*a*) **Method of mixtures** Take a copper calorimeter with a wire-gauze stirrer fig 34. Weigh and fill it about one-half with water at a temperature of about 10^0C, above the atmospheric temperature. Weigh it again. Take ice nearly 1/8th of the mass of water in the calorimeter. Wrap it in a flannel, which will prevent its further melting. Break it into pieces, neither too big nor too small. Dry the pieces by a blotting paper and drop them quickly into the calorimeter. Stir it and note the lowest temperature reached, of the mixture. This temperature should be about as much below the atmospheric temperature as the initial temperature of the water in the calorimeter was above it. Remove the thermometer and weigh it again. Equate heat lost by warm water and calorimeter in cooling from the initial temperature to the final temperature, with that absorbed by ice.

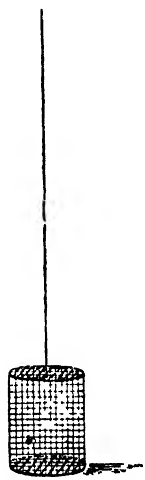


FIG. 34

It should be clearly noted that **ice absorbs heat in two instalments,** *first in being converted from ice into water without any rise of temperature and secondly in raising the temperature of water formed by the fusion of ice, from 0°C. to the final temperature of the mixture.*

Enter results in the following manner :—

1. Weight of empty calorimeter and stirrer $= w_1$
2. „ „ „ etc. + warm water $= w_2$
3. Temperature of water and cal $= t_1$
4. Temperature of the contents of cal. after mixing ice. $= t_2$
5. Weight of cal. + water + ice added $= w_3$

Heat lost = Heat gained

$$[(w_2 - w_1) + w_1 \times \cdot 1](t_1 - t_2) = L(w_3 - w_2) + (w_3 - w_2)t_2$$

$$\therefore \quad L = \frac{[(w_2 - w_1) + w_1 \times \cdot 1](t_1 - t_2)}{(w_3 - w_2)} - t_2$$

The latent heat of fusion of ice is found to be 80 therms per gram.

(*b*) **Method of Ice Calorimeters.** Ice calorimeters are used for the purpose of either finding the latent heat of fusion of ice when the specific heat of a given substance is known; or assuming the latent heat of fusion of ice as 80, they can be used to determine the specific heat of the substance.

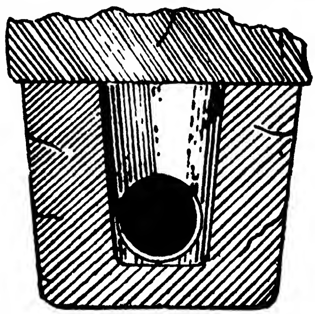


FIG. 35

1. **Black's Ice Calorimeter.**—Take a block of ice about 4″×4″×4″ and make a nearly spherical cavity in it. Take another slab of ice and make its surface smooth, so as to serve as an air-tight cover for it. This constitutes Black's Ice Calorimeter, fig. 35.

The determination of latent heat of fusion of ice or specific heat of a given substance, is carried on in the following manner :—The cavity is thoroughly dried by a blotting paper. A solid of mass *m* is heated to a known temperature *t*, say that of steam, by holding it in the steam-jacket of a hypsometer. It is quickly transferred to the cavity and

the covering slab is laid over it. After some minutes, the solid mass will attain the temperature of ice *i.e.* 0^0C. and a certain amount of ice will be converted into water at 0^0C. by the heat given up by the solid.

The whole quantity of water formed by the melting of ice is very carefully weighed by transferring it to a beaker previously weighed, the cavity and the solid being thoroughly dried by means of weighed filter papers. The results are entered thus :—

1. Weight of solid $=w_1$
2. Initial temperature of hot solid $=t_1$
3. Weight of beaker and filter papers $=w_2$
4. ,, ,, ,, ,, after pouring water formed from the melting of ice $=w_3$

Heat lost=Heat gained

$$w_1st_1=L(w_3-w_2).$$

This method does not require any elaborate apparatus and the use of the thermometer is also limited to taking one reading only. If s, the specific heat of the solid is known, L can be calculated; and if L be assumed as 80, then the specific heat of the solid can be calculated from the above equation.

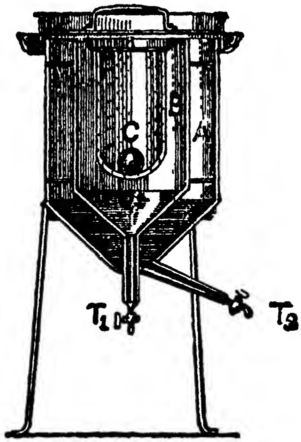


FIG. 36

(2) Laplace and Lavoisier's Ice Calorimeter. It consists as shown in figure 36 of three copper vessels; the outermost A, the inner B and the innermost C. The space between A and B as well as that between B and C is thoroughly packed with ice. Tubes T_1 and T_2 drain the water from inside B and A respectively. The external chamber between A and B prevents any heat from being communicated to the ice in B from external sources, and thus acts as a guard. The whole apparatus as shown is kept for some time, till water altogether ceases to flow from the tube T_1. At this stage the heated solid, as in Black's Ice Calorimeter, is placed in the innermost vessel C and the lids closed. The heat, com-

municated from the solid, melts some ice in B and the water so formed is collected by the tube T_1 in a previously weighed beaker. The calculations are similar to those in Black's Ice Calorimeter.

Both the above calorimeters, except being of historical interest, are of little practical value. In both cases, it is rather difficult to find the mass of ice melted.

(3) Bunsen's Ice Calorimeter. It is designed on the principle that when ice is converted into water at 0^0C., a diminution in volume occurs. Knowing exactly the diminution in volume, which 1 gm. of ice undergoes and also the total diminution, we can find out the number of grammes of ice melted and hence the quantity of heat absorbed.

It consists of a thin-walled test tube A fig 37, fused to a big glass cylinder BC; the lower end of which communicates with a bent tube $CDEF$. At F the tube communicates with a capillary tube of uniform bore, having a scale along it.

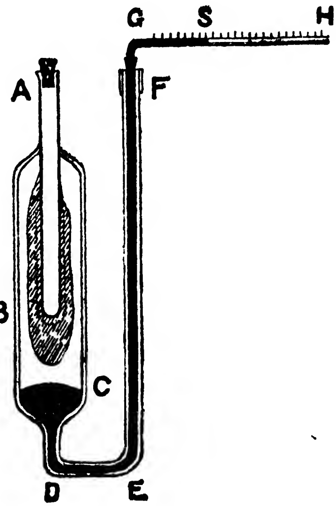


Fig. 37

The upper part of BC contains pure distilled water, while the lower part contains mercury, which also fills the bent tube and a portion of the capillary tube.

In order to use the instrument, the whole is kept in melting ice for several hours, so that the temperature of calorimeter and its contents may become 0^0C. A little quantity of solid carbon dioxide is introduced in A to freeze some water in B, due to local overcooling. This little ice-sheath is made to grow to the required size by pouring ether into A and blowing a gentle current of air through it. After this a quantity of cold water is introduced in A, so as to equalize the temperature in B to 0^0C., even if it had gone down. The whole instrument is still kept in melting ice. Now to determine the specific heat, a small weighed quantity of the solid is heated to a known temperature and then

quickly dropped in A. Heat is communicated to the water contained in it, which becomes denser (for water has its maximum density at 4°C), goes down and gives its heat to ice in B which melts and the process continues till the temperature of all is 0°C. Contraction takes place due to this melting of ice and the mercury-thread in the capillary tube recedes. By its reading, total diminution in volume is ascertained. Let it be v, then the total mass of ice melted will be equal to $\frac{v}{\cdot 09}$ grams; for ·09 c.c. is the decrease, when 1 gm. of ice melts. Heat to melt it, is given by the hot solid, therefore

$$MST = L \frac{v}{\cdot 09}$$

where M=mass of solid, S=the specific heat and T its temperature. If L *i.e.* the latent heat of water is known, then we can find out S the specific heat.

This calorimeter is difficult to set up, but is very accurate and sensitive. It is capable of measuring upto 1/10th of a therm. It is used for the measurement of specific heats of substances procurable in small quantities. No thermometer is needed, no radiation-correction to be applied and no water-equivalent is to be determined.

The fact that the latent heat of water is very high, is of very great use in the economy of Nature. Water freezes slowly and thus during frosty weather, only the upper layers of lakes, seas etc. are frozen. Thus aquatic life is preserved. Snow melts slowly and thus prevents the countryside from being flooded. If the latent heat of water were low, the height of the snow-line would have increased.

115. Freezing Mixtures. To change solid into liquid state, heat is required; this principle is made use of in freezing mixtures. Pour a quantity of ammonium nitrate into a beaker of water, observe that the temperature falls considerably due to the latent heat required to convert solid ammonium nitrate into liquid state—being derived from water, which is thus cooled. The following freezing mixtures are commonly used:—

(1) Common salt is mixed with pounded ice, both change into the liquid state and the resultant temperature goes down to −22°C.

(2) Calcium chloride and ice mixed in the proportion of 4 : 3 produce a temperature, of -51°C. nearly.

SUMMARY

1. The process of conversion of solid into liquid state is called **Liquefaction** or *Fusion* and the reverse process is called **Freezing** or *Solidification*.

2. The amount of heat required to convert one gram of a solid substance into the liquid state is called the **latent heat** *of fusion of that substance.*

3. The melting of ice under increased pressure and its re-solidification under reduced pressure is known as **Regelation.**

4. Freezing mixtures are produced by mixing a soluble salt in pounded ice.

EXAMPLES

1. What amount of heat will be required to melt 15 lbs of lead, the initial temperature of lead being 15°C.

$$\left\{\begin{array}{l}\text{Given sp. heat of lead}='031\\ \text{Melting point of lead}=320^{\circ}\text{C.}\\ \text{and latent heat of laed}=507\end{array}\right\}$$

The quantity of heat required to raise its temp. from 15° to $320^{\circ}=15\times305\times{}^{\cdot}031=141{}^{\cdot}825$ lbs. cals.

The quantity of heat required to melt 15 lbs. of lead $=15\times507=7605$ lbs. cals.

$\therefore$ the total heat required$=7746{}^{\cdot}825$ lbs. cals.

2. ·87 gramme of a substance is heated to 98·6°C. and then dropped into Bunsen's Ice Calorimeter. The contraction observed is 7·9 cubic mms. Find the sp. heat of the substance ($L=80$ and contraction per gm. of ice melted $={}^{\cdot}09$ cubic centimetres).

The quantity of ice melted$=\dfrac{{}^{\cdot}0079}{{}^{\cdot}09}$ gms.$={}^{\cdot}087$ gm.

The quantity of heat absorbed$=80\times{}^{\cdot}087$
$=6{}^{\cdot}96$ therms.

Heat given out by the solid$={}^{\cdot}87\times98{}^{\cdot}6\times S$

$$\therefore\quad S=\frac{6{}^{\cdot}96}{{}^{\cdot}87\times98{}^{\cdot}6}={}^{\cdot}0814$$

3. 10 gms. of ice at -10°C. are mixed with 120 gms. of water at 80°C. Find the final temperature of the mixture?

4. A copper calorimeter weighing 100 gms. contains 100 gms. of water at 16^{0}C.; 20 gms. of ice at -10^{0}C. are poured into it. Will all the ice melt? If so, what will be the final temperature of the mixture?

$$\left.\begin{array}{l}\text{Given sp. heat of ice} = 0{\cdot}5 \\ \text{and ,, ,, ,, copper} = {\cdot}094\end{array}\right\}$$ (Sen. Camb. Loc.)

5. 10 gms of water at 90^{0}C. are placed in the tube of a Bunsen's Ice Calorimeter and the contraction noticed is 1·09 c.c. Find the sp. gravity of ice.

6. A calorimeter weighing 50 gms. contains 100 gms. of a mixture of ice and water, 1000 gms. of copper at 90^{0}C. are poured into it and the final temperature is observed to be 10^{0}C. Find the quantity of ice in the mixture. Sp. heat of copper = 0·1

7. If 1 c.c. of water in freezing becomes 1·09 c.c. of ice and the introduction of 10 gms. of a substance at 100°C, into a Bunsen's Calorimeter, causes the end of the column of mercury to move through 260 mms. in a tube 1 sq. mm. in section; find the sp. heat of the substance.

8. Heat is continuously applied to a mass of ice at -10°C. until it becomes steam at 100°C. Trace the changes in volume and temperature.

CHAPTER VI

CHANGE OF STATE (*continued*)

Evaporation and Ebullition

116. Evaporation. The molecules of a substance are presumed to be in a state of motion. In solids this motion of molecules is about a constant mean position; in liquids this is not restricted to a mean position, but is hindered by frequent collisions with other molecules; and in gases the motion is much more rapid due to lesser number of molecules in the same space.

The heat contained in a body is supposed to be merely the sum of the kinetic energies of its various molecules; and the effect of giving heat to a body is to throw its molecules in a violent state of agitation.

In a liquid at ordinary temperature, the molecules near the surface of a liquid are restrained ordinarily from going out by the straining force of surface tension; but even then some of the molecules, having greater velocity than the rest, manage to get out of the liquid. The higher the temperature, the greater the number of molecules, which so escape. This phenomenon is spoken of as *evaporation. It is the process by which a liquid is converted into vapour at all temperatures.*

Experiment. Take a little quantity of ether in a shallow dish, see that it evaporates away quickly. Repeat the experiment with warm water. Notice that it takes a long time for the liquid to evaporate.

117. Ebullition or Boiling. We have seen above that evaporation continues at all temperatures, it takes place at the surface and is necessarily a slow process. If however, a liquid be heated, it is noticed that at

first its temperature continues to rise till bubbles of the gas are seen to rise *throughout its mass* and the liquid is quickly converted into vapour form, the *temperature remaining constant* till the whole of the liquid is so converted. This is called *boiling or ebullition* *It is the process of conversion of a liquid into vapour form at a definite temperature and throughout its mass.* The definite temperature is called the *boiling point* and is constant for the given liquid under the same pressure. It is the temperature at which the vapour of the given liquid exerts pressure, equal to that of the atmosphere.

Laws of Ebullition:—

1. Every liquid begins to boil at a definite temperature under normal pressure.

2. Every liquid requires per gram a definite amount of heat, to convert it into vapour without any rise of temperature; and that amount of heat is known as the latent heat of vaporization.

3. The temperature during boiling remains constant till the whole of the liquid is converted into vapour.

4 The boiling point of a liquid is raised by increased pressure and it is lowered by a fall of pressure.

Influence of pressure on the boiling point of a liquid. Take a tube with one end closed, bend the closed end in the form of a U as shown in fig. 38. Fill the bend and limb *B* with mercury, taking care that the level of mercury in *A* is lower than that in *B*, introduce a little water in the limb *B*. Hold this in a flask containing water as shown, so that the bend is clearly above the level of the liquid.

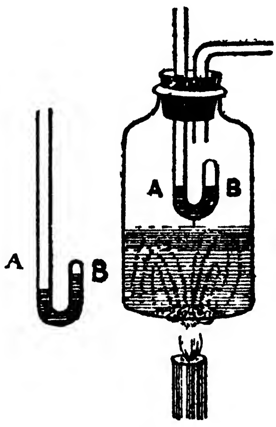


Fig. 38

Heat the flask till the liquid in it begins to boil

freely; observe that the mercury stands at equal heights in both the limbs, showing that the pressure of aqueous vapour in *B* is equal to the atmospheric pressure. This proves that at the boiling point, vapour of a liquid exerts the same pressure as exists on its surface.

Thus by increasing or decreasing the pressure on the surface of a liquid, it may be made to boil at any temperature. The following beautiful experiment showing the boiling of water at a temperature considerably below 100°C., under decreased pressure, is due to Franklin.

Heat water in a strong flask; when it begins to boil freely and the whole of the air has been driven away by the steam, cork the bottle, remove the burner and invert the flask on a ring as shown in fig. 39. Allow the flask to cool in this position, till boiling stops altogether.

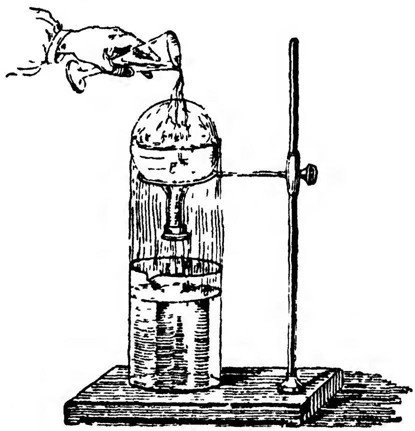


Fig. 39

At this time the space above the liquid is filled with vapour, which exerts pressure and does not allow the liquid to boil at this low temperature. Condense the vapours by sprinkling cold water on the bottom with a sponge The pressure due to the vapour on the liquid surface will be released and the liquid will be seen to boil vigorously, till the space above is again filled with it. By sprinkling more water, the process may be repeated.

At high altitudes, the pressure of the atmosphere is much below the normal and water begins to boil below 100°C. Thus at Simla, which is 6700 feet above the sea-level, water boils at 93·5°C. In order to cook

certain kinds of food, it is necessary to raise the temperature to 100°C. For this, **Papin's Digester** is used to raise the boiling point of water by increased pressure.

It consists of a strong metal vessel with a lid held in position by a screw. In the lid is a valve closed by means of a lever, carrying a movable weight. On heating the water, the vapour given off by the liquid does not escape till its pressure is sufficient to lift the valve; thus it exerts increased pressure on the surface of the liquid and thereby increases its boiling point. By adjusting the position of the movable weight along the lever, water may be made to boil at any convenient temperature.

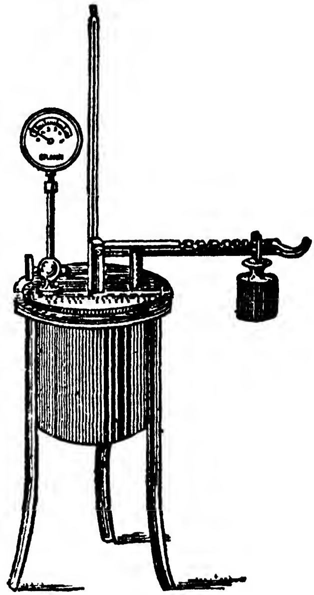


FIG. 40

118. Determination of latent heat of steam. Heat water in a tin vessel, provided with a delivery tube. In the meanwhile, take a copper calorimeter with a stirrer and cover and weigh it. Fill it about two-thirds with cold water and weigh it again, the temperature of water being about 10° below the atmospheric temperature. Note the temperature of cold water. When steam begins to come out freely, dry the mouth of the tube with a little cotton-wool and dip it in the liquid contained in the calorimeter, so as to pass steam into it. Continue passing steam till the temperature rises through about 20°C., *i.e.* it is about 10°C. above the atmospheric temperature. Remove the calorimeter and note the highest temperature reached. Allow the calorimeter to cool and weigh it again Note the atmospheric pressure. Enter your results thus:—

1. Weight of cal., stirrer & cover $= w_1$
2. " " " " + cold water $= w_2$
3. Temperature of cold water $= t_1$
4. Temperature of water after passing steam ... ... $= t_2$
5. Weight of cal. + stirrer and condensed steam . . . $= w_3$
6. Pressure of Atmosphere $= A$ mms. of mercury column

∴ Temperature of steam =

$$100 - {\cdot}037(760 - A) = t_s$$

Heat lost=Heat gained.

$$L(w_3 - w_2) + (w_3 - w_2)(t_s - t_2) = [(w_2 - w_1) + w_1 \times {\cdot}1](t_2 - t_1).$$

From this the value of L, the latent heat of steam, is obtained. It is equal to 540 therms.

Two points deserve special notice. The temperature of steam is not to be assumed as 100°C. but should be obtained by the formula $t_s = 100 - {\cdot}037(760 - A)$. Steam loses heat in two instalments; it gives out a quantity of heat in being condensed from steam into water, at the same temperature and then it gives out heat in being cooled from the temperature of steam to the final temperature of the mixture.

119. Evaporation The conversion of a liquid into vapour state at ordinary temperatures is known as evaporation. If it takes place in a closed space, a limit is soon reached; the vapour on the surface of the liquid becomes saturated and no more liquid can be converted into vapour state. At that time as many molecules return to the liquid as leave the liquid. If however, the space above the liquid is not closed, but is unlimited; evaporation continues steadily, but its rate is accelerated by the following factors:—

1. High temperature of the liquid,
2. High temperature of the atmosphere,
3. Low boiling point of the liquid,
4. Largeness of the exposed surface of a liquid,
5. Low pressure of the atmosphere,
6. Dryness of the atmosphere, and

7. Removal of the air in contact with the liquid surface.

Whenever evaporation takes place, heat is required for the purpose If it be not supplied from any external source, the heat necessary for the purpose is derived from the liquid itself and thus its temperature falls. It is well illustrated by **Wollaston's cryophorus,** which consists as shown in fig. 41 of a long bent tube with a bulb at each end. it contains a small quantity of water with its vapour only and no air. All the water is transferred into the bulb *A*, while *B* is kept in freezing mixture. The vapour in *B* condenses and the liquid in *A* evaporates so quickly as to lower the temperature of the liquid in it to 0°C. and freeze it.

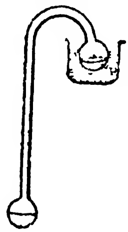


Fig 41

The same principle is used in refrigerating machines for the manufacture of ice by the evaporation of liquid ammonia.

119. (*a*) **Carre's Freezing Machine.** It consists of a boiler *H* (Fig. 41 *a*), which contains a *strong solution of ammonia.* This boiler communicates by a bent tube with the space between two concentric chambers of the conical double-walled vessel called the freezer The whole apparatus is made of strong galvanised iron plate and can bear a pressure of about 8 atmospheres. A tube is generally inserted in the upper part of the boiler. this is filled with oil and carries a thermometer to denote the temperature. Due to its high boiling point, oil is used to ensure thorough contact between the thermometer and the tube. The inner chamber of the freezer receives a metal vessel, containing the water to be frozen.

The process of freezing consists of two separate operations. *First*, the boiler is heated to a temperature of about 130°C., while the freezer is surrounded by a cold-water bath. The ammonia gas is expelled from the solution in the boiler and condenses in the space between the two concentric chambers of the freezer, due to its own high pressure and the comparatively low temperature. In this process a small quantity

of water also condenses along with the gas. When sufficient quantity of ammonia gas has been condensed, the *second* stage of the operation begins. The freezer is covered thoroughly from outside with flannel or other non-conducting material, the cylinder containing the water to be frozen is placed in the interior chamber of the freezer and the boiler is surrounded by a cold-water bath. As the boiler cools, the ammonia gas **dissolves** in the water and the liquid ammonia in the jacket of the freezer rapidly evaporates. This

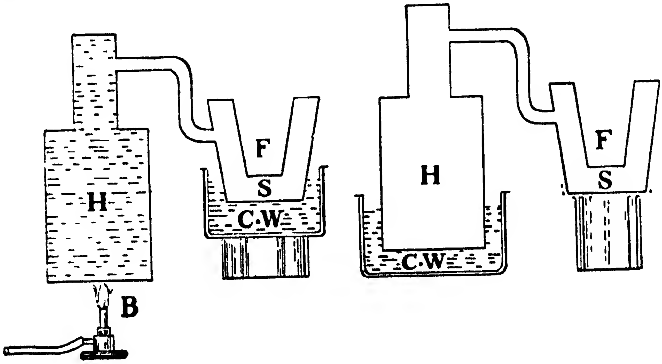


FIG. 41 (*a*)

process continues, *due to the absorption of ammonia by water in the boiler*. During the process of evaporation of the ammonia gas, the temperature of the freezer falls and the water in the cylinder solidifies. To ensure good contact between the water cylinder and the interior of the freezer, a liquid of low freezing point is poured in between them. The apparatus is used only when small quantities of ice are required as in about $1^1/_2$ hours some two seers of ice are thus produced.

119. (*b*) **Ammonia Ice Plant.** It is a vapour-compression refrigerating machine. The cooling is produced by alternate condensation and evaporation of the fluid used. The fluid takes in heat from the substance to be cooled, during evaporation at a low pressure; it gives out heat, during condensation at a high pressure. The fluid used must not have a very low vapour-pressure at low temperatures, or a very high vapour-pressure at high temperatures. The fluids most commonly used are ammonia and carbonic acid. Ammonia has a convenient range of

vapour-pressures; but it acts chemically on copper and brass. For this reason all parts of ammonia plant, that have to come in contact with the working substance, are made of iron. The apparatus as shown in the diagram consists of two coils of metal pipe *A* and *B*, joined *above* through a compression pump and *below* through the narrow regulating valve.* The coil *A* is surrounded by a vessel containing salt solution. This vessel has outlets at the top and bottom, communicating with a large tank in which are placed the metal vessels containing the water to be frozen.

To freeze water, the pipes are partly filled with ammonia, which has been liquefied by pressure and cooling, and the compression-pump piston is set in motion. When the piston descends, some of the liquid ammonia in *A* rapidly evaporates, the 'latent heat' required for the purpose is taken from the salt solution, and its temperature falls. When the piston moves up, the ammonia vapour is compressed and driven into the coil *B*, where it is liquefied by being cooled by the cold water circulating round it The process is repeated till the temperature of the salt solution falls below zero degree and thus water freezes to form ice.

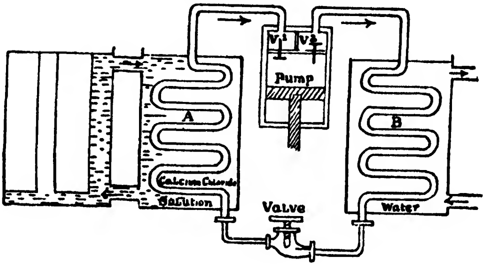


FIG. 41 (*b*)

SUMMARY

1. The conversion of a liquid into vapour form at *ordinary* temperatures is known as **Evaporation** The conversion of a liquid into vapour form at the *boiling-point* temperature of the liquid is known as **ebullition.**

2. At boiling-point temperature, the vapour of a given liquid exerts pressure equal to the atmospheric pressure.

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*In the compression pump at the bottom, are two valves v_1 and v_2 communicating separately with pipes *A* and *B* The valves open in opposite directions, *i e.* v_2 opens when pressure is applied by the upward motion of the piston and v_1 opens when vacuum is produced by the downward motion of the piston. See Fig. 41 (*b*).

3. The boiling point of a liquid falls with the fall of pressure.

4. The quantity of heat required, to convert 1 gm. of water at its boiling point into vapour at the same temperature, is called **latent heat of steam** and is equal to 540 therms.

EXAMPLES

1. Find the amount of heat required to convert 20 gms. of ice into steam at 100°C.

(*i*) Heat required to convert ice into water at 0^0C.
$=20\times 80=1600$ cals.

(*ii*) Heat required to raise its temperature from 0^0 to 100^0C. $=20\times 100=2000$ cals.

(*iii*) Heat required to convert water into steam $=20\times 540=10800$ cals.

$\therefore$ the total heat required $=14400$ cals.

2. A vessel, the mass of which may be neglected, contains 250 gms. of ice at 0^0C. Steam at 100^0C. is passed into it. Find the total quantity of water in the vessel· (*a*) when the ice has just melted and (*b*) when the temperature has risen to 100^0C.

(*a*) Let x gms. of steam be passed to convert the whole of ice into water, then

$x(100+540)=250\times 80$

or $x=31\cdot 4$ gms.

$\therefore$ the total amount of water will be $250+31\cdot 4$
$=281\cdot 4$ gms.

(*b*) Let y be the wt. of steam condensed in the second case, then $y\times 540=250(80+100)$

or $y=83\cdot 6$ gms.

$\therefore$ the total quantity of water in the calorimeter
$=250+83\cdot 6$
$=333\cdot 6$ gms.

3. How many grams of ice can be melted by 40 gms. of steam at 200^0C.? (*P.U.* 1925)

4. 7 grams of ice float in water in a calorimeter of thermal capacity 5 calories. When 4·5 grams of steam at 100^0C. are passed into the calorimeter, the final temperature becomes 50^0C. How much water was there in the calorimeter?

5. 50 gms. of steam at 100^0C. are passed into a mixture of 100 gms. of ice and 200 gms. of water. Find the rise of temperature, the latent heat of steam being 527.

6. A calorimeter weighing 70 gms. contains 150 gms. of a mixture of ice and water. Steam is passed into it, till the final temperature reaches 10^0C. The weight of the steam condensed is found to be 10 gms. Find the amount of ice in the mixture.

7. Explain the construction and action of some kind of practical freezing machine, that does not require a freezing mixture. (P.U. 1931)

CHAPTER VII

CHANGE OF STATE (*concluded*)

Properties of Vapours

120. Saturated and unsaturated vapours. We have already described what evaporation means, but we have not as yet studied in detail the various properties of the vapour so formed. For this purpose, the instrument shown in fig. 42 is most useful. It consists of a wide glass tubing about 36 inches long, graduated in centimetres. It is closed at the upper end by a tap S, above which is a small funnel F. The other end is connected by a rubber tubing to a wide-mouthed glass tube R. The tap is of a special form as shown separately. The hole is not right through it, but is half deep only. When a liquid is placed in the funnel F, each time the tap is turned on completely, a small quantity of it is discharged into A without the air outside communicating with it. In order to put the instrument ready for use, the tap S is removed and mercury poured in R. It is then raised till the mercury reaches the level of S in A. The tap is put in position and a liquid, to be experimented on, say water, is poured into the funnel F. The reservoir tube R is then lowered till a vacuum of a few cubic centimetres is produced over the surface of mercury

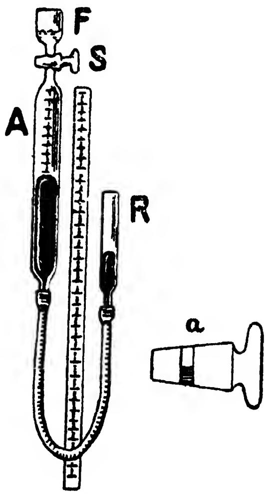


FIG. 42

in A. At this time the difference in the heights of mercury in A and R will be equal to the barometric height.

Turn the tap so as to drop a little quantity of water into A, it will evaporate immediately and exert pressure as will be shown by the lowering of the mercury-level in A. If at this stage, the volume be decreased by raising R, the product of the pressure and volume of the vapour will remain fairly constant. This vapour which obeys Boyle's law is called *Unsaturated vapour.*

If more water be introduced in A by turning on the tap several times, only a part will evaporate and some of it will remain in the liquid state. If at this stage the volume be decreased by raising R, it will be noticed that the pressure does not increase but that a part of the vapour is condensed. This vapour is called *saturated vapour*. The pressure of this vapour is dependent only on the temperature and *not* on the volume, as it does in the case of unsaturated vapours.

Thus unsaturated vapour is that vapour which, at a given temperature, is not exerting its maximum pressure. Its pressure can be increased by decreasing the volume and it can take more of the liquid in the vapour state.

Saturated vapour is that, which is exerting the maximum pressure at the given temperature. Its pressure does not increase with decrease of volume and it cannot take more of the liquid into the vapour state for the same volume and temperature. The pressure exerted by the saturated vapour of a liquid is called the vapour-pressure or vapour-tension of that liquid, at the given temperature.

Behaviour of unsaturated and saturated vapours under changes of volume and temperature.

(*a*) It has been observed above that the pressure of unsaturated vapour depends like that of a gas on the volume which it is made to occupy. By surrounding A with a hot bath, we can observe that its pressure, like that of a gas, increases with rise of temperature and decreases with the fall of temperature, till it becomes

saturated. When the vapour becomes saturated, the fall of pressure with cooling is most marked. Thus we can say that unsaturated vapours behave like gases and approximately obey Boyle's and Charles' laws.

The behaviour of unsaturated vapour is shown graphically in fig. 43 (*a*). From *A* to *B* the vapour is unsaturated and the pressure-volume curve is just like that of a gas. At *B* the volume is diminished so much that the vapour is saturated and the pressure, on decrease of volume, does not increase but remains constant, as is shown by the horizontal line *BC*.

Similarly fig 43 (*b*) shows that if unsaturated vapour

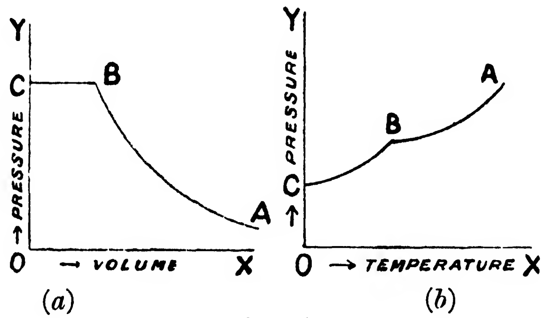


FIG. 43

be cooled, then at first the vapour behaves like a gas. Its pressure diminishes with fall of temperature as is shown by the line *AB*, which if produced backwards, will cut the axis of temperature at -273^0C. This being the temperature, at which a gas cooled at constant volume exerts no pressure. At *B* the vapour is exerting its maximum pressure and any diminution in temperature is accompanied by a very rapid fall in pressure, as is shown by the curve *BC*, which indicates that saturated vapour does not obey Charles' Law.

(*b*) If the volume of saturated vapour be decreased, its pressure will remain the same but some of the vapour will be condensed as is shown by the horizontal line in fig. 43 (*a*). If however, the volume be increased, and the vapour is in contact with the mother-liquid

its pressure will remain the same and more of the liquid will be converted into vapour form. When the vapour is not in contact with its mother-liquid, increase of volume will result in the vapour becoming unsaturated; and its pressure will fall as shown by the curve BA in fig. 43 (a).

If the temperature of saturated vapour *in the presence* of its liquid be raised, the pressure of the vapour will rise with temperature and more of the liquid will be converted into vapour, as shown by the curve CB in fig. 43 (b). If however, the temperature be increased *in the absence* of its liquid, the pressure of the vapour will rise correspondingly less with rise of temperature, than if the liquid were present; and the vapour will become unsaturated. When the temperature of saturated vapour is lowered, its pressure is diminished more rapidly and condensation takes place.

121. Pressure exerted by a mixture of vapours and gases. Dalton and later on Regnault, experimenting with the apparatus shown in fig. 42, determined the pressure exerted by a vapour. (*i*) in vacuum and (*ii*) when the same space was occupied by other permanent gases or vapours, not having any chemical affinity with one another. Their conclusion is *that the pressure exerted by a vapour is the same whether the space is empty or filled with some other gas or vapour.* This statement put in the following way is known as Dalton's law:—

Dalton's law of partial pressures. *The mass of vapour, which can be contained in a given space, is independent of the presence of other gases or vapours; and the pressure of the mixture of gases and vapours, having no chemical affinity for one another, is the sum of the pressures, which each constituent would separately exert, if it occupied the whole space alone.*

The above law is proved by the apparatus of fig. 42 in the following manner:—

The tap S is removed to introduce dry air into A, till the level of mercury in A and R is the same. The

tap S is put into position and water poured in the funnel F. The reservoir R is lowered to lower the level in A and water is introduced into it by turning on the tap several times. After a little while, R is raised to bring the mercury-level in A to its former position. The level of R will be higher. The difference of levels of mercury in A and R is entirely due to the presence of saturated vapour of water at the room temperature; for the pressure of air in A must be the same as before, as it occupies the same volume. From Regnault's tables, this difference is found to correspond exactly with the saturation pressure of water at the room temperature.

SUMMARY

1. An **unsaturated** vapour does *not* exert its maximum pressure at the given temperature and *is capable* of taking more liquid into the vapour state.

2. A **saturated** vapour *exerts* its maximum pressure at the given temperature and *is incapable* of taking more of the liquid into the vapour state.

3. The pressure of a mixture of gases and vapours, having no chemical affinity for one another, is the sum of the pressures, which each constituent would separately exert, if it occupied the whole space alone.

CHAPTER VIII

HYGROMETRY

122. Humidity. The atmosphere always contains some water-vapour due to the evaporation, that is continually going on from the surface of water, which covers a very large portion of the Earth's surface. The amount of aqueous vapour present in the atmosphere varies considerably with the locality and even at the same place the amount may vary from time to time. To understand fully the weather conditions, it becomes important for meteorological observations to know the state of atmosphere as regards moisture. In order to be complete, the knowledge of atmosphere regarding moisture requires information as to the amount of vapour: (*i*) actually present in the atmosphere and (*ii*) the amount which will saturate the atmosphere with water-vapour at the given temperature. The second determination is necessary, for our feeling of dampness of the air depends not only on the amount of vapour present, but also on that, which will saturate it. Thus on a hot day, the atmosphere may contain more aqueous vapour than on a cold day, yet the former seems to be drier. This appears to be so, because on a hot day atmosphere is capable of taking more of the liquid in the vapour state.

The ratio of the mass of water-vapour actually present in the atmosphere in a given volume, to the mass of water-vapour, which will saturate the same volume of the atmosphere at that temperature, is called Humidity or Relative humidity of the Atmosphere.

123. Hygrometers: (*i*) **Chemical.** Humidity is determined directly by a chemical hygrometer, which consists of a large wide-mouthed bottle with a stop-cock near the bottom, fig. 44; three U-shaped tubes A,

B and C, containing phosphorus penta-oxide or pumice-stone soaked in strong sulphuric acid are connected to it as shown in the figure. The aspirator R is filled with water and tubes A and B are weighed. (Let their mass be m_1). The stop-cock of R is opened; water flows out, air is sucked in from E and its moisture is caught in the tubes A and B, while passing through them.

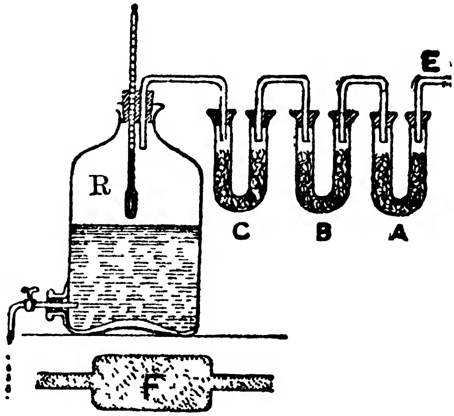


FIG. 44

When sufficient air has been drawn in, the stop-cock is closed and the tubes A and B are weighed again. (Let their mass be m_2). C acts simply as a guard against any vapour coming in from R. The aspirator R is once more filled with water and the tubes A and B again put in position. Now the open end E is connected to a wide tube F, containing small glass tubes charged with water and the stop-cock of R is opened to admit, through A and B, air saturated with water-vapour. When as much air has been drawn in, as was drawn previously, the stop-cock is closed and the tubes A and B are weighed again. (Let their mass be m_3). Then the relative humidity is given by $\frac{m_2-m_1}{m_3-m_2}$, for (m_2-m_1) denotes the mass of water-vapour actually present in the given volume of atmosphere and (m_3-m_2) denotes the mass of water-vapour, which saturates the same volume at the same temperature.

This method of finding relative humidity is extremely tedious and is never used for the purpose in actual practice. Indirect methods involving the determi-

nation of dew-point are more convenient. The principles underlying these methods are the following:—

(1) Dew-point is the temperature at which the aqueous vapour actually present in the atmosphere, is sufficient to saturate it.

(2) The mass in gms. of saturated aqueous vapour, per cubic metre at any temperature, is equal to the pressure of saturated vapour in mms. of mercury column at the same temperature.

(3) *Thus relative humidity may also be defined as the ratio of the saturation pressure at dew-point to the saturation pressure at the existing temperature.*

Saturation pressures at various temperatures have been found out by Regnault. Thus to find humidity all that is required is to find the dew-point. For relative humidity will be given by *the ratio of saturation pressure* of aqueous vapour at the *temperature of dew-point to saturation pressure* of aqueous vapour at the *temperature of the atmosphere.*

The instruments devised for the purpose of measuring dew-point are called hygrometers. In these instruments a surface is gradually cooled down, till dew begins to be deposited, when the temperature of the surface gives the dew-point.

(*ii*) **Daniell's Hygrometer.** It consists of a glass tube bent twice at right angles and having a bulb at each end. Inside this is ether, its vapour and no air. The bulb *B* contains a polished silver cap and a thermometer; while *A* has a muslin piece over it. To find the dew-point, the whole of ether is transferred to the bulb *B* and ether is slowly poured on the muslin covering the bulb *A*. Rapid evaporation, resulting in the lowering of temperature of *A*, takes place. The ether

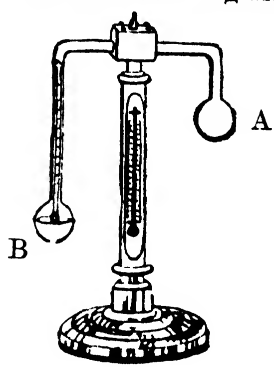


FIG. 45

vapour inside A condenses, while ether in the bulb B evaporates and produces cooling. When the temperature of B falls to the dew-point, a thin film of vapour condenses over the polished silver surface and the temperature t_1 is noted. Cooling is stopped and the thin film of moisture begins to disappear; the temperature t_2 at which it disappears is again noted. The dew-point is given by the mean of the temperatures at which dew is formed and at which it disappears, *i.e.* $t_d = \frac{t_1 + t_2}{2}$.

Daniell's hygrometer has the following defects and is therefore now seldom used:—

1. It is not possible to regulate the temperature of cooling.

2. Glass being a bad conductor, the temperature inside and outside the bulb will not be the same

3. The observer has to stand near the instrument and thus his breath may affect the formation of dew.

(*iii*) **Regnault's Hygrometer.** This is the most important dew-point hygrometer, chiefly in vogue and the defects noticed in Daniell's instrument are avoided herein.

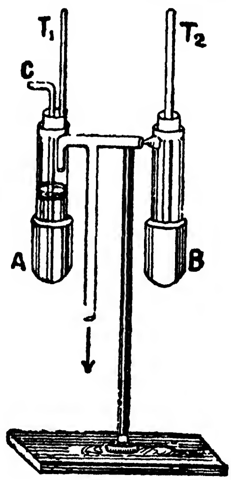


FIG. 46

It consists of a test tube A of which the lower part is removed and replaced by a silver cap, which is highly polished from outside. The mouth of the tube is closed with a rubber cork, which carries a thermometer T_1 and a tube C running down to the bottom of the silver cap. The interior of the test tube A is put into communication with an aspirator by a side tube. A tube B similar to A, but having no side tube is also supported on the same stand for purposes of reference.

Ether is poured into the tube A and a steady current of air is drawn through the ether, by allowing

water to flow out from the aspirator. This steady current of air evaporates the ether and the temperature of A falls slowly. The observations are taken from a distance by a cathetometer and the temperature of thermometer T_1 is noted as soon as dew appears on the silver cap. The temperature of T_2 in the tube B, gives the atmospheric temperature. The following advantages are claimed for this hygrometer:—

(1) The flow of air can be regulated with great nicety.

(2) The errors due to the breath of the observer are avoided, as the observations are taken from a distance by means of a cathetometer.

(3) The temperature given by thermometer T_1 must be the same as that of the silver cap, because it is a good conductor of heat.

(4) The bubbling air produces uniformity of temperature throughout the whole mass

Wet and Dry bulb Hygrometer. On account of its simplicity and great ease, with which it can be used to determine humidity and dew-point by the help of tables, it is extensively used for the purpose. It consists of two thermometers exactly alike, mounted near each other on a common stand. The bulb of one of them is kept moist by a muslin piece, one end of which is kept dipping in a small vessel containing water.

FIG. 47

To use the instrument, the muslin is moistened. After a little while, the temperatures, when they are stationary, of both the thermometers, are noted. The temperature given by the wet-bulb thermometer will, unless the atmosphere is thoroughly saturated, be invariably less than that of the dry-bulb thermometer; because evaporation goes on from the surface of the wet-bulb and heat for the purpose is derived from the air and the muslin piece surrounding the bulb. These cool and keep the temperature of the wet-bulb thermometer lower than that of the atmosphere. On a dry

day, evaporation will be brisk and the difference between the temperatures of two thermometers will be great ; while on a damp day, evaporation will be slow and the corresponding difference small. Thus the difference in temperatures of the two thermometers depends upon the humidity. Tables have been constructed to give the aqueous vapour-pressure at various temperature-differences and from them dew-point and humidity can be calculated.

124. Atmospheric phenomena. We have already observed that aqueous vapour is distributed throughout the atmosphere. Whenever this vapour comes in contact with a cold object, it is condensed in small droplets known as *dew*. For the formation of dew, the temperature of the cold object evidently must be below or at the dew-point.

Mist or Fog. When the temperature of a large quantity of air containing vapour is gradually lowered, as does happen at the evening time during cold weather; a temperature is reached, when the air becomes saturated with water-vapour, which condenses on small particles of dust or smoke forming small drops of water. These are spoken of as *Mist* or *Fog*.

Cloud. A cloud like mist or fog is formed by the condensation of large quantities of water-vapour in the upper regions of the atmosphere. This condensation does not necessarily take place due to low temperature; but it generally takes place, when a stream of air saturated with vapour, blowing against a high mountain, ascends upwards. It expands, cools and forms clouds. When these small droplets forming a cloud coalesce to form large drops, they fall down by the action of gravity as *rain*.

Snow. When the temperature of a cloud falls below 0°C. by cold air currents, icy crystals called *snow* fall down. It is the direct conversion of vapour into solid form.

Hail. When rain-drops pass through cold regions on their downward descent, they get frozen and are

called *hailstones.*

SUMMARY

Humidity. It is the ratio of the water-vapour, actually present in the atmosphere in a given volume to the quantity of water-vapour, which will saturate the same volume at the same temperature.

Hygrometers are instruments to determine the amount of vapour present in a given volume of the atmosphere.

Dew-point. It is the temperature at which the water-vapour present in the atmosphere will saturate it.

Dew-point hygrometers. These are designed to measure the dew-point.

EXAMPLES

1. On a day when the barometer is 760 mms. high, the temperature of the air is 20°C. and the relative humidity is 50%; what fraction of the whole pressure of the air is due to water-vapour? The saturation pressure at 20°C. is 18 mms. (Sen. Camb. Local)

Let the existing pressure due to water-vapour be f.

Then $\cdot 5 = \frac{f}{F} = \frac{f}{18}$ or $f = 9$mms.

$\therefore$ the fraction of the whole pressure $= \frac{9}{760}$.

2. A quantity of hydrogen collected over water measures 150 c.cs. at 10°C. The barometric pressure=750 mms. and saturation pressure at 10°C.=9·0 mms. Find the volume of dry hydrogen at N.T.P.

The pressure due to hydrogen=750−9·0=741 mms.

$\therefore$ volume of dry hydrogen at N.T.P. $= \frac{150 \times 741}{760} \times \frac{273}{283}$

$= 141 \cdot 0$ c cs.

3. The dew-point on a certain day being found to be 12°C. and the temp. of air 16·5°C. Find the relative humidity.

Aqueous pressure at 12°C.=13·48 mms. of mercury column

„ „ „ 16° =13·64 „ „ „ „

and „ „ „ 17° =14·42 „ „ „ „

4. A quantity of hydrogen collected over water occupies 350 c.cs. at 15°C. and 742·7 mms. pressure. Calculate the volume of dry hydrogen at N.T.P. Aqueous pressure at 15°C.=12·7 mms.

EXAMINATION QUESTIONS VI

1. Explain the construction and principle of a constant volume air thermometer. What are its advantages over a mercurial thermometer?

2. What conditions must be fulfilled in order that a Centigrade and a Fahrenheit thermometer, having the same bore, may have their scale-divisions of equal length.

3. What is meant by absolute zero? The air within a half-inflated balloon occupies a volume of 1000 c.cs. at 25°C. and 76 cms. pressure. What will be its volume, after the balloon has risen to a height, where the pressure is 40 cms. and the temperature -8°C?

4 Describe a method of finding the co-efficient of apparent expansion of a liquid with a weight thermometer.

5. Why does an island in mid-ocean undergo less extremes of temperature than an inland region?

6. In a Bunsen's calorimeter, the bore of the capillary tube is 1·5 sq. mm and the mercury surface is displaced through 5 cms. when 8 grams of a substance at 75°C. are introduced. Find the specific heat of the substance?

7. Distinguish between saturated and unsaturated vapours. Describe their behaviour, when temperature and volume are changed in turn

8. What do you understand by humidity and dew-point? Describe Regnault's hygrometer and how it is used to determine humidity.

9. Enunciate Dalton's law of partial pressures. How will you experimentally verify the same?

10. What is meant by the specific heat of a substance? What methods would you employ to determine the specific heats of a solid and of a liquid? (*Lond. Mat. 1895*)

CHAPTER IX

TRANSFERENCE OF HEAT

125. Modes of Transmission of Heat. Heat is transferred from a place at a higher temperature to one at a lower temperature by three distinct modes, known as (*i*) **Conduction**, (*ii*) **Convection** and (*iii*) **Radiation.**

Conduction is the process of transference of heat from one part of a body to another without any relative alteration of its particles; the intermediate particles being heated however, in the meanwhile.

Experiment Take a rod of metal, keep its one end in a Bunsen's flame and hold the other in your hand. After a while, it will begin to feel warm; and it will feel warmer, if touched near to the hot end.

In this process molecular kinetic energy is communicated from particle to particle; the molecules oscillate in their own orbits, but do not leave their places. This process takes place generally in solids.

Convection is the process of transference of heat, from one part of a body to another, by the actual motion of the hot particles.

Experiment. Take a round-bottomed glass flask half-filled with water and place a few crystals of potassium permanganate into it. Heat the flask by a small Bunsen's burner. Notice that the water, which is heated at the bottom, gets coloured as it passes by the crystals, rises up in the middle of the flask and its place is occupied by the cold water from the sides. The hot water as it rises up gets cooled by mixing with cold water and again descends down the sides of the flask.

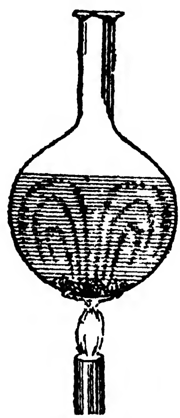


FIG. 48

In this process, which takes place in fluids, the hotter portions become lighter due to expansion and the colder portions come to take their places. Thus heat is transmitted by currents of hot fluid, going away from the source of heat and those of cold fluid approaching it.

Radiation is the process by which heat is transferred from a hotter body to a colder one, by means of waves in ether, which is supposed to pervade the whole interstellar and inter-molecular spaces.

Experiment. Heat an iron ball till it becomes red-hot. cover it with a glass bell-jar and hold an ether thermoscope near it. Notice that the thermoscope is affected, although it cannot be heated by conduction and convection.

In this process the presence of any material medium is not essential; for instance the heat of the Sun is communicated to us in this way. It does not travel as heat, but as waves in ether, from a distance of nine crores of miles, which separates the Sun from us.

126. Conduction. All bodies conduct heat to a greater or lesser extent. Metals are good conductors of heat, while substances such as stone, marble, slate, etc, are very bad conductors of heat. Substances like wood, paper and leather are partial conductors.

The difference in the conductivity of different

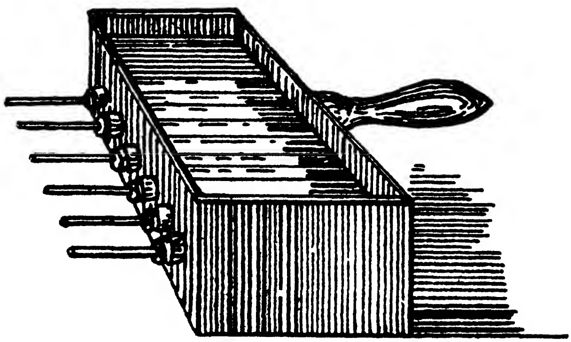


Fig. 49

substances is illustrated by *Ingenhausz's apparatus.*

It consists of a metal-can having a number of holes in its side. Rods of equal length and equal area of cross-section but of different materials pass through the holes; these are covered over with a thin layer of wax Boiling water is poured into the can and the lengths, along which the wax melts, are measured. *Then the conductivities of different materials are directly proportional to the squares of the lengths, along which the wax melts.*

The difference in conductivity is also illustrated as follows:—A thin paper is wrapped once round a compound rod of wood and copper and its middle point is held over a Bunsen's flame. It is noticed that the paper over the wooden part gets charred, while the one over the metal, remains unaffected. The reason is that copper being a good conductor, conducts away heat very quickly and does not allow the paper to be affected; while wood, being a bad conductor, does not conduct heat. That metals conduct away heat very quickly is strikingly illustrated also by the following experiments:—

(*a*) Lower a coil of copper wire over the flame of a spirit lamp, so as just to enclose it, notice that the flame is extinguished The explanation is that copper, being a good conductor, absorbs so much heat as to lower the temperature below the ignition point of spirit.

(*b*) Place a piece of wire-gauze, a little distance above the opening of a Bunsen's burner. Turn on the gas and light it above the gauze. Notice that the flame does not strike down through the gauze The explanation is the same as given above.

On the same principle, Sir Humphrey Davy constructed his safety-lamp to be used in mines, where marsh gas exists. This is an ordinary oil lamp, the flame of which is surrounded by a thick wire-gauze, closed at the top by a metal plate. If marsh gas goes into the flame through the meshes of the gauze, it burns inside;

FIG 50

but due to the good conductivity of the metal, it does not raise the outside temperature to ignition point quickly. This inside burning of the marsh gas serves as danger-signal for the miners, to stop work and arrange a for the proper ventilation of the mine.

Stationary and Variable States. If one end of a

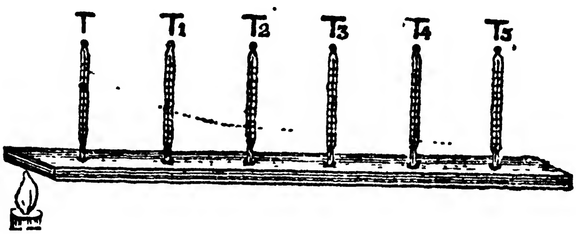


FIG. 51

rod be kept at a constant high temperature and the temperatures of various points along its length be noted, it will be seen that at first the temperature continues to increase along the length of the rod; this state is called the **variable state.** After some time the temperatures of the various points remain steady, though they fall as we proceed from the hot to the cold end. This state is called the **Stationary State**. When this condition is reached, it should not be supposed that conduction has ceased. As the bar is at a temperature higher than that of the atmosphere, it is giving out heat by convection and radiation. The temperature of any point (say T_1) will be steady, when the heat supplied to it by conduction from the hotter side is equal to the loss of heat by convection and radiation from its surface and by conduction to the colder side. The temperature at any point, say T_1, will evidently be higher in a good than in a bad conductor; or the distances along the bars of various materials for the same fall of temperature will be greater in a good than in a bad conductor. Theoretically such distances can be proved to vary directly as the square root of the conductivity of the material, which result forms

the basis of Ingenhausz's experiment described on page 272, to compare conductivities.

127. Thermal conductivity. Consider the case of a large plate of a substance, having parallel faces $ABCD$ and $EFGH$; if the face $ABCD$ be kept at a temperature higher than that of the opposite face, heat will flow by conduction from the front to the opposite face. After some time, when stationary state is reached, imagine a small lamina a in the middle being separated from the rest of the plate. The flow of heat through the lamina will be parallel to the arrow, *i. e.* from the front to the opposite face and there will be absolutely no lateral loss; since the edges will be at the same temperature, as the remainder of the plate in the same plane. Thus heat, entering one face, will leave the opposite face. The total quantity of heat, which flows in a given time through such a lamina, is seen to be

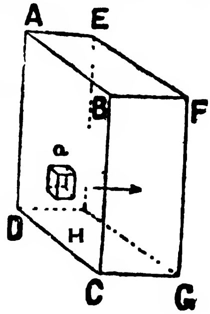


FIG. 52

(*i*) *directly proportional to the area of the lamina,*

(*ii*) *directly proportional to the difference of temperature* $(\theta_1-\theta_2)$ *between the two faces,*

(*iii*) *directly proportional to the time* t, *and*

(*iv*) *inversely proportional to the length of the lamina from one face to the opposite.*

Thus Q, the quantity of heat passing from one face of the lamina to the opposite, will be given by the mathematical expression

$$Q \propto \frac{a(\theta_1-\theta_2)t}{l}$$

which is equivalent to $Q = K\dfrac{a(\theta_1-\theta_2)}{l}t$, where K is a constant, depending on the nature of the material of the lamina. It is called the thermal conductivity or the co-efficient of conductivity.

In the above equation, suppose $a=1$ sq. cm.

$\theta_1 - \theta_2 = 1°C.$, $t=1$ sec. and $l=1$ cm.; then $Q=k$.

Thus the thermal conductivity of a substance is the quantity of heat, which flows in 1 sec. from one face of a unit cube to the opposite, when the difference of temperatures between the opposite faces is equal to 1°C.

The equation $Q=k\dfrac{a(\theta_1-\theta_2)t}{l}$ is very important, as it enables us to find the conductivity of any substance.

127 (*a*) **Conductivity of Liquids.** When a liquid is heated from below, expansion takes place and its density decreases. This decrease in density causes the heated portions of the liquid to rise to the surface, while the colder parts come down. In this way, convection currents are set up, which heat the whole mass of the liquid. If however, the liquid be heated from its upper surface, the flow of heat downwards is generally slow due to its bad conductivity, except in the case of mercury or other metals in molten state.

The bad conductivity of water is shown in the following way:—

Experiment.—Take a test-tube, fill it about three-fourths with ice-cold water, attach a piece of ice to a piece of lead and make it sink to the bottom of the test-tube. Heat the test-tube near its top by slightly inclining it. It will be noticed that water at the top may boil, while ice at the bottom will remain in the solid state.

The conductivity of liquids was first determined by **Despretz.** The principle is the same as that applied in *Ingenhausz's* method for determining the conductivity of solids. Despretz apparatus consisted of a cylindrical wooden vessel *V* about a metre long and 20 cms. diameter. Holes were drilled in the wall of the cylinder at equal intervals and thermometer

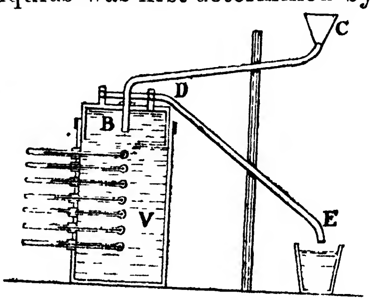


Fig. 52 (*a*)

bulbs were inserted through these along the axis of the cylinder as shown in the figure. The vessel was filled with the liquid to be experimented upon. On the top of the liquid was placed a thin copper box *B*, which was filled with hot water, kept at constant temperature. After some time the upper thermometers began to indicate a slow increase of temperature; and it was after about 40 hours in the case of water that stationary state was reached This flow of heat could not possibly be due to conduction of heat along the sides of the feebly conducting vessel, because the temperature along the axis was higher than that near the sides of the cylinder.

Despretz filled the vessel in turn with liquids of conductivities k_1, k_2, k_3 etc. and found that the distribution of temperature along the axis of the cylinder under stationary state was identical with that of a metal bar under similar conditions. He found lengths l_1, l_2, l_3 etc., corresponding to equal differences of temperature and showed that $\frac{k_1}{l_1^2}=\frac{k_2}{l_2^2}=\frac{k_3}{l_3^2}$, etc.

127. (*b*) **Conductivity of Gases.** The determination of conductivity of gases is a very difficult problem, because in their case the transference of heat takes place not only by convection currents but also by radiation. Of all the gases, hydrogen is the best conductor and this is shown in the following manner.—

Experiment.—A thin platinum wire is stretched inside a tube, as shown in fig 52 (*b*). On passing a current, the wire becomes red-hot when the tube is filled with air. On exhausting the tube, its brightness decreases. On introducing hydrogen, it becomes altogether dull, which shows that heat is readily conducted by hydrogen.

Gases are very bad conductors of heat and special methods have to be used to determine their conductivity.

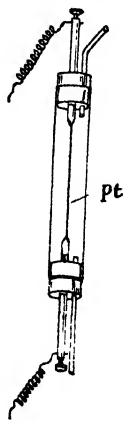


Fig. 52 (*b*)

In order to determine the conductivity

of a gas, the rate of cooling of a thermometer in an enclosure is observed: (*i*) when it is totally exhausted and loses heat by radiation alone; (*ii*) when it contains the gas to be experimented upon at *low pressure*, and it loses heat both by conduction and radiation. From the difference in the rates of cooling, the conductivity can be determined. The convection effects are avoided in both cases; for it is found experimentally that at low pressures, convection cannot take place.

Gases are extremely bad conductors of heat. It is the presence of air in small cavities, that renders wool and fur clothes bad conductors of heat.

Table of conductivities at 18°C.

Silver	·974	Water	·00136
Copper	·916	Air	·000048
Iron	·147	Hydrogen	·00033
Steel	·115		

128. Convection. From what has been said already, it is clear that convection currents tend to equalize the temperature, throughout the whole mass of a fluid. Heat is transferred more rapidly by convection than by conduction. Thus, it is possible to boil water in a thin paper vessel without its being scorched.

Experiment. Take a small thin paper box. Suspend it by fine wires. Half fill it with water and heat it from the bottom. Water will boil, without the paper being scorched. This shows that heat is taken from the paper by water with great rapidity.

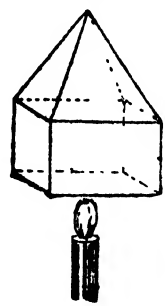


Fig. 53

Ventilation. The ventilation of buildings or furnaces depends upon convection currents in air. Fresh air comes from the side openings, while hot air goes away from the top chimneys and thus circulation of air is maintained.

Experiment. Place a burning candle in a shallow basin, containing a little quantity of water and place a lamp chimney over the candle flame. Notice the flame is

extinguished after a short time; because oxygen, essential for combustion, cannot enter from the top, wherefrom hot gases are leaving and there is no other passage for it to enter. Now repeat the above experiment, holding a piece of card-board in the upper part so as to divide it into two halves. The candle continues to burn, because hot gases escape through one half and fresh air enters through the other half. The outward and inward rush of air may be demonstrated by holding a piece of thin paper near the two halves.

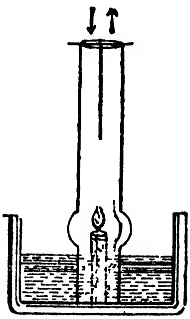


FIG. 54

Winds. These are huge convection currents set up in the atmosphere, due to unequal heating of the Earth's surface. Whenever a portion of the Earth becomes heated more highly than the rest, air in contact with it gets heated and rises up. Colder air from the surroundings rushes in to take its place, which constitutes wind.

Land and Sea Breezes. During daytime the land becomes hot, while the sea due to high specific heat of water does not become so. Thus air in contact with the Earth gets heated, rises up and its place is taken by the wind coming from the sea-side which constitutes *sea-breeze.* During night-time, the phenomenon is reversed; because the Earth becomes cold very soon due to radiation, while the sea-water remains hot and air in contact with it also becomes hot and rises up. Wind blows from the Earth towards the sea and constitutes *land-breeze.*

129. Radiation. It has been remarked already that in the process of radiation, heat does not travel as such through the intervening space; but as waves, propagated through ether with the velocity of light, *i. e.* 186000 miles per sec. These waves are produced at the surface of the hot body by the vibratory motion of its molecules. When these waves fall on another body, they increase the vibratory motion of its molecules and thus give heat to it.

Hence we see, the emission of heat by radiation is a process of transformation of heat into wave-energy; and the absorption of heat is the reverse process of transformation of wave-energy into heat-energy. The radiation and absorption thus depend upon the nature of surface layers of the source and the absorbing body respectively.

129 (a) Properties of Thermal Radiations:—

(i) *Thermal radiations can be transmitted through vacuum.* The Sun's heat is transmitted through space, unoccupied by matter.

Sir Humphrey Davy showed that an ether thermoscope, placed outside the receiver of an air-pump, was affected as much by a platinum wire heated to redness by an electric current, placed inside an exhausted receiver, as when it was full of air.

(ii) *Thermal radiations are transmitted in straight lines.* In front of a heated iron ball, place two screens having small holes in them. Place a thermopile facing the hot iron ball in such a way that the ball, the thermopile and the two holes are in one straight line. Notice that the thermopile is affected by the heat of the ball, as is shown by the deflection of the galvanometer. Displace one of the screens and notice that there is no indication of the rise of temperature.

(iii) *Thermal radiations are transmitted with the same velocity as light.* At the time of total eclipse of the Sun, the heat-supply ceases as soon as the light is extinguished.

(iv) *Thermal radiations are reflected from polished surfaces, like light.*

C is a polished tin sheet, *A* and *B* are two tubes, *H* is a hot iron ball placed at the mouth of the tube *A*.

T is an ether thermoscope placed at the end of the other tube *B*.

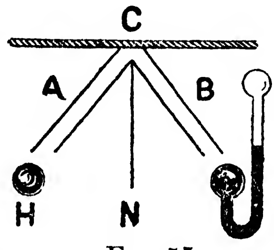


Fig. 55

Notice that the thermoscope

shows a rise of temperature when the two tubes are equally inclined to the normal.

(*v*) *Thermal radiations can be refracted, like light; i. e. their path deviates as they go from one medium to another.* A convex lens refracts sun-rays to its principal focus; and a piece of black paper placed there begins to burn, showing that heat-rays are concentrated like light-rays at the principal focus of the lens.

(*vi*) If a continuous spectrum of light be obtained by means of a rock-salt or carbon bisulphide prism and a thermopile be moved from the violet to the red end, it is seen that the heating effect increases as we approach the red end; and in fact beyond the red end, heating effect is most marked. This shows that thermal radiations extend beyond the red portion of the visible spectrum, they suffer less deviation and so have *longer wave-lengths, than even the red portion of* the visible spectrum.

(*vii*) *The thermal radiations received by a surface of given area are inversely proportional to the squares of the distances between the source and the surface.*

Place a thermopile at varying distances from an electric-glow lamp coated with lamp-black and notice the deflections. They will be inversely proportional to the squares of its distances from the lamp, *provided the thermopile is not placed too near it.*

130. Diathermancy. When white light is passed through a substance, waves of a particular wave-length are absorbed, while those of other wave-lengths are transmitted. Thus glass transmits the visible portion of the spectrum and absorbs the ultra-violet and infra-red portions. Similarly, a solution of iodine in carbon bisulphide transmits the infra-red rays, but is opaque to actinic rays. *A substance, which transmits radiations of a certain wave-length is said to be* **diathermanous** *for those radiations; and a substance, which absorbs radiations of a particular wave-length is said to be* **athermanous** *for those radiations.*

The diathermancy of a body depends greatly on the nature and temperature of the source of radiation. Thus glass, which is practically diathermanous to sunlight is athermanous to radiations emitted from bodies below white heat. It can hardly cut off heat rays coming from the Sun, but can effectively absorb the radiations coming from glowing coal and becomes hot itself. On account of this property, it is used for fire screens and green-houses. It allows sun-rays of short wave-lengths to go into the green-house; but does not allow long waves radiated by the plants to come out. Moist air behaves more or less like glass; *i.e.* it allows free passage to waves of short wave-lengths, but absorbs radiations of long wave-lengths. A solution of iodine in carbon bisulphide is diatherma-nous to heat rays, but is perfectly athermanous to light rays.

Rock-salt is very transparent to thermal radiations of all wave-lengths, except such as are emitted by heated rock-salt. This is in accordance with the law that a *substance is opaque to radiations, which it emits itself when heated.*

131. Prevost's Theory of Exchanges. Suppose a hot body *A* is surrounded by a cold enclosure *B*, which is totally exhausted of air, so that no conduction or convection can take place. It is noticed that the temperature of *A* continuously falls and that of *B* rises, till the temperature of both is the same. If the temperature of *B* be still further decreased, it is noticed that the temperature of *A* again begins to fall, till equalization of temperature takes place.

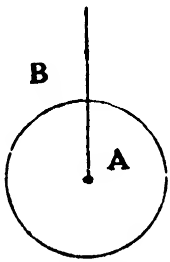


FIG. 56

The explanation of the above is as follows:—The hot body *A* gives out thermal radiations and at the same time absorbs heat radiations. When its tempera-ture is higher than that of the surroundings, it gives out more thermal radiations than it absorbs. Its temperature falls and that of its surroundings increases,

till equalization of temperature *does* take place. After that the amount of thermal radiations given out equals the amount absorbed and no rise or fall of temperature is noticed. Thus a system of exchange continues, though there may be no rise or fall of temperature. This is known as **Prevost's Theory of Exchanges.**

According to this, if the temperature of a body remains constant, *the energy given out by its vibrating molecules in the form of thermal radiations in ether, is exactly counterbalanced by the absorption of an equal amount of energy.* Further it follows from the above theory that the absorbing and radiating powers of a body must be the same. For if the absorbing power were greater than the radiating power, a rise of temperature should be the result; and if radiating power were greater than the absorbing power, a fall of temperature would be the result. *Hence a black substance which is a good absorber is a good radiator, and a polished surface which is a bad absorber is a bad radiator.*

132. It was proved by Leslie that some substances notably lamp-black, emit heat more copiously than others, under similar conditions. The radiating body employed by him is known as Leslie's cube and consists of a cubical vessel of tin, filled with hot water. It can be coated with any substance, whose emissivity is desired to be studied. It can be rotated round a vertical axis, so that any one face can be allowed to radiate energy against the face of a thermopile and the amounts of he t radiated per second by different substances compared. By experiments of this kind, it is observed that a polished face emits little heat, while a black one emits radiant heat very copiously. Polished surfaces are less efficient radiators; as radiations are reflected when passing from inside outwards or from outside inwards, and thus they can neither go out nor penetrate inside. The polished surface is a good reflector and therefore a poor radiator and a poor absorber.

132. (*a*) **Emission and Absorption of Radiation.** It has been proved above on the theory of exchanges that the absorbing and radiating powers of a body

must be equal, and is experimentally proved by Ritchie's apparatus. It consists of a differential air thermometer with the modification that the two bulbs are replaced by two equal and similar flat cylindrical tin boxes. In between them and equidistant from both is placed a tin vessel C; one face of it, say that opposite B, is blackened, while the other is polished, and the face of A towards C is blackened.

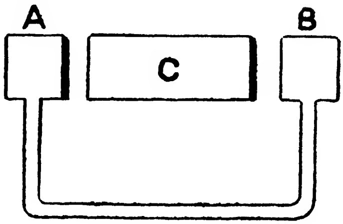


Fig. 57

Experiment. Fill the vessel with boiling water and notice there is no movement of the liquid in the stem. This shows that A and B are at the same temperature The greater emissive power of the face opposite B is compensated by the greater absorbing power of the face A

Emissivity. It is the quantity of energy given up in one second by one sq. cm of the surface, when the difference of temperatures between it and its surroundings is equal to 1°C.

132 (*b*) **Dewar's flask.** The fact that polished surfaces are very poor radiators is made use of in thermos-flasks, which are double-walled vessels of glass highly polished both from within and without. The space between the two walls is thoroughly exhausted to prevent loss of heat by conduction and convection. Such flasks are used to keep liquid-air, hot tea or ice. They do not allow any transference of heat and therefore the substance is kept for a long time at the same temperature.

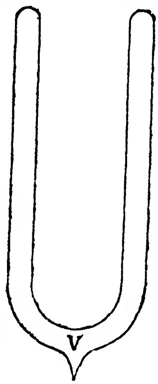


Fig. 57 (*a*)

133. Law of Cooling. Newton formulated the law that the heat radiated by a body per second is proportional to the difference of temperature between it and its surroundings. The law fairly represents the experimental results at ordinary temperatures, but does not hold good at high temperatures.

It may be clearly understood that the quantities of

heat given out by two bodies are directly proportional to the differences of temperatures between them and their surroundings, *provided all other conditions are identical.*

The rate of cooling depends upon various factors:—

The conditions favourable for the cooling of a hot body are: (*i*) Great difference of temperature between the body and its surroundings.

(*ii*) The large extent of the area of the surface exposed.

(*iii*) Nature of the exposed surface. Thus a black calorimeter cools more quickly than a polished one.

(*iv*) Constant renewal of the gas or any other substance in contact with the cooling body.

(*v*) Low specific heat of the cooling body.

(*vi*) Constant stirring.

133 (*a*) **Specific heat by cooling.** Newton's Law is made use of, for finding the specific heat of liquids. The apparatus employed for the purpose consists as shown in fig. 58, of a double-walled vessel, the space between the two being filled with water to keep the temperature constant. A calorimeter is weighed when empty, then a certain volume (say 100 c cs.) of hot water is poured into it and freely suspended in the double-walled vessel as shown. Temperature is observed every minute, a graph *EF* fig. 59, showing fall of temperature with time is drawn and the calorimeter along with water, after being cooled, is weighed again. The experiment is repeated

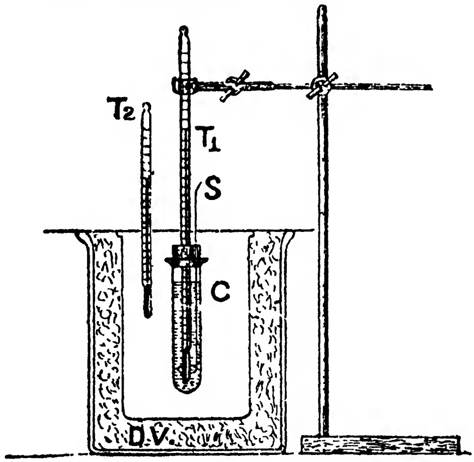


FIG. 58

with the given liquid in the calorimeter, taking care that the quantity *by volume* of the liquid used, is equal to that of water. The initial temperature of both is the same and a graph *EG* showing fall of temperature with time is drawn. Thus we get two graphs on the same paper and with the same scales. From these graphs, we can easily find the intervals of time θ_1 and θ_2, in which the water and the liquid cool through the same range of temperature (say t_1 to t_2).

Then if w_1=wt. of empty calorimeter

w_2= " " " " +water

w_3= " " " " +liquid

and s=specific heat of the liquid

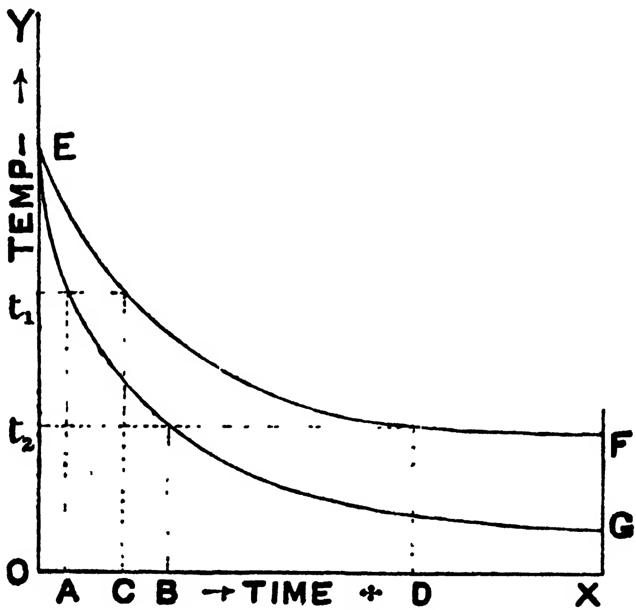


Fig. 59

Then as the cooling takes place in both cases, under identical conditions, we must have by Newton's Law,

$$\frac{[(w_2-w_1)+w_1\times\cdot 1](t_1-t_2)}{\theta_1}=\frac{[s(w_3-w_1)+w_1\times\cdot 1](t_1-t_2)}{\theta_2},$$

i.e. the quantity of heat lost by water per second should be equal to that lost by the liquid during the same interval. Here ·1 represents the sp. heat of copper, of which the calorimeter is supposed to be made up. In order to accelerate the rate of cooling, the outside of the calorimeter is blackened.

SUMMARY

1. **Conduction** is the process of transference of heat from one part of a body to another without any relative alteration of its particles, the intermediate particles being heated in the meanwhile.

2. **Convection** is the process of transference of heat from one part of a body to another by the actual motion of the particles.

3. **Radiation** is the process of transference of heat by waves in ether from a hotter body to a cooler one.

4. **Co-efficient of thermal conductivity** is the quantity of heat, which passes through one face of a unit cube to the opposite face in one second, when the difference of temperature is 1°C.

5. **Conductivities** of different metals are proportional to the squares of the distances through which the wax melts, when one of the ends of bars of equal sectional area, is maintained at a uniform temperature.

6. **Diathermanous.** Substances, which do not absorb radiations of a particular wave-length are said to be diathermanous for those radiations.

7. Substances, which absorb thermal radiations, are called **Athermanous.**

8. **Newton's Law of Cooling.** The rate at which a body loses heat is directly proportional to the difference of temperature between it and its surroundings.

9. **Emissivity.** It is the quantity of energy radiated per second per unit difference of temperature between it and its surroundings.

10. **Prevost's Theory of Exchanges.** A body emits radiations at all temperatures. A body at constant temperature gives out as much energy as it absorbs.

EXAMPLES

1. Find the amount of ice, which will melt in 5 minutes by heat conveyed along a bar 20 cms. long, 5 cms. broad and 5 cms. thick, one end of which is kept at 100°C. and the other end in ice. Given conductivity = ·8 (*P.U.* 1921).

$$Q=k\ \frac{A(\theta_1-\theta_2)t}{l}$$

$$Q=\frac{\cdot 8\times 25\times 200\times 5\times 60}{20}=30000 \text{ calories}$$

$$\therefore \text{ amt. of ice melted}=\frac{30000}{80}=375 \text{ gms.}$$

2. How much steam per minute is generated in a boiler ·5 cms. thick, 2 sq. metres area of fire chamber, if K the conductivity be ·164 and the temperature of the two faces of the boiler plate be 200°C. and 120°C. (L=522)

$$Q=\cdot 164\frac{2\times 100\times 100\times 80\times 60}{5}=31488000 \text{ therms}$$

$$\therefore \text{ quantity of steam generated}=\frac{31488000}{522}=60321\cdot 8 \text{ gms.}$$

3 A metal vessel 1 sq metre in area and 15 mms thick plates, is filled with melting ice and is kept surrounded by steam at 100°C. How much ice will melt in one hour? The conductivity of metal=·2.

4. A glass vessel with an area of 100 sq. cms. and 1·5 mms. thick. is filled with ice and placed in a vessel kept at 100°C. How many gms. of ice will melt per minute, when the flow of heat has become steady ? K for glass=·00185 (Science and Arts).

5. Steam at 100°C. is passed into an iron cylinder, 15 mms. thick and 100 sq. cms area, water at 100°C. collects at the rate of 100 gms per minute. What is the temperature of the outside, k=·2?

6. Water kept at 10°C. is separated from ice by a plate of iron 1 cm. thick and 100 sq cms. in area. If 36,000 gms. of ice are melted in one hour, what is the thermal conductivity of the iron ?

7. If one touches the pan containing a loaf of bread in a hot oven, he receives a much more severe burn, than if he touches the bread itself, although the two are at the same temperature. Explain.

8. When a room is heated by a fireplace, which of the three methods of heat transference plays the most important role ?

9. What is radiant heat? Can opaque substances transmit radiant heat and transparent ones absorb it? Give examples. (P. U. 1931)

CHAPTER X

MECHANICAL THEORY OF HEAT
&
Steam and Internal Combustion Engines.

134. Theories of Heat · The Caloric Theory.—Upto the beginning of the 19th century, heat was believed to be an elastic fluid called *caloric* To explain the various facts, the caloric was presumed to be weightless, indestructible, self-repellent and having an attraction for matter.

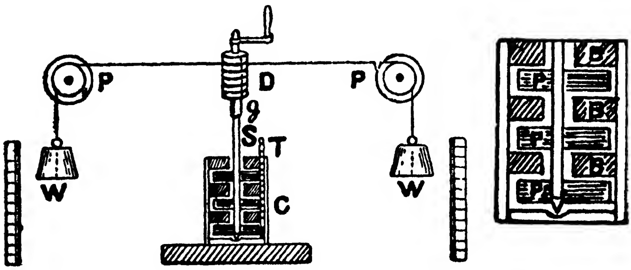
In the year 1798 Count Rumford, an American in the service of Bavarian Government, while engaged in boring cannons at Munich, was struck by the great heat, evolved during the process.

To examine the matter more closely, he took a cylinder of gun-metal, bored a small hole into it to hold the thermometer, covered it with flannel to prevent loss of heat, rotated it by horse-power and pressed a blunt borer against one end. After a little time, he noticed that the temperature of the gun-metal had risen through 70°F. There was evidently no limit to the quantity of heat, which could be evolved from those bodies in this way. He concluded therefore, that heat could not possibly be a form of matter (*caloric*), such as the ancient philosophers thought, ***but it was a form of Motion.***

Later, in the year 1799, Sir Humphrey Davy showed that when two pieces of ice are rubbed together, water is produced, showing that heat is produced by rubbing. From this he concluded, ***that heat is a form of molecular energy.***

Inspite of so strong an evidence to the contrary, the spirit of conservatism was difficult to be uprooted

and the old caloric theory continued to hold its own till the year 1840, when Dr. Joule of Manchester actually determined the amount of work necessary to get one calorie of heat. In Rumford's and Davy's experiments, the bodies changed their character during the operations ; but in Joule's experiment, heat was evolved by stirring water, the conditions at the start and at the conclusion were the same, except that the temperature had risen.



(*a*) Fig. 60 (*b*)

Joule's experiment. The apparatus used by Joule is diagrammatically shown in fig. 60 (*a*). The calorimeter *C* was fitted with *baffle* plates, having spaces in between, to permit the paddles to rotate. The calorimeter with its cover was weighed and after pouring suitable quantity of water, it was weighed again. The water-tight lid had two apertures, one for the spindle *S* and the other for a sensitive thermometer *T*. The spindle carried paddles which rotated between the *baffle* plates Fig. 60 (*b*), and thus churned the water. The spindle *S* could be attached by a screw *g* to the drum *D*, round which two pieces of cords were wound side by side, in such a manner that both left it at the same level but in opposite directions. These cords passed over frictionless pulleys and carried equal weights at the ends. The distances, through which the weights descended, were given by the scales attached.

The observations were taken in the following manner:—The spindle was attached to the drum by

the screw and the weights were allowed to fall through a measured height. The paddles rotated, their kinetic energy was spent in churning the water, whose temperature rose.

When the weights had reached the lowest point, the spindle was disconnected from the drum by taking out the screw g; and the weights were again raised to their former positions, by rotating the handle of the drum. This operations was repeated several times, till the temperature of the calorimeter and its contents had risen through an appreciable range.

The following observations were taken to find the amount of work in ergs, required to produce one therm of heat.

w_1 = weight of empty calorimeter
w_2 = " " " + water
t_1 = initial temperature
M = mass of each weight
t_2 = final temperature
h = distance through which the weights fell
n = number of times of fall
v = velocity of the mass M on reaching the lowest point.

Then, neglecting corrections due to friction, elasticity of cords, cooling of the calorimeter due to radiation and other factors, we have the energy spent in producing heat

$$=\left(2Mgh-2\times\frac{1}{2}Mv^2\right)\times n \text{ ergs}$$

and the heat produced

$$=\{(w_2-w_1)+w_1\times\cdot 1\}(t_2-t_1) \text{ therms}$$

$\therefore$ quantity of work for one therm

$$=\frac{n(2Mgh-Mv^2)}{(t_2-t_1)\{(w_2-w_1)+w_1\times\cdot 1\}}=J.$$

The value so obtained was $4{\cdot}8\times 10^7$ ergs.

Joule's experiment clearly demonstrated that heat was a form of energy and could be obtained by the conversion of mechanical energy. This statement is often called the *First Law of Thermodynamics* and is

defined by Maxwell as follows :—

When work is transformed into heat or heat into work, the quantity of work is mechanically equivalent to the quantity of heat.

According to the Kinetic Theory, heat is energy possessed by a body, and it is manifested by the motion of its molecules. In a solid, the molecules do not change their places relatively; but vibrate in their own orbits and heat throws them into more violent agitation. In the case of liquids, heat besides producing the above effect causes currents of molecules to travel from hotter to colder parts. While in the case of gases, heat enhances still more the very rapid state of motion, in which the gaseous molecules are, in the ordinary state; and thus increases the pressure, which is supposed to be due to the bombardment of the molecules on the walls of the vessel.

The electric method of finding J is described later in Current Electricity.

135. Steam Engine.

The action in a steam engine may be understood by reference to Fig. 61. The steam from the boiler enters into the valve-chest by the tube S. In the valve-chest are three holes towards the side, facing the cylinder; h_2 communicates with the upper part, h_1 with the lower part of the cylinder and the middle one, with the exhaust pump or condenser. There is a D-shaped valve, which fits the middle hole of the exhaust pump and either of the two holes of the cylinder. The valve works by the eccentric rod R' in such a manner, that when the piston P moves upwards in the cylinder, the valve moves downwards, and *vice versa*. Thus when the piston in the cylinder moves upwards, the valve moves downwards, exhausts the steam on the lower side of the piston and allows the steam to play on its upper surface. And when the piston reaches downwards, the valve moves upwards, exhausts the steam from the upper surface and allows steam to exert pressure on the lower surface.

This forward and backward motion of the piston in the cylinder is converted by the driving rod *R* into

- C = Crank
- W = Fly-wheel
- F = Shaft
- h_2 = upper hole in the cylinder
- h_1 = lower hole in the cylinder
- A = Cylinder
- P = Piston
- R = Connecting Rod
- G = Cross head pin or gudgeon
- E = Crank-pin
- V = Valve
- R′ = Eccentric
- S = Steam pipe
- V.C. = Valve-chest
- P.R. = Piston-rod

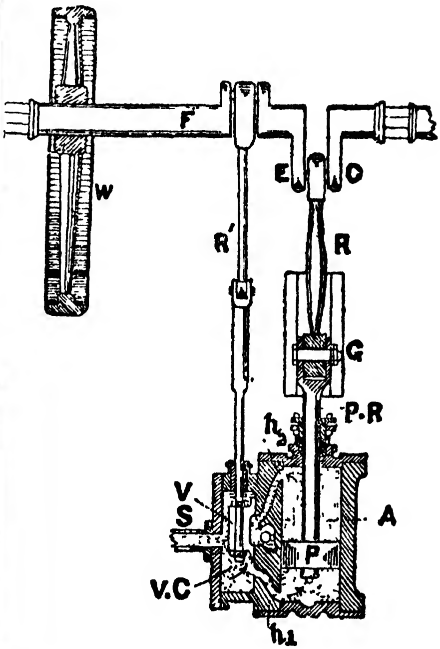


Fig. 61

rotatory motion of the shaft *F* by the eccentric *E*. The shaft has a large heavy wheel *W*, known as the fly-wheel, to render the motion of the shaft uniform and to move it (*i.e.* the shaft) smoothly over the dead points, by virtue of its great momentum, when the piston reaches the lowest and the highest points in the cylinder.

To control the speed, a governor, not shown in the figure, is attached to the shaft and that controls the amount of steam in the valve-chest.

The motion of the shaft can then be communicated by pulleys and belts to any desired place.

Such an engine is called a double-stroke reciprocating engine.

The essential parts are: (*i*) cylinder, (*ii*) piston and (*iii*) some means of controlling the steam. The piston is a movable division-plate, constrained to move axially within the bore of the cylinder. It must fit exactly into the cylinder and should not allow steam to pass from one side to the other.

The Horse-power of an engine is given by the formula $\dfrac{P.a.l.n}{550}$,

where P=the average pressure of the steam in lbs. weight per *sq. inch*,
a=area of the piston in *sq. inches*,
l=length of the piston-stroke in *feet*, and
n=number of single strokes per second.

A single stroke means the motion of the piston either from bottom to top or from top to bottom, in the piston chamber.

136. Internal-combustion Engine. In these engines, the fuel, supplying the energy, is burnt inside the engine cylinder. The engine of a motor car as shown in fig. 62, is a model of an internal-combustion engine. A and B are two cylinders, in which pistons C and D work. These pistons are directly connected to the main shaft. Water-jacket F, in which cold water circulates, keeps the cylinders cool. Water in the jackets, when heated, passes down through the radiator R, which is fixed in front of the car and is cooled by the wind generated by the fan working inside.

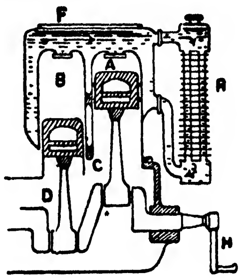


FIG. 62

Working. When the piston is at its lowest point, as shown in A fig. 62, there is always some space left between the piston and the base of the cylinder. This space is called *explosion-*

chamber. In this chamber, near the base of the cylinder are two valves, not shown in the figure. Through one of these called the *throttle valve or the inlet valve*, the fuel is admitted, while through the second, known as the *exhaust valve*, the product of combustion is expelled.

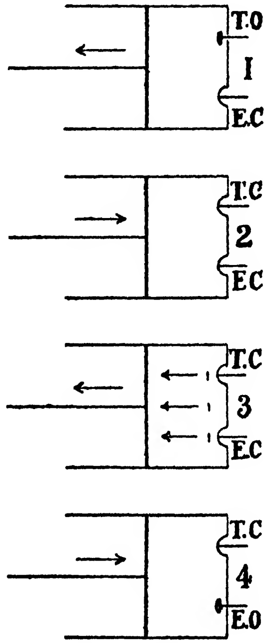


FIG. 63

Four strokes of the piston, as shown in fig 63, complete a cycle of operations. At the first stroke fig. 63 (1), the piston moves out, the throttle-valve (*T. O*) opens and the fuel enters the explosion-chamber through the valve. At the second stroke fig. 63 (2), the piston moves in, the throttle-valve (*T.C*) is closed and the fuel is compressed in the small space. At the end of this stroke, the fuel is exploded by an electric spark. At the third stroke fig. 63 (3), both the valves are closed, the piston is pushed out by the great pressure of the exploded fuel. At the fourth stroke fig. 63 (4), the exhaust-valve *E.O* opens, the piston is pushed in and the product of combustion is driven out. The engine has now completed one cycle.

Gases or vapours of petrol or benzine are used as fuel.

The liquids are generally converted into vapour, and mixed with proper quantity of air in an apparatus called carburettor. The fuel is then known as gasoline.

These engines are light, easy to start and more economical. They are extensively used in motors, æroplanes and light-boats.

SUMMARY

Caloric. Ancients believed that heat was due to a weightless and self-repellent elastic fluid.

Rumford and Joule have shown conclusively that mechanical work can be converted into heat and *vice versa.*

J. the mechanical equivalent of heat is equal to $4{\cdot}2 \times 10^7$ ergs, *i e.* to get one therm, work equal to 4·2 Joules is required.

First Law of Thermodynamics.—When work is transformed into heat or heat into work, the quantity of heat is mechanically equivalent to heat.

EXAMPLES

1. A cannon-ball, the mass of which is 100 kilograms, is projected with a velocity of 500 metres per sec. Find what amount of heat would be produced, if the ball were suddenly stopped (*Science and Arts*, 1899)

$$K\ E. = {}^1/_2 \times 100000 \times 50000 \times 50000$$

∴ Therms generated will be equal to

$$\frac{1 \times 25 \times 10^{13}}{2 \times 4\ 2 \times 10^7} = 2{\cdot}98 \times 10^6 \text{ therms}$$

2. With what velocity should a lead bullet at 50°C. strike against an obstacle in order that the heat produced by the arrest of its motion, if all were preserved within the bullet, might be just sufficient to melt it; sp heat of lead = ·031, melting-point of lead = 335°C and latent heat of fusion = 5·37.

(*i*) Quantity of heat required to raise the temp from 50°C. to 335°C. $= m \times 031 \times 285 = 8\ 835m$

(*ii*) Quantity of heat required to melt it

$$= m \times 5\ 37 \qquad = 5\ 37m$$

∴ Total quantity of heat $= 14{\cdot}205m$.

Let the velocity be v

then $\quad {}^1/_2 mv^2 = 4{\cdot}2 \times 10^7 \times 14{\cdot}205 \times m$

$$v = 3\ 45 \times 10^4 \text{ cms per sec.}$$

3. Water falls from a height of 100 metres If the whole of the energy is changed into heat and is retained by water, what is the rise of temperature?

4. From what height must a bullet be dropped in order that it may be completely melted by the heat generated by the impact, assuming that 4/5 of the heat generated remains in the bullet.

Heat required to raise 1 gm. of lead from the initial temperature and to melt it = 15 therms, $g = 980$ cms. per sec. per sec. (S. and Arts, 1899)

5. An engine, working at 622·4 H. P., keeps a train running at constant speed on level ground for 5 minutes. How much heat is produced?—assuming that the missing energy is converted into heat. Mechanical Equivalent =

778 ft. lbs. of work.

6. Meteorites are small, cold bodies moving about in space. Why do they become luminous, when they enter the Earth's atmosphere?

7. 120 gms. of water are heated from 0° to 100°C. If the heat-energy put into the water could all have been made to do useful work, how high would a cannon-ball, 250 gms. go?

EXAMINATION QUESTIONS VII

1. State clearly the properties of saturated and unsaturated vapours. What is Dalton's law of partial pressures?

If water boils at 99°C when the pressure is 73 cms. What is the saturation pressure of steam at 101°C.?

2. Describe Dulong and Petit's method of determining the co-efficient of real expansion of mercury. Deduce the formula used

3. How would you study the hygrometric state of air at any place? Define dew-point, relative humidity and saturation vapour-pressure.

4. Describe some accurate method of determining Joule's Mechanical Equivalent of Heat.

5. How would you construct an absolute scale of temperature?

A ball of copper sp. heat ·1, weighing 400 gms. is transferred from a furnace to 1000 gms. of water at 20°C. The temperature of water rises to 50°C. Calculate the temperature of the furnace. (P U. 1923)

6. There are two specific heats of a gas. Explain the above expression, giving in detail a method to find the specific heat of air at constant volume

7. Describe Ingenhausz's apparatus for comparing thermal conductivities of metals. State the formula employed.

Discuss the advantages and defects of the following thermometric substances.—mercury, alcohol and air.

8 Describe some form of maximum and minimum thermometer.

9. How would you determine the linear co-efficient of expansion of a metallic rod? Define co-efficient of linear expansion.

10. How would you prove that the density of water is maximum at 4°C.? What is the importance of the above fact in Nature?

11. Describe Bunsen's Ice Calorimeter. What advantages has this method over others?

12. Discuss in detail the laws of change of state from

solid to liquid. What is regelation?

13. Define latent heat. How can the latent heat of steam be determined experimentally? Discuss the precautions and the sources of error. How to eliminate or correct them?

14. Write short notes on :—Conduction, Convection, Radiation, Diathermancy, Ignition temperature and Davy's Safety-lamp.

15. Explain with a clear diagram the principle of a steam engine.

15 (*a*) State Newton's Law of Cooling. What is the difference between specific heat and thermal capacity of a body?

16. Describe an experiment for determining the specific heat of a liquid.

17. What is the difference between heat and light? State the laws of thermal radiations, describing briefly how they can be proved?

18. Deduce an expression connecting volume and pressure of a gas with temperature. Explain isothermal and adiabatic changes.

19. Give the weight-thermometer method of finding the apparent co-efficient of expansion of a liquid. The density of mercury at 0°C. is 13·6 and at 100°C. it is 13·35. Calculate the co-efficient of absolute expansion of mercury.

20. What is Prevost's Theory of Exchanges? Give an experiment to explain the theory.

21. Define emissivity of a surface. Describe an experiment to prove that absorbing power of a surface is equal to its emissive power.

22. Define co-efficient of thermal conductivity of a substance. A current of dry steam is maintained through a glass steam-trap, surrounded by melting ice on all sides. Determine the thickness of glass walls and the steam condensed per minute.

Given Total area of glass trap =100 sq. cms.
Ice melted per minute =75 grams.
Co-efficient of thermal conductivity of glass =·001

(P. U. 1929)

23. Discuss the development of the Mechanical Theory of Heat.

24. Describe some experimental method for determining thermal conductivity of a solid.

25. Describe the construction and graduation of a gas thermometer. Explain how you would use it to find the melting-point of naphthalene.

LIGHT

CHAPTER I

Fundamental Properties of Light and Photometry

137. Introductory—Light is the external cause of our impression of vision. Objects, excepting those which emit light themselves, cannot be seen, unless light falls on them. Thus objects lying in a dark room are invisible; and to make them visible, we require something, which we call *light.*

Objects, which emit light themselves, such as a burning candle, an electric lamp or the Sun, are called *self-luminous.*

Homogeneous medium is one, which has the same density and properties throughout its mass.

A transparent substance like glass, is that which allows light to pass freely and objects can be clearly seen through it.

An opaque substance like wood, is that which cuts off all the light and objects cannot be clearly seen through it.

A translucent substance like greased paper, is that which allows some light to pass through it; but the outline of objects cannot be clearly seen through it.

Ray is the term applied to the straight path, along which light from a *point* travels.

A collection of rays proceeding from or to a point, is termed a **pencil** of light.

A collection of rays, proceeding from various points of a luminous object, is called a *beam of light.*

138. Rectilinear Propagation of Light :—*light travels in straight lines through a homogeneous medium.* A small opaque obstacle held in front of the eye, cuts off all the rays coming from the other side. This shows that light does not bend.

Experiment. Take two screens having very small holes in them. Arrange them in a line with a candle flame. An eye placed in this line behind the screens will see the flame; but a slight displacement of either the flame or the eye or either of the two screens, will make the flame invisible.

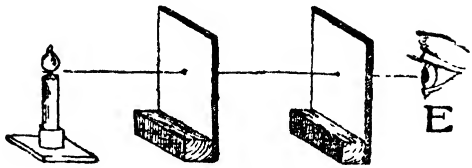


Fig. 1

Experiment. Place a white screen of paper or cloth opposite a small hole in the shutter of a dark room. An inverted image of the objects outside, will be formed on it. The image will be more distinct, if the hole is smaller.

The explanation of the above is furnished by the fact that light travels in straight lines. Consider an object *AB*, placed in front of a small hole. A ray starting from *A* passes through *O* and is incident at *A′*, thus *A′* gets light from *A* and from no other point. Similarly *B′* gets light from *B*. Thus an inverted image is seen.

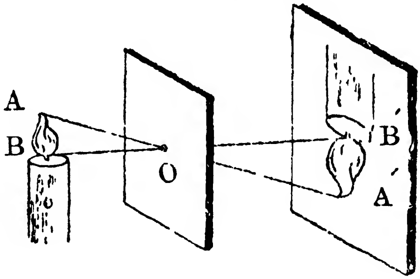


Fig. 2

This forms the basis of a *Pin-hole camera*, an instrument used to take photographs of the landscape. It consists of a closed camera with a small hole in front and a ground-glass screen behind.

In order to get a sharp and well-defined image, it is essential that the *hole should be very narrow.*

A big hole may be supposed to consist of a large number of small holes. Each one of which will give rise to a separate image of its own; thus the various images will overlap and the resulting image will be blurred and confused.

The advantage of this instrument over a photographic camera is that it is cheap and requires no focussing; but the images formed are not bright

enough to effect a photographic plate as quickly as those formed by a photographic camera.

If the distance of the object from the hole be denoted by u and that of the image by v; we have, from the similarity of triangles OAB and $OA'B'$ (fig. 2),

$$\frac{A'B'}{AB} = \frac{v}{u}$$

$$i.e. \quad \frac{\text{image}}{\text{object}} = \frac{\text{distance of image}}{\text{distance of object}}.$$

Thus to get a magnified image, either u should be decreased or v increased. But by magnifying the image, its brightness decreases.

Shadows. Whenever an opaque object is held in the path of rays of light, they are cut off and a region, where no light falls, is created behind the opaque object. This is called a shadow. It is evident that the production of shadows is the result of rectilinear propagation of light.

Experiment. Let L (fig. 3) be a point-source of light and an opaque sphere O be held between it and the screen S. The region AB on the screen does not get any ray and thus it will be dark; while all points above A and below B of the screen will be bright, because they get light from the luminous source.

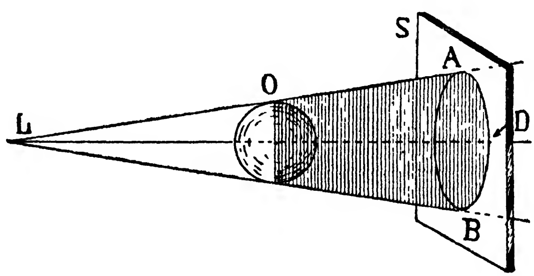


FIG. 3

Generally the source of light is not a point but has a definite boundary and so the shadow is not very sharply defined. Figure 4 illustrates this.

AB is the region of total darkness, while points above

C and below *D* are fully illuminated ; but in the regions *AC* and *BD*, there is only partial shadow. The shadow gradually fades from *A* to *C* and from *B* to *D* respectively.

The region of total darkness, such as *AB* in fig. 4,

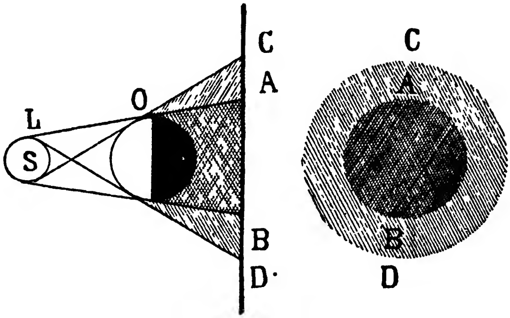


FIG. 4

is called *Umbra* and the region of partial darkness, such as *AC* or *BD* in the above figure, is called *Penumbra*.

Experiment. Let the source of light *L* be very large in comparison to *O*, the opaque obstacle, then the umbra will be a cone having its apex at *C*, Fig 5. And if a screen be placed in the position *AB*, there will be a small area of deep shadow surrounded by a region of partial darkness. If

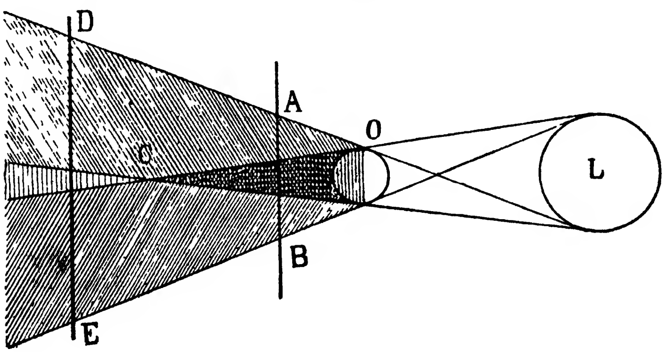


FIG. 5

however, the screen be shifted to *DE*, there will be no complete shadow but merely a confused half-shadow.

It may be remarked here, that on careful examination, it will be noticed in each case, that the shadows cast by opaque objects have no sharp defined edges; but on the contrary, show a gradual transition from total shadow to total brightness. This effect is called *diffraction* and is due to very slight bending of light.

Eclipses. The Solar Eclipse is the direct outcome of the shadow cast by the Moon on the Earth. In fig. 6, when the Moon occupies the position m_1, between the Earth and the Sun, and all the three are in the same plane and straight line; the Moon casts a shadow and prevents the light of the Sun from falling on certain areas of the Earth. In these areas, the Sun will be either invisible or partially visible and this is called the Solar Eclipse.

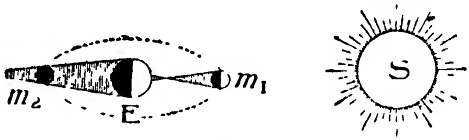


FIG. 6

The Lunar Eclipse is caused by the shadow of the Earth on the Moon. When the Moon occupies the position m_2 (Fig. 6), with respect to the Earth and the Sun, the Earth obstructs the sunlight from reaching the Moon, which has no light of its own, and appears to be dark. This is called Lunar Eclipse. Here also the condition necessary is, that all the three heavenly bodies should be in the same plane and straight line.

139. Intensity of illumination. The *amount of light*, which falls upon *unit area* of a surface *in one second*, provided the light is *incident normally*, is called the *intensity of illumination* of that surface. It is impossible to measure intensity of illumination absolutely, because our eye is incapable of giving any numerical value of the brightness of a surface. It can however, give us an idea of the relative brightness of two surfaces and of the same surface, when illumined by two different sources.

Law of Inverse Squares. *The intensity of illumination of a surface, due to a point-source of light, varies*

inversely as the square of the distance of the surface from the source.

Let L be a luminous source of light, giving a quantity of light L per second. Suppose this source to be situated at the centre of the sphere of radius r. Then the whole of the light L, given out by the source, will fall on the surface of the sphere and be incident normally. Thus intensity I_1 *i.e.* the quantity of light falling on unit area of the surface of the small sphere, will be equal to $\frac{L}{4\pi r^2}$, the area of the surface of a sphere being $4\pi r^2$. Suppose this small sphere of radius r is now replaced by a bigger sphere of radius R, whose centre also coincides with L; then by the above reasoning, the intensity I_2 on the surface of the bigger sphere will be equal to $\frac{L}{4\pi R^2}$.

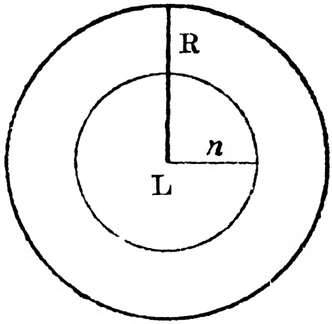


FIG. 7

$$\text{Thus we have } I_1=\frac{L}{4\pi r^2} \text{ and } I_2=\frac{L}{4\pi R^2},$$

$$\text{or } \frac{I_1}{I_2}=\frac{R^2}{r^2}.$$

The intensity, due to a source at a given distance, varies inversely as the square of the distance of the surface from the source.

Illuminating power of a Source*.—*Illuminating*

---

* It should be carefully noted, that illuminating powers of two sources of light are *proportional* to their intensities at 1 cm. from them.

$$\text{Thus} \qquad I_1=\frac{L_1}{4\pi} \text{ and } I_2=\frac{L_2}{4\pi} \text{ or } \frac{I_1}{I_2}=\frac{L_1}{L_2}.$$

For this reason, illuminating power is also defined as the intensity of illumination produced by the source at 1 cm. from it.

power of a source of light is the total light, emitted by it per second.

The illuminating powers of two sources of light, situated at distances d_1 and d_2 from a screen, where they produce equal intensities, are directly proportional to the squares of their distances from that screen.

Let L_1 and L_2 be the sources of which the illuminating powers are L_1 and L_2; and d_1 and d_2, their distances from the screen.

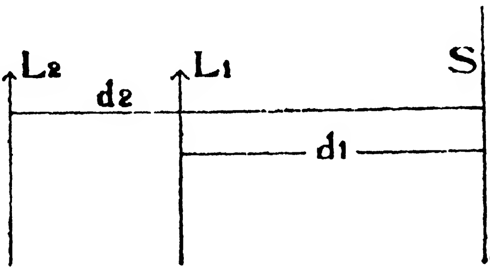


Fig. 8

Then I_1, the intensity due to L_1 on the screen S will be $\frac{L_1}{4\pi d_1^2}$ and I_2, the intensity due to L_2 on the same screen will be $\frac{L_2}{4\pi d_2^2}$. If I_1 be equal to I_2,

$$\text{then} \qquad \frac{L_1}{4\pi d_1^2} = \frac{L_2}{4\pi d_2^2},$$

$$\text{or} \qquad \frac{L_1}{L_2} = \frac{d_1^2}{d_2^2}.$$

If two sources of light produce the same intensity of illumination on a common surface, their illuminating powers are directly proportional to the squares of their distances from that surface.

The above equation affords us a method of comparing the illuminating powers of two sources of light. The process is called *Photometry* and the instruments by which this comparison is effected are called **Photometers.**

Rumford's Photometer. It consists (Fig. 9) of a cylindrical rod R, fixed in front of a vertical ground-glass screen AB. The two sources of light, L_1 and L_2,

are placed to the right of AB in such positions, that each one of them casts a separate shadow of the rod R on AB. The distances of the sources are so adjusted, that the shadows are of equal depth, lie side by side and do not overlap.

$$\text{Then } \frac{L_1}{L_2} = \frac{d_1^2}{d_2^2}.$$

Good comparison is effected by Rumford's photometer; and a totally dark room is *not* an absolute necessity.

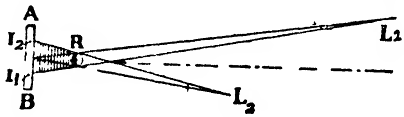


FIG. 9

Bunsen's grease-spot photometer:—

It consists of a plain piece of white unglazed paper, a small circular area of which is made translucent by a greased spot. This photometer is placed between the two sources of light and its position altered, until the greased spot and the rest of the paper appear to be equally illuminated, then $\frac{L_1}{L_2} = \frac{d_1^2}{d_2^2}$, where d_1 is the distance of L_1 from the photometer and d_2 that of L_2.

Theory of the above photometer is as follows:—

Let q_1 and q_2 be the quantities of light per sq. cm., falling on opposite faces of the photometer, from the sources L_1 and L_2 respectively. Suppose the unglazed paper reflects a fraction x of the light and transmits the remainder $1-x$; while the greased spot reflects a fraction y of the light and transmits the remainder $1-y$.

Then on seeing the photometer from the side of the source L_1, the light which will reach the eye from the unglazed paper will be $[q_1x + q_2(1-x)]$ and that which will reach the eye from the greased spot will be $[q_1y + q_2(1-y)]$. When however, the two appear equally bright, we must have

$$q_1x + q_2(1-x) = q_1y + q_2(1-y)$$

$$q_1(x-y)=q_2(x-y);$$

or dividing by $(x-y)$,

$$q_1=q_2$$

$$\therefore \quad \frac{L_1}{4\pi d_1^2}=\frac{L_2}{4\pi d_2^2},$$

$$\text{or} \quad \frac{L_1}{L_2}=\frac{d_1^2}{d_2^2}.$$

Joly's photometer. It consists of two similar, plane and parallel slabs of paraffin-wax, with a sheet of tin-foil in between them. The photometer is placed between the two sources and its position adjusted, till on looking sideways, both the slabs appear equally bright, then

$$\frac{L_1}{L_2}=\frac{d_1^2}{d_2^2}.$$

Light standard The standard of light used for comparison is the light, emitted by a sperm candle, weighing 6 to the pound and burning 120 grains per hour The brightness depends upon several factors, such as pressure, presence of vapour in the atmosphere and other causes, and is liable to fluctuations.

SUMMARY

1 Light is the external cause of our impression of vision

2. Light travels in straight lines through a homogeneous medium

3 Substances, which allow light to pass through them are called **transparent**, while those, which cut off light completely, are known as **opaque**, and those, which allow light to pass through them, but through which substances cannot be seen distinctly, are known as **translucent**.

4 Umbra means the region of total darkness, while penumbra means the region of partial darkness.

5 The quantity of light falling normally in one second on a unit area is called **intensity of illumination**, while the quantity of light emitted by a source per second is called **illuminating power**

6 The intensity of illumination of a surface varies inversely as the square of its distance from the source.

7. The illuminating powers of two sources vary directly.

as the squares of their distances from the screen, over which they produce the same intensity of illumination.

EXAMPLES

1. A lamp produces intensity of illumination on a screen 20 metres from it, equivalent to that of 1·2 candles at 50 cms. Find the candle-power of the lamp? (L.U.)

2. Two sources of light, each 2 candle-powers, are placed on the same side of a Bunsen's grease-spot photometer. One is at a distance of one foot from the spot and the other at 2 feet. Where must a third source of 5 candle-powers be placed in order that the appearance of each side of the photometer may be the same? (L.U.)

3. A lamp produces a certain intensity of illumination at a distance of 50 cms. from it. On placing a sheet of glass between the lamp and the screen, the former must be moved 10 cms. towards the latter to produce the same intensity of illumination. What percentage of light is stopped by the glass?

4. Two sides of a photometer look alike, when the distances of the sources are 1 foot and 3 feet respectively. What should be the distance of the fainter source, when the brighter one is at 5 feet?

5. The diameter of the Sun being 900,000 miles, its distance from the Earth 90,000,000 miles and the diameter of the Moon 2,100 miles. Find the distance of the Earth from the Moon at the time of total Solar Eclipse on a single point on the Earth's surface. (P.U.)

6. Qutub Minar appears as big as a pencil 6″ long, when the pencil is 5 feet from the eye. If the Minar be at a distance of $^1/_2$ mile, find its height.

7. A circular uniform source of light is 2 inches in diameter and a sphere of the same size is placed 3 feet away. Find the sizes of the umbra and penumbra, 3 feet away from the sphere.

CHAPTER II

REFLECTION

140. Laws of Reflection. When a narrow beam of light falls on a smooth polished surface, it is sent back in the very same medium according to definite laws, known as the laws of reflection.

(*i*) The angle of incidence is equal to the angle of reflection.

(*ii*) The incident ray, the reflected ray and the normal to the reflecting surface at the point of incidence, lie in the same plane.

When the surface, from which reflection takes place, is rough, the reflected rays go in various directions and the light is said to be *scattered* or *diffused*.

Experiment. Let *AB* (fig. 10) be a plane mirror perpendicular to the plane of paper. Fix two pins *C* and *D* to denote the incident ray. On looking from the other side, fix pins *E* and *F* in such a way, that they are in a *line* with the *images* of *C* and *D*. Join *CD*, *EF* and from the point of intersection of these draw perpendicular to *AB*.

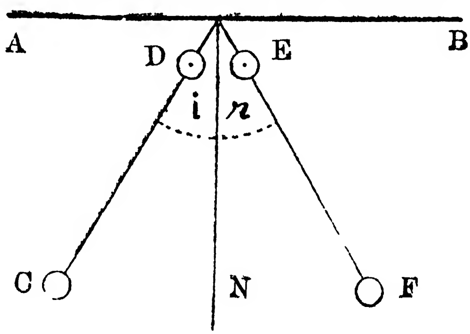


FIG. 10

Ray *CD*, which proceeds *towards* the mirror is called the *Incident ray*; ray *EF*, which proceeds *from* the mirror is called the *Reflected ray* and the perpendicular *N* is called the *Normal*. The angle, which the

incident ray makes with the *normal*, is called the *angle of Incidence* and the angle which the *reflected ray* makes with the *normal*, is called the angle of reflection.

Measure the angle of incidence $\angle i$ and the angle of reflection $\angle r$ and you will see that they are equal, *i.e.* $\angle i = \angle r$. This proves the first law. Further the bases of the pins C and D denoting the incident ray, the bases of the pins E and F denoting the reflected ray and the normal N, all lie in the plane of the paper; and thus they are in the same plane.

Experiment The laws of reflection may further be verified with the help of the instrument called Reflection and Refraction apparatus shown in fig 11 It consists of a vertical circle graduated in degrees A small mirror AB is fixed at its centre and in a plane perpendicular to that of the circle Two tubes T_1 and T_2 are fixed on its circumference and their axes are directed towards the centre of the circle The zero of the scale coincides with O, where CO is the normal to the plane mirror Hold a candle flame near the tube T_1, so that rays through it, fall on the mirror, look through the other tube and move it till the reflected image of the candle flame is visible Note the angles, which each tube makes with the normal and see that *they are equal* *Moreover the plane of the circle contains the incident ray, the reflected ray and the normal* Thus both the laws are proved.

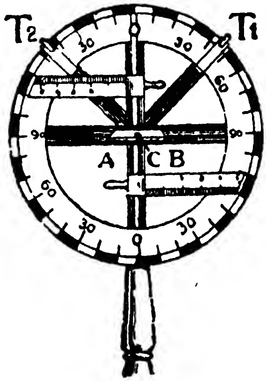


FIG. 11

Image in a plane mirror. Let MM' (fig. 12) be a plane mirror supposed perpendicular to the plane of the paper and let O be a luminous object situated in front of it. Let any two rays OA and OB starting from O be incident at A and B respectively. These incident rays give rise to the reflected rays, AC and BD, which when produced backwards, meet at O'. Thus O' is the image of O.

$\angle OAN = \angle CAN$, by laws of reflection or $\angle i = \angle r$,

$\therefore \quad \angle OAM = \angle CAM'$, being complementary of equal angles, $\angle i$ and $\angle r$ respectively.

But $\angle CAM' = \angle O'AM$, being vertically opposite angles,

$\therefore \quad \angle OAM' = \angle O'AM'$, being supplementary of equal angles,

and $\angle OAM = \angle O'AM$.

By similar reasoning $\angle OBA = \angle O'BA$.

In the triangles OAB and $O'AB$, AB is common and two angles of the one are respectively equal to two angles of the other

$\therefore$ the triangles are congruent

Thus $OA = O'A$ (i)

Again join OO' and in the two triangles OAP and $O'AP$,

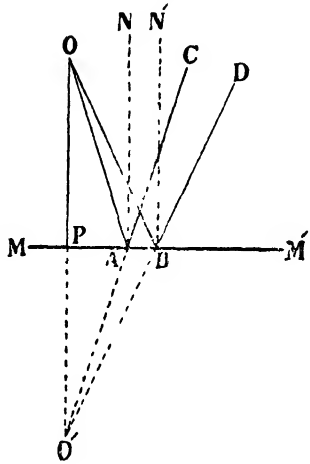


FIG 12

$OA = O'A$ proved in equation (i) above,

$\angle OAP = \angle O'AP$, ,, above,

and AP is common,

$\therefore$ the triangles are congruent;

Hence $OP = O'P$ and $\angle OPA = \angle O'PA = 90°$.

The image of an object lies as far behind the mirror as the object is in front of it, and the line joining the object and the image intersects the reflecting surface at right angles

The rays proceeding from the object are reflected from the mirror, but they *appear to come* from O', a point behind the mirror, therefore O' is called the *virtual image of O*

A virtual image is that which cannot be obtained on a screen and the rays do not actually pass through the point, where the image appears to be. Images formed by plane mirrors are always virtual.

To trace the course of rays for an object of finite size placed in front of a mirror.—

Let AB be the object placed in front of a mirror. A gives rise to the image A', as far behind the mirror as A is in front of it and similarly B gives rise to the image B'. Thus the reflected image $A'B'$ is of the same size as the object AB. The course of rays by which the image is seen by the eye E is shown in fig. 13.

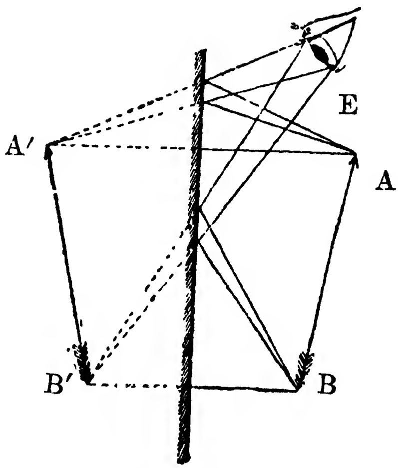


Fig. 13

141. Principle of Parallax. If an eye be placed in such a position that any two objects or two images or an object and an image (real or virtual), appear in a straight line; then on moving the eye to one side, the thing, which lies nearer to the observer, always appears to move more to the opposite side. The principle is of great importance in so far as it enables us to judge, which of the two things is nearer to us. If two things are situated at one and the same point, then on moving the eye one way or the other, no displacement whatsoever is noticed between them. The student can easily verify this by holding a pencil in front of his eye, in such a way that some distant well-defined object such as a telegraphic pole or the string of an electric lamp is covered over by the pencil. On moving his eye to the left, it will be noticed that the pencil will be displaced more towards the right.

142. To find the position of the image of an object placed in front of a mirror.

Experiment. Let M_1M_2 be a mirror placed perpendicular to the plane of the paper. Let a pin be fixed at O to denote the object. Fix a pin B at random near the mirror and then

fix a pin A, farther from the mirror in such a way that the *pins* A, B and the *image* of O appear in one straight line. Repeat the process on the other side, so that the *pins* C, D and the *image* of O appear in another straight line. Remove the mirror and draw lines joining AB and DC. Their point of intersection gives the position of the image of O. Because the image of O lies on AB as well as on DC, by construction; therefore it must lie on O', the intersection of both these lines. Join OO', intersecting the mirror at N. Show that $ON = O'N$ and that OO' is normal to M_1M_2.

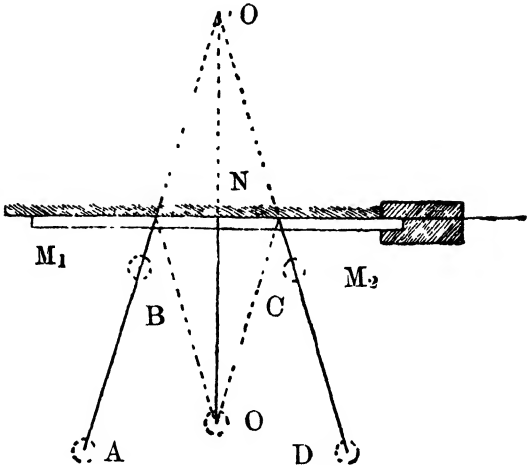


FIG. 14

Experiment. Let M_1M_2 be a mirror scratched in the middle, and placed perpendicular to the plane of the paper. Fix a pin O in front of it to denote the object. On looking in the mirror, the image of the lower portion of the pin will be visible, the image of the upper portion will be missing, because the silvered portion of the mirror is

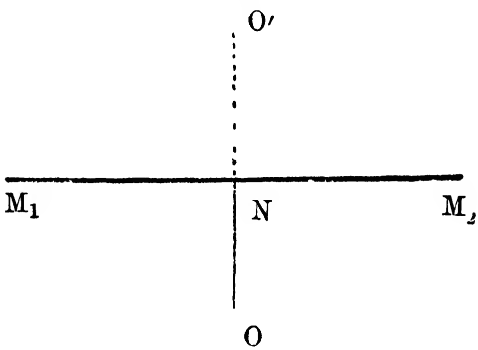


FIG. 15

scratched opposite to it. Now fix a pin behind the mirror in such a way, that its upper portion is visible through the scratched portion of the mirror and adjust its position at O', till there is no parallax between *it* and the *image* of the front pin, then the image of O is formed at O'. Remove the mirror and join OO'. Show that $ON=O'N$ and OO' is perpendicular to M_1M_2.

Rotating mirror. *When a mirror is rotated through any angle θ, the reflected beam rotates through double the angle, i.e.* 2θ.

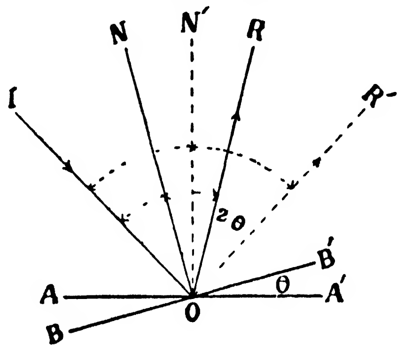


FIG. 16

Let BB' be the first position of the mirror, IO the incident ray, OR the reflected ray and ON the normal to the mirror in that position; then if the angle of incidence ION be denoted by $\angle i$; the $\angle IOR$ between the incident ray and the reflected ray will be equal to $\angle 2i$. Let the mirror be rotated through an angle θ to the position AA', the normal will be shifted to the position ON', so that the $\angle NON'$ between the two normals will be θ. The new angle of incidence ION' will become $(i+\theta)$; and $\angle IOR'$ between the incident ray and the new reflected ray will be $2(i+\theta)$.

$\therefore$ $\angle ROR'$ *i.e.* the angle between the old and new reflected rays will be $2(i+\theta)-2i=2\theta$.

Lateral Inversion. In a plane mirror, the right of an object appears as the left of the image and the left of the object appears as the right of the image; this is called lateral inversion, as a consequence of this, the print appears inverted.

Deviation produced by reflection. The incident ray CD, fig. 10, would have gone in a straight line if the mirror AB were absent. On account of reflection

from the mirror, its course is changed to EF. Thus its path is deviated through an angle $\pi - 2\iota$.

143. Inclined mirrors. When rays from a luminous point are reflected from a mirror, they appear to proceed from a point as far behind the mirror, as the object is in front of it If these rays fall on a second mirror, an *image* of the first image will be formed at the back of the second mirror and so on. It should be noted however, that in order to get an image of an object or image from a mirror, *that* object or image *must lie in front of the reflecting surface.* When the object or image lies behind a mirror. no further image can be obtained from that mirror.

(a) Two mirrors perpendicular to each other. Let XA and XB be two mirrors at right angles to each

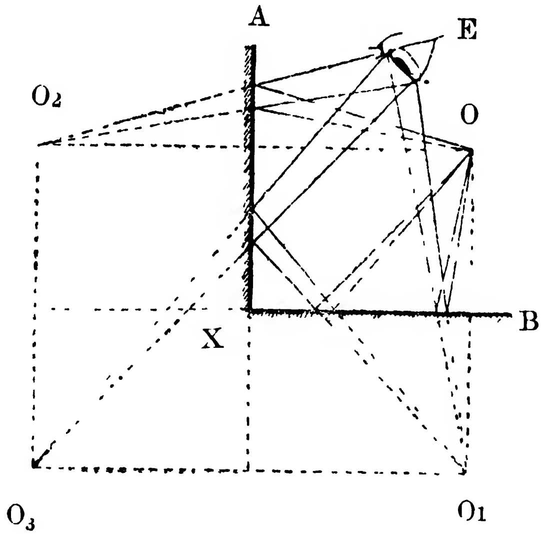


Fig. 17

other and O a luminous object situated between them. An eye placed at E will see three images O_1, O_2 and O_3 of the object. The course of rays is shown in fig. 17. The object gives rise to image O_1 by reflection from

XB and to O_2 by reflection from XA. The image O_3 is formed by the coalescence of the image of O_1 in XA and of O_2 in XB and is formed by two reflections. O_3 being behind both the mirrors, no further image can be produced. Thus the number of images seen when mirrors are inclined at 90° is 3.

But $3 = \frac{360}{90} - 1$. The actual course of rays is shown in figure 17.

(b) Two mirrors inclined at 60°.

Let XA and XB be two mirrors, inclined to each other at an angle of 60°. Let O be a luminous object situated between them. With X as centre and XO as radius describe a circle. From O draw normals on XB and XA to intersect the circumference of the circle at O_1 and O_2. Then O_1 and O_2 will be the images of O formed by one reflection from XB and XA respectively.

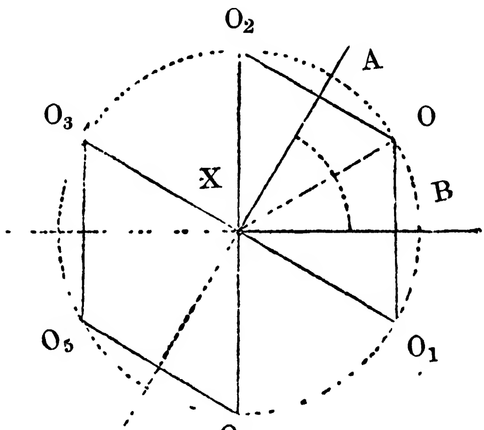


Fig. 18

Now since O_1 lies in front of XA, its image O_3 will be formed in XA by two reflections and the position of O_3 is obtained by drawing a normal from O_1 on XA. The point of intersection of this with the circumference gives the position of O_3 (by simple geometry).

Similarly O_2 lies in front of XB, its image O_4 will be formed in XB by two reflections and its position is obtained by drawing the normal form O_2 on XB.

Where this intersects the circumference, that gives the position of O_4 (as before).

Again O_3 lies in front of XB and O_4 in front of XA; each of them will give rise to an image by three reflections and their positions can be found by the above construction. These two images overlap and give rise to one image O_5, which lies behind both the mirrors and thus no further image can be produced.

$$\text{But } 5=\frac{360}{60}-1.$$

Generalising from the above two cases, we come to the conclusion that n the number of images is always equal to $\frac{360}{\theta}-1$, where θ is the angle between the two mirrors; that is,

$$n=\frac{2\pi}{\theta}-1, \text{ when } \frac{2\pi}{\theta} \text{ is an even number,}$$

$$\text{or} \quad n=\frac{2\pi}{\theta}, \text{ when } \frac{2\pi}{\theta} \text{ is an odd number.}$$

For in the latter case, the two final images are not superposed; as does happen, when $\frac{2\pi}{\theta}$ is even.

(*c*) **Two mirrors parallel to each other.**

If the two mirrors are parallel, then θ is zero and

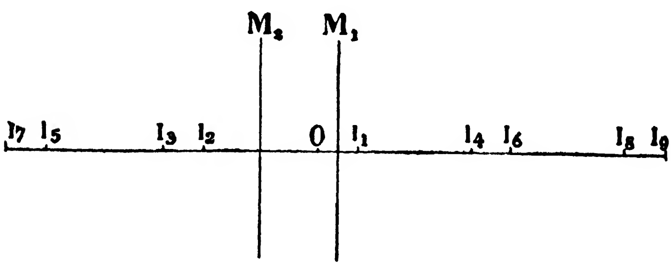


FIG. 19

from the formula, the number of images should theoretically be infinite; but light is weakened at each reflection, so that the images, situated very far off,

become too dim to be seen.

O gives rise to I_1 by reflection from M_1 and to I_2 by reflection from M_2. I_1 being in front of M_2 gives rise to I_3 and I_2 being in front of M_1 gives rise to I_4. I_3 and I_4 in turn give rise to other images.

144. Kaleidoscope. It consists of 3 plane mirrors inclined to each other at an angle of 60°, so as to form an equilateral triangle enclosed within a tube. On one side of the tube is a small hole for the observer to look in; while the other side is closed with 2 pieces of ground glass, with a number of coloured glass pieces in between them. Due to multiple reflections, the coloured glass pieces together with their images, form beautiful symmetrical patterns, which are of use to designers.

145. Reflection from spherical surfaces.

Definitions. A reflecting surface, which is part of a sphere, is known as a **spherical mirror**. When the inner surface reflects light and the outer surface is silvered, the mirror is called **concave**; if however, the inner surface is silvered and the outer surface reflects light, the mirror is called **convex**. The centre of the sphere of which the reflecting surface forms a part is called the **centre of curvature**. The boundary of the mirror is usually circular. The diameter of the circular boundary is called the *Aperture* and the centre of the circular boundary is called the *Pole*. Line joining the *pole* and the *centre of curvature* is called the **Principal axis**. Any line passing through the centre of curvature and incident on the reflecting surface is called a **secondary axis.**

The distance of any point on the reflecting surface from the centre of curvature is called the **Radius of curvature. Principal focus** is a point on the principal axis, half way between the pole and the centre of curvature such that in the case of a concave mirror, the rays which have their course parallel to the principal axis before incidence, converge to it after reflection; while in the case of a convex mirror, rays parallel to the principal axis appear to diverge from it after reflection.

Distances measured from the pole in the direction opposite to that in which the incident ray travels are denoted by **positive** *sign, while those measured from the pole in the same direction in which the incident ray proceeds are denoted by* **negative** *sign.*

Principal focus is half way between the Pole and the Centre of curvature.

Let AX be a ray parallel to the principal axis incident at the point X of a concave mirror. Join X to C, the centre of curvature. Then XC is normal at the point X, because every radius is normal to the circumference, where it is incident. From X let the reflected ray proceed in the direction XF, where F is the principal focus.

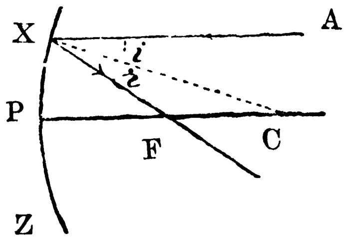


FIG. 20

Now $\angle AXC = \angle$ of incidence,

and $\angle CXF = \angle$ of reflection,

$\therefore$ by laws of reflection, $\angle AXC = \angle CXF$;

but $\angle AXC = \angle XCF$, being alternate angles;

$$\therefore \quad \angle XCF = \angle CXF,$$

$$\text{or} \quad CF = XF,$$

If X is very near to P, then $XF = PF$ and therefore $CF = PF$, very approximately. Thus F is midway between P and C.

The distance of the principal focus from the pole is called the **focal length**. It is denoted by f and is equal to $\frac{r}{2}$, *i. e.* half the radius of curvature.

146. Determination of focal length.

Let XPZ fig. 21 (i), denote a concave mirror with P as the pole, C as the centre of curvature and O as the point-object lying on the principal axis. Let a ray OX be incident at X. Join XC and this gives the radius at the point X. The reflected ray will proceed from X

in the direction of XI making an angle r with XC,

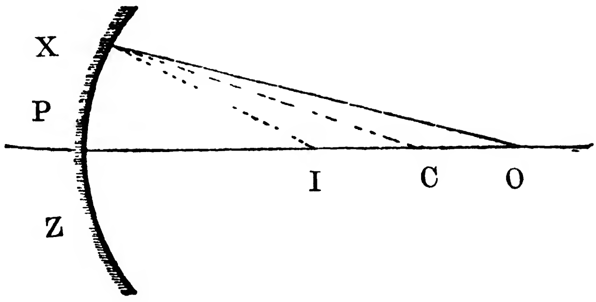


FIG. 21 (*i*)

equal to the angle of incidence, then I will be the image of O.

Now in the triangle OXI, $\angle OXI$ is bisected by the line XC at the vertex; therefore C intersects the base OI into two segments OC and CI, which bear the same ratio to each other, as exists between the two other sides of the triangle.

$$\therefore \quad \frac{OX}{IX}=\frac{OC}{CI} \quad \ldots\ldots\ldots\ldots \quad (i)$$

Let the distance of O from the pole P be denoted by u,
and that of I " " " by v,
and that of C from the pole P be denoted by r.

Then if the aperture be supposed very small and the distances u, v and r very large in comparison to it;

we have $OX=OP$ and $IX=IP$,

or $\dfrac{OP}{IP}=\dfrac{(OP-CP)}{(CP-IP)}$ [by equation (*i*) above], for

X would be very near to P when the aperture is small.

$$\text{or } \frac{u}{v}=\frac{u-r}{r-v} \quad \ldots\ldots\ldots \quad (ia)$$

or $ur-uv=uv-vr$, by cross-multiplication,

or $ur+vr=2uv$.

Dividing both sides by uvr, we have

$$\frac{1}{v}+\frac{1}{u}=\frac{2}{r}=\frac{1}{f}, \text{ for } f=\frac{r}{2} \text{ (proved above).}$$

Thus we have $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ ....(ii)

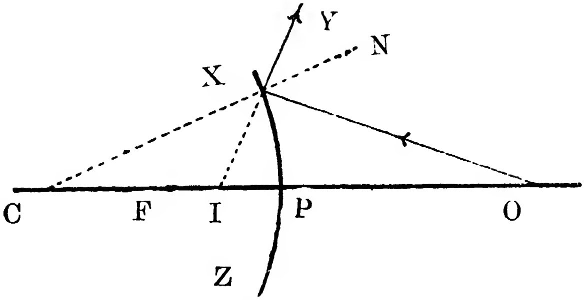


FIG 21 (*ii*)

Again let XPZ, fig. 21 (*ii*), denote a convex mirror with P as the pole, C as the centre of curvature and O as the point-object on the principal axis. Let a ray OX be incident at the point X. Join XC and produce it to N; then XN is the normal at the point X, because XN is simply the prolongation of the radius XC. The reflected ray will go in the direction XY, making an $\angle r$ with N equal to the angle of incidence OXN. Produce YX backwards to intersect the principal axis at I, then I is the *virtual* image of O.

In the triangle OXI the external angle OXY at the apex is bisected by XN, therefore this bisector intersects the base in two segments OC and IC, which bear the same ratio to each other as exists between the two other sides of the triangle.

$$\therefore \quad \frac{OX}{XI}=\frac{OC}{IC} \text{ .... } (iii)$$

Then with the presumptions made in the case of concave mirrors and recalling that distances measured from P towards O are positive, while those measured towards C are negative,

we have $\frac{OP}{IP}=\frac{OP+PC}{PC-IP}$,

$$\text{or} \qquad \frac{u}{-v} = \frac{u+(-r)}{-r-(-v)},$$

$$\text{or} \qquad \frac{u}{-v} = \frac{u-r}{v-r} \quad \ldots\ldots\ldots\ldots\ldots\ldots (iiia)$$

$$\text{or} \quad uv - ur = -uv + vr,$$

$$\text{or} \qquad 2uv = ur + vr.$$

Dividing as before by uvr, we get

$$\frac{2}{r} = \frac{1}{v} + \frac{1}{u} \quad \ldots\ldots \quad \ldots \qquad (iv)$$

$$\text{or} \; \frac{1}{f} = \frac{1}{v} + \frac{1}{u}, \text{ because } f = \frac{r}{2}.$$

147. Position of the image by drawing the course of rays.

So far we have been considering point-objects and images. When however, the object has a finite size, its image can be easily obtained by drawing the course of rays. To get the image, two rays are drawn from a point: one parallel to the principal axis which converges to *the principal focus* in the case of a concave mirror, or diverges from it in the case of a convex mirror; and a second ray proceeding from the point towards the centre of curvature. The point of intersection of the two gives us the position of the image. Fig. 22 shows the

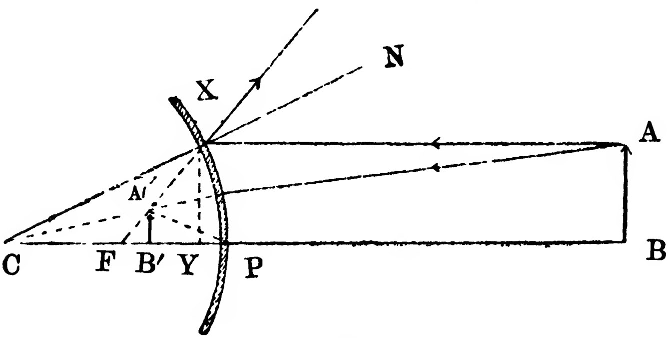


FIG. 22

(virtual) image $A'B'$ of the object AB formed by reflection from a convex mirror. The student should

himself draw the course of rays when the object is situated at varying distances from the mirror and show that in the case of a convex mirror, the image is always *virtual, erect, diminished* and *behind the mirror.*

Fig. 23 gives the (real) image $A'B'$ of the object AB formed by reflection from a concave mirror. The student should draw the course of rays when the object is situated at varying distances from the mirror and show that in the case of a concave mirror, the image:—

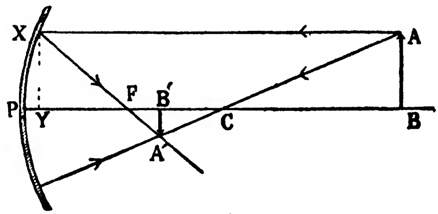


FIG. 23

(*i*) is real, inverted, diminished and in front of the mirror, when the object is at a distance greater than *r* from the pole.

(*ii*) is real, inverted, equal in size, in front of the mirror and coinciding with the object, when the object is at the centre of curvature.

(*iii*) is real, inverted, magnified and in front of the mirror, when the object is between the centre of curvature and the principal focus.

(*iv*) is real, inverted, magnified and in front at infinity, when the object is at a distance equal to *f* from the pole.

(*v*) is *virtual, erect, magnified* and *behind* the mirror, when the object is at a distance less than *f* from the pole ; and

(*vi*) is *virtual, erect, of equal size* and *behind* the mirror coinciding with the object, when the object is situated on the pole of the mirror.

148. Relative position of image and object (treated mathematically.)

The graphic method of finding the relative position of image and object has been given above. Now the same relationship can be easily deduced by the

application of the formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}, \text{ or } \frac{1}{v}=\frac{1}{f}-\frac{1}{u}.$$

(*i*) In the case of a convex mirror, f is negative; therefore for all positive values of u, v must be a negative quantity and less than u; *i.e.* the image must be virtual, erect and diminished.

$$\text{If } u=\infty;\ v=-f;$$

and if u is less than infinity, then v must be less than f;

$$\text{for } \frac{1}{v}=-\left\{\frac{1}{f}+\frac{1}{u}\right\}.$$

Thus in a convex mirror, as the object moves from infinity to the pole, the image moves from the principal focus to the pole:—

(*ii*) In the case of a concave mirror:—

(*a*) If the object is at infinity, then as f is positive and $u=\infty$

$$\frac{1}{v}+\frac{1}{\infty}=\frac{1}{f}, \therefore v=f.$$

i.e. the image is at the principal focus.

(*b*) When the object is at the centre of curvature,

$$u=2f,$$

$$\frac{1}{v}=\frac{1}{f}-\frac{1}{2f}=\frac{1}{2f} \text{ or } v=2f=u.$$

The object and the image coincide.

(*c*) When the object is at the principal focus,

$$u=f,$$

$\dfrac{1}{v}=\dfrac{1}{f}-\dfrac{1}{f}=0$ or $v=\infty$. The image is at infinity.

(*d*) When the object is at the pole, $u=0$,

$$\frac{1}{v}=\frac{1}{f}-\infty$$

or $v=-0$; the image is virtual and is behind the mirror.

149. Conjugate foci. In figs. 21 (*i*) and (*ii*), rays from an object at O, after reflection from a mirror XPZ, form an image at I. A ray such as IX in a concave mirror

fig. 21 (*i*), or YX in a convex mirror fig. 21 (*ii*), is reflected along XO. Thus an object real or virtual at I gives rise to an image at O; and the points O and I are such that an object at one of them gives rise to an image at the other. The points O and I are termed *Conjugate foci*.

150. Magnification. In the case of mirrors and lenses, magnification is defined as the ratio of the *linear* dimensions of the image to the *linear* dimensions of the object.

Thus in figs. 22 and 23, the ratio $\frac{A'B'}{AB}$ is called magnification. When distances are measured from the axis in the upward direction, they are positive; while distances measured in the downward direction are negative. Thus magnification in a convex mirror is positive; while in the case of a concave mirror (Fig. 23), it is negative.

(*a*) To get an expression for magnification in terms of u, v, f and r, we have the right-angled triangles $CA'B'$ and CAB (Figs. 22 and 23) similar,

$$\text{therefore}\ \frac{A'B'}{AB}=\frac{CB'}{CB},$$

$$\text{or}\ \frac{I}{O}=\frac{-(r-v)}{-(r-u)},\ \text{in fig. 22 for a convex mirror,}$$

$$\text{or}\ \frac{I}{O}=\frac{v-r}{u-r}=\frac{-v}{u},\ \text{from equation } (iiia)\ \text{P. 322.}$$

As $\frac{I}{O}$ is positive; but v is negative in the case of a convex mirror, so a negative sign is attached to v to make the ratio $\frac{v}{u}$ positive.

In the case of a concave mirror,

$$\frac{-I}{O}=\frac{r-v}{u-r},\ \text{in fig. 23 for a concave mirror.}$$

$$\text{Or} \frac{-I}{O} = \frac{r-v}{u-r} = \frac{v}{u}, \text{ from equation } (i\ a) \text{ P. 320.}$$

$$\text{or} \frac{I}{O} = \frac{-v}{u}.$$

As $\frac{I}{O}$ is negative, but v and u are positive, so a negative sign is attached to v, to make the ratio $\frac{v}{u}$ negative.

Thus magnification is equal to $-\frac{v}{u}$ *for both convex and concave mirrors.*

(b) Again referring to figs. 22 and 23, we have the triangles $FA'B'$ and FXY similar;

therefore, we have $\frac{I}{O} = \frac{FB'}{FY}$, for both convex and concave mirrors.

Now for a convex mirror, fig. 22,

$$\text{we have} \frac{I}{O} = \frac{-(f-v)}{-f},$$

$$\text{or} \frac{I}{O} = \frac{v-f}{-f},$$

$$\text{or} \frac{I}{O} = -\frac{(v-f)}{f}.$$

And for a concave mirror, fig. 23,

$$\text{we have } -\frac{I}{O} = \frac{v-f}{f},$$

$$\text{or} \frac{I}{O} = -\frac{v-f}{f}.$$

Thus magnification is also equal to $-\frac{(v-f)}{f}$ *for both the mirrors.*

(c) Again, we have $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ for both convex and concave mirrors.

Or multiplying both sides by u, we get

$$\frac{u}{v}+1=\frac{u}{f}.$$

$$\text{or}\ \frac{u}{v}=\frac{u}{f}-1=\frac{u-f}{f}.$$

Inverting both sides, we get $\frac{v}{u}=\frac{f}{u-f}$,

$$\text{or}\ \frac{-v}{u}=\frac{-f}{u-f};$$

$$\text{but}\ \frac{-v}{u}=\frac{I}{O}\ \therefore \frac{I}{O}=-\frac{f}{u-f}.$$

Hence magnification is also equal to $-\frac{f}{u-f}$, *for both kinds of mirrors.*

(*d*) Again from equations (*i a*) and (*iii a*), we have

$$\frac{-v}{u}=\frac{I}{O}=-\frac{r-v}{u-r}.$$

Thus magnification is also equal to $-\frac{r-v}{u-r}$, *for both kinds of mirrors.*

SUMMARY

1. **Laws of Reflection.** Angle of incidence is equal to the angle of reflection; and the incident ray, the reflected ray and the normal lie in the same plane.

2. **Image** is formed as far behind a plane mirror, as the object is in front of it.

3. **Real image** is one, which can be obtained on a screen; while a virtual image is that, which cannot be obtained on a screen.

4. The reflected ray is turned through double the angle, through which a mirror is rotated.

5. Two mirrors inclined to each other at an angle θ, give rise to a number of images given by the formula, $n=\frac{360}{\theta}-1$.

6. Focal length of a spherical mirror is given by

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}.$$

7. Magnification is defined as the ratio of the linear dimensions of the image to the linear dimensions of the object; and is equal to $-\frac{v}{u}$.

EXAMPLES

1. The image formed by a convex mirror of focal length 30 cms. is a quarter of the size of the object. What is the distance of the object from the mirror? (*P.U.* 1926)

$$\frac{I}{O}=\frac{1}{4}. \quad \text{But } \frac{I}{O}=-\frac{v}{u},$$

$$\therefore \quad \frac{-v}{u}=\frac{1}{4}, \qquad \text{or} \quad v=\frac{-u}{4}.$$

Substituting this value in the equation, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$,

$$\text{we get } \frac{1}{\frac{-u}{4}}+\frac{1}{u}=\frac{1}{f},$$

$$\text{or } \frac{-4}{u}+\frac{1}{u}=\frac{1}{f}=-\frac{1}{30},$$

$$\therefore \quad u=90 \text{ cms.}$$

2. The image formed by a concave mirror of 30 cms. radius of curvature, is twice as big as the object. Find the distance of the object from the mirror?

When the image is real, it is always inverted :—

$$\frac{I}{O}=\frac{-2}{1}, \text{ but } \frac{I}{O}=\frac{-v}{u},$$

$$\therefore \quad -2=\frac{-v}{u}, \text{ or } v=2u.$$

$$\because \left(\frac{1}{v}+\frac{1}{u}=\frac{1}{f}\right)$$

$$\text{or} \quad \frac{1}{u}+\frac{1}{2u}=\frac{1}{15}$$

$$\text{or} \quad \frac{3}{2u}=\frac{1}{15}$$

$$\text{or} \quad 45=2u$$

$$\text{or} \quad \frac{45}{2}=u.$$

When the image is virtual.—

$$\frac{I}{O}=\frac{2}{1}, \text{ but } \frac{I}{O}=\frac{-v}{u},$$

$$\therefore \quad 2=\frac{-v}{u}, \quad \text{or} \quad v=-2u$$

$$\therefore \quad \left(\frac{1}{v}+\frac{1}{u}\right)=\frac{1}{f}$$

$$\text{or} \quad \frac{1}{u}+\left(-\frac{1}{2u}\right)=\frac{1}{15}$$

$$\text{or} \quad \frac{2-1}{2u}=\frac{1}{15}$$

$$\text{or} \quad \frac{1}{2u}=\frac{1}{15}$$

$$\text{or} \quad u=7{\cdot}5$$

3. A concave mirror is to be used to give an image of an illuminated slit, on a screen 15 cms. from it, 1·25 times the length of the slit and parallel to it. Find the focal length of the mirror?

4. An object 3 cms. high is placed at a distance of 120 cms. from a convex spherical mirror of 30 cms focal length. Find the size of the image (*P U* 1920)

5. An object placed in front of a mirror gives rise to an image twice as large as itself The object is moved 5 cms. towards the mirror The magnification becomes 3 times. Find the focal length.

6. Show that the size of a plane mirror should be at least half the size of an observer, if he wishes to see his full-size image

7. If the full moon's disc subtends an angle of one degree at the Earth's surface, find the focal length of the concave mirror, which would produce a real image of the moon upon a screen, the diameter of the image being 3 cms.

8 An object is placed 4 inches from a concave mirror of focal length 8 inches. How far away is the image and what is its magnification?

CHAPTER III

REFRACTION

151. When a ray of light passes from one transparent medium to another transparent medium of *different density*, its path undergoes a change of direction at the surface of separation of the two media. This phenomenon is spoken of as *Refraction.*

Experiment. Allow a beam of Sunlight AB, fig. 24, to fall on the surface of eosine-solution contained in a rectangular glass vessel. The path of the beam in the liquid is marked by green streak BC, showing that the ray has been bent on being incident at the point B. AB is called the incident ray, BC the refracted ray and BN the normal.

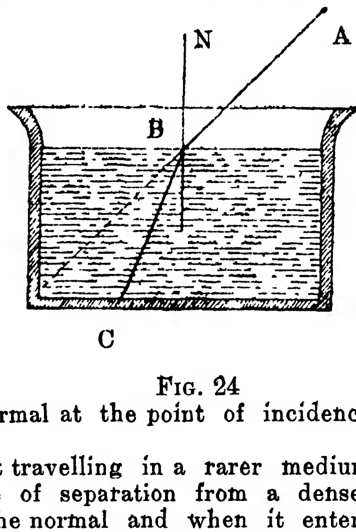


FIG. 24

Laws of Refraction. The bent ray always obeys the following laws:—

1. The incident ray, the refracted ray and the normal at the point of incidence lie in the *same plane.*

2. When a ray of light travelling in a rarer medium is incident on the surface of separation from a denser medium, it bends *towards* the normal and when it enters from a denser to a rarer medium, it bends *away from* the normal.

3 If the rarer medium consist of vacuum (or even air), then the ratio of the sine of the angle of incidence to the sine of the angle of refraction, is constant for the two given media and is called the *refractive index.* It is denoted by the Greek letter μ. Thus $\mu = \frac{\sin i}{\sin \gamma}$.

The last law is sometimes called *Snell's Law*, after the name of the Dutch Philosopher, who first discovered it.

The laws of refraction are verified in the following manner:—

Experiment. Place a glass slab *ABCD* on a drawing board and draw its outline. Fix two pins *E* and *J*, to denote the incident ray; and two pins *K* and *H* on the other side, in a line with the images of *E* and *J*.

Join *EJ* to intersect the face *AB* at *F* and join *HK* to intersect the face *CD* at *G*. Join *FG* and draw normals *FN* and *GM*.

Line *EJF* denotes the *incident* ray.

FG denotes the *refracted* ray, and

GKH denotes the *emergent* ray.

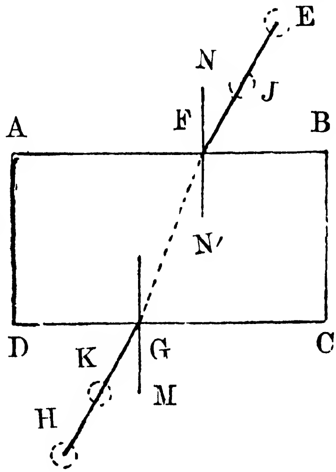


Fig. 25

All the above lie in the plane of the paper and so do the normals *FN* and *GM*. Thus the first law of refraction is proved. Measure the angle *EFN*, the angle of incidence and the angle *GFN′*, the angle of refraction, it will be seen that the latter is smaller than the former, showing that the ray has bent towards the normal, on entering glass a denser medium. From the tables, get the values of the sines of the angles of incidence and refraction and see that their ratio is 1·5, the refractive index of glass. The last result can be verified by taking one more ray, which is incident at a different angle and showing that the ratio of the sine of angle of incidence to the sine of angle of refraction, is still the same; whatever the angle of incidence, provided the media remain the same.

The laws of refraction can also be demonstrated with the help of the Refraction and Reflection apparatus of fig. 11. Instead of keeping the reflecting surface at the centre, water is contained in a semi-circular vessel, Fig. 26.

The incident ray AB falls on the plane surface of water and is refracted along BC. On leaving the water it is not bent, because it travels along a radius and is thus perpendicular to the curved surface. By varying the angle of incidence, corresponding angles of refraction can be found and the relation $\sin i/\sin r = \mu$ (constant), can be proved.

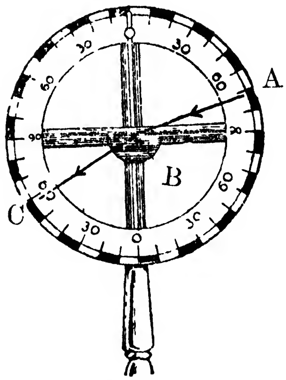


FIG. 26

152. Methods of finding refractive indices of solids and liquids.

The refractive index of a solid is obtained by the method described in the experiment of fig 25; and that of a liquid can be found by Refraction and Reflection apparatus, fig. 26; but accurate methods are based on finding the apparent thickness or depth of the given substance.

Let $DEFG$ be a glass plate or water contained in a rectangular vessel. Let O be the object situated just at the bottom. To get the position of its image, we take two rays emanating from it: one OC incident normally on the surface of separation, which will go in a straight line and a second ray OA, making an angle OAN' with the normal on the surface of separation. This ray will go in the direction AB, because it is travelling from a denser to a rarer medium and is thus refracted away from the normal. Produce this ray backwards to intersect OC at the point O'. Then O' is the *virtual* image of O.

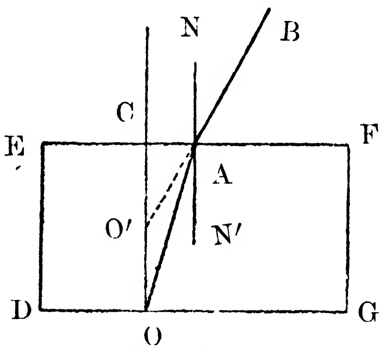


FIG. 27

As the ray is travelling from the denser medium to air, we have $\frac{1}{\mu}=\text{Sin } \angle OAN' / \text{Sin } \angle BAN$.

Now $\angle OAN'=\angle AOC$, being alternate angles.
and $\angle BAN=\angle AO'C$, " corresponding angles.

$$\therefore \quad \frac{1}{\mu}=\frac{\text{Sin } \angle AOC}{\text{Sin } \angle AO'C}$$

If the angles are small; as they must be in actual practice, because both the rays must enter the pupil of the observer's eye, if the image of the object is to be seen at all, then the sine, the circular measure and the tangent of an angle are very nearly equal. Therefore substituting tangents for sines, we have

$$\frac{1}{\mu}=\frac{\frac{CA}{CO}}{\frac{CA}{CO'}}=\frac{CO'}{CO},$$

$$\text{or} \quad \mu=\frac{CO}{CO'}=\frac{\text{Real thickness}}{\text{Apparent thickness}}$$

$$\text{or} \quad \mu=\frac{\text{Real depth}}{\text{Apparent depth}}=\frac{t}{t'}$$

$$=\frac{CO}{CO-OO'}=\text{thickness} / (\text{thickness } \textit{minus}$$

distance through which the image appears to be raised up).

The apparent thickness of a block for rays, for which the angle of incidence is extremely small, is equal to real thickness, divided by the refractive index of the substance;

$$\textit{i.e. } t'=\frac{t}{\mu}.$$

And refractive index is given by the *quotient* of true thickness or depth, divided by apparent thickness or depth.

$$\mu=\frac{t}{t'}=\frac{\text{Real depth}}{\text{Apparent depth}}.$$

Experiment.—Refractive index of glass. Take a glass slab and place it on a line drawn on a drawing board, fig. 28. Measure its depth t and hold a needle N at a place so that there is no parallax between it and the image of the line, seen through the glass slab. Measure its height h above the line, then the refractive index of glass $= \frac{t}{t-h}$.

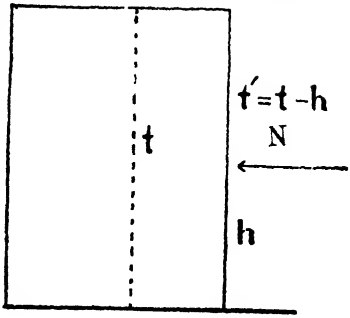


Fig. 28

Refractive index of water. Take a beaker of glass and make a mark at its bottom. Focus a travelling microscope to see it distinctly. Let the reading of the microscope be r_1. Pour water in the beaker gently and move the microscope to focus it again on the image of that mark, as seen through the water Let the reading be r_2. Sprinkle a little lycopodium powder on the surface of water and focus the microscope over it. Let the reading be r_3, then $\mu = \frac{\text{Real depth}}{\text{Apparent depth}} = \frac{r_3 - r_1}{r_3 - r_2}$.

153. Refraction through a Compound plate.

Let a ray BC travelling in air be incident at C, fig. 29, the surface of separation of air from water; let its path in water be denoted by CD. At D, it enters from water into glass and let its path in glass be DE. From E it emerges again into air and its path is indicated by EF. *Experiment shows that the emergent ray EF is always parallel to the incident ray BC, provided the medium above and below the compound plate is the same*

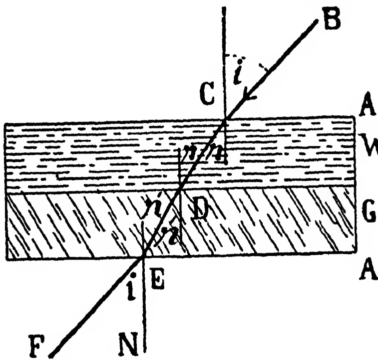


Fig. 29

and provided also the two faces are parallel.

Then denoting the refractive index from air to water by $a\mu w$,

the refractive index from air to glass by $a\mu g$,
and ,, ,, ,, ,, water to glass by $w\mu g$;

We have $a\mu w = \dfrac{Sin\ i}{Sin\ r}$ .. (*i*), where i is $\angle$ of incidence at C and r is $\angle$ of refraction in water.

$w\mu g = \dfrac{Sin\ r}{Sin\ r'}$ .. (*ii*), where r' is $\angle$ of incidence at D in water and r' is $\angle$ of refraction in glass.

and $g\mu a = \dfrac{Sin\ r'}{Sin\ i}$ .. (*iii*), where r' is $\angle$ of incidence at E in glass and i is $\angle$ of refraction (or emergence) in air.

or $a\mu g = \dfrac{Sin\ i}{Sin\ r'}$. (*iv*), by simply inverting equation no. (*iii*).

Dividing equation no. (*iv*) by (*i*), we get

$$a\mu g / a\mu w = \frac{Sin\ r}{Sin\ r'} = w\mu g.$$

Thus the refractive index of glass divided by the refractive index of water gives us the refractive index of glass with respect to water.

The refractive index from one medium A to another medium B is equal to the refractive index of B divided by that of A.

Multiplying equations *i*, *ii* and *iii*, we get

$$a\mu w.\ w\mu g.\ g\mu a = 1.$$

154. Total Internal Reflection. When a ray of light is refracted from a denser to a rarer medium, the refracted ray is bent away from the normal and the angle of refraction becomes greater than the angle of incidence.

Thus if O fig. 30 be a point in a denser medium,

the ray OA will go in the direction AA', OD will go in the direction of DD' and OE will go in the direction of EE'. As the angle of incidence increases, the corresponding angle of refraction also increases; till for a particular angle of incidence, such as that of the ray OF fig. 30, the corresponding angle of refraction

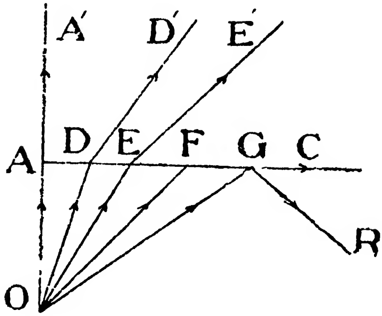


FIG. 30

becomes 90° and the refracted ray just grazes along FC, the surface of separation of the two media *This particular angle of incidence is called the* **critical angle** for the two media, which is defined thus.—When a ray of light is travelling from a *denser to a rarer medium*, then that *angle of incidence*, for which the *corresponding angle of refraction is equal to* 90°, is called the **critical angle.**

If the rarer medium be air, then

$\frac{\sin C}{\sin 90} = \frac{1}{\mu}$, where μ is the refractive index of the denser medium and C the critical angle.

$$\therefore \mu = \frac{1}{\sin C}, \text{ for } \sin 90^\circ = 1.$$

Thus if we find out the critical angle, *the inverse of its sine, gives us the refractive index of that substance.*

If the angle of incidence is greater than the critical angle, then the ray cannot be refracted but is reflected back in the same medium, obeying the laws of reflection. This phenomenon is called **total internal reflection.**

Total internal reflection takes place, when a ray of light travelling from a denser to a rarer medium is incident at the surface of separation, at an angle greater than the critical angle for the two media.

This fact is of utmost utility, when light is required to be reflected without any loss of intensity. The simplest and the best method of doing so is to take a right angled glass-prism ABC. Light enters its one face AB normally and is thus incident at an angle of 45°, on the longest side AC. As 45° is more than the critical angle (41·5°) for glass and air, it is totally reflected there and emerges out of the face BC normally.

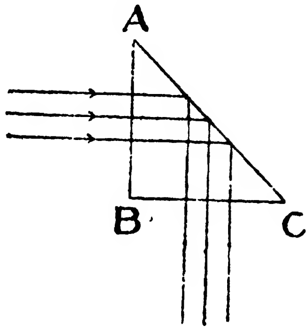


FIG. 31

Mirage. When sand in a desert is strongly heated by the Sun's rays, air in contact with it is heated and becomes lighter as compared to the air in the upper strata of the atmosphere. Then an observer E fig. 32, sees the reflection of distant terrestrial objects such as the foliage, due to total reflection, as would be seen in the surface of a calm lake. This phenomenon is

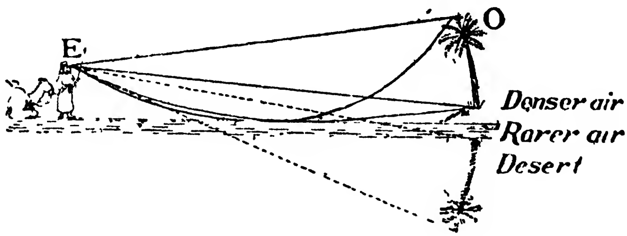


FIG. 32

called mirage. It may be noted that total reflection takes place, because the rays travelling in denser upper air are incident at its surface of separation from the lighter lower air, at an angle greater than the critical.

155. Refraction through a Prism. A prism is a portion of a transparent medium, enclosed between two planes inclined to each other at any angle. The angle between the two planes is spoken of as the angle of the

prism. Let ABC be a prism, DE the incident ray, BF the refracted ray and FG the emergent ray. Produce DE to H and produce FG backwards to intersect DE produced at I. Angle HIG represents the angle through which the incident ray is deviated and this angle is called the angle of deviation. Draw EN and FM normals at the points E and F respectively and produce them to meet at O.

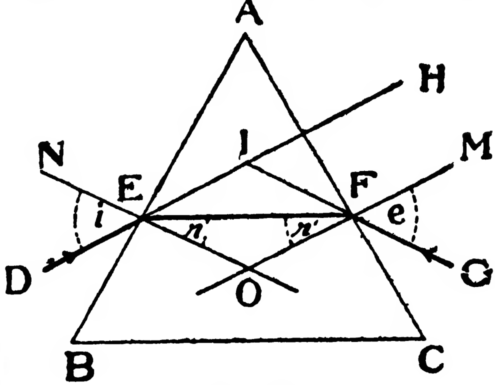


FIG. 83

$\angle DEN = \angle$ of incidence i, $\angle FEO = \angle$ of refraction r; $\angle EFO = r'$, the $\angle$ of incidence from glass; $\angle MFG = \angle$ of emergence e.

Then $\angle IEF = \angle IEO - \angle FEO = i - r$

and $\angle IFE = \angle IFO - \angle EFO = e - r'$

But $\angle HIG = \angle IEF + \angle IFO$; because in the triangle IEF, the external angle must be equal to the two opposite internal angles.

$\therefore$ angle of deviation D *i.e.* $\angle HIG = (i-r)+(e-r')$.

The value of the angle of deviation depends for a given prism upon the angle of incidence i. It is found by experiment that for a certain value of i, the deviation has the least value. This smallest deviation, which a given prism can have, is known as the *angle of minimum deviation.*

Further in the position of minimum deviation, the angle of incidence i and the angle of emergence e, must be equal and the ray EF inside the prism, must be

equally inclined to the prism.*

Then D the deviation$=2(i-r)$.......... ....(i)

For when the prism is placed in the position of minimum deviation, then $r=r'$ and $i=e$. Again $AEOF$ is a quadrilateral of which the two angles at E and F are each equal to a right angle (by construction), therefore $\angle A+\angle O$ must be equal to 2 *rt.* angles,

but $\angle O+\angle OEF+\angle OFE=2rt.\ \angle s.$

i.e. $\angle A+\angle O=\angle O+\angle r+\angle r'.$

$\therefore\ \angle A$ of the prism$=2r$,

or $r=\frac{A}{2}$. . . . . . . (ii)

Substituting this value of r in equation no. (i) we have $D=2i-A$

or $i=\frac{D+A}{2}$ (iii)

But $\mu=\frac{Sin\ i}{Sin\ r}$. Hence $\mu=\frac{Sin\ \frac{D+A}{2}}{Sin\ \frac{A}{2}}$.

If the $\angle A$ of the prism be small, deviation will also be small; then the sine of an angle may be taken very nearly equal to its circular measure.

Then $\mu=\frac{D+A}{A}$, (provided the $\angle$s are very small)

or $\mu=\frac{D}{A}+1$ or $\mu-1=\frac{D}{A}$,

or $D=(\mu-1)A$

*For suppose i and e to be unequal and $\angle\ HIG$ to be minimum. Then if the emergent ray FG be reversed so as to become the incident ray, the new emergent ray will be denoted by ED and the $\angle$ of deviation will still remain the same. It follows that for two $\angle\, s$ of incidence i and e, the deviation must have the least value; but this fact is contrary to actual experience. Hence the only conclusion is $i=e$. If $i=e$, then $r=r'$, for $\frac{Sin\ i}{Sin\ r}=\frac{Sin\ e}{Sin\ r'}=\mu$, the refractive index of the substance of the prism. And if $i=e$, $r=r'$; *i.e.* the ray must be equally inclined in the prism, to its two faces.

i.e. the deviation for a small-angled prism is equal to the product of the angle of the prism and the difference between its refractive index and unity.

For glass, $\mu=1\cdot5$

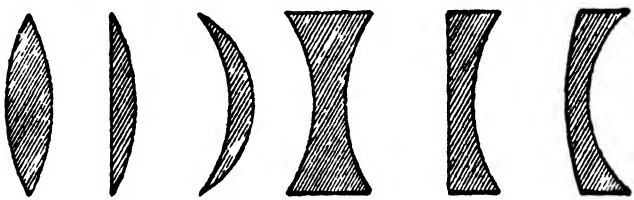
$\therefore$ deviation=Half of the angle of the prism.

156. Lenses. *A lens is a portion of a transparent medium such as glass, bounded by two definite geometrical surfaces.* The surfaces are generally spherical; but one of them may be plane, which is then considered to be a small part of a spherical surface of infinite radius.

Line joining the two centres of curvature of the two surfaces of the lens is called the **Principal Axis.** When one surface is plane, the principal axis is obtained by drawing the normal on the plane surface, from the centre of curvature of the spherical surface.

The diameter of the boundary of a lens is called the **aperture** of the lens.

Lenses are divided into two main classes, known as **converging or convex lenses** and **diverging or concave lenses.** Convex lenses are thicker in the centre and thinner at the edges, while concave lenses are thinner in the centre and thicker at the edges. Another distinguishing feature of these lenses is this: that if a lens be held close to the eye and moved away from it, then the image of a distant object seen through it moves in the *same direction* as the lens if it is *concave* and in



a b c d e f

Converging lenses Diverging lenses

FIG. 34

the opposite direction if it is *convex*.

Each of the above two classes includes three kinds of lenses:

a is called double convex or bi-convex,
b is called plano-convex or convexo-plane,
c is called concavo-convex,
d is called double concave or bi-concave,
e is called plano-concave or concavo-plane,
and *f* is called convexo-concave.

When a beam of light parallel to the principal axis falls on a convex lens, it converges to a point called the **Principal Focus** and in the case of a concave lens, it appears to diverge from that point.

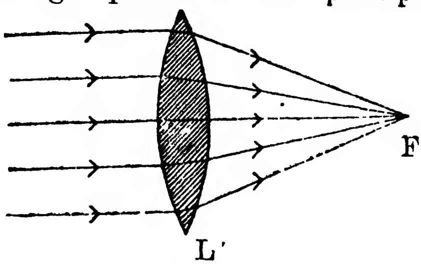


FIG. 35 (*i*)

Thus *Principal focus* is a point on the principal axis, to which in the case of a convex lens converge and from which in the case of a concave lens diverge, all the rays, which are parallel to the principal axis before incidence. The distance of the principal focus from the optical centre of the lens is called the **focal length.**

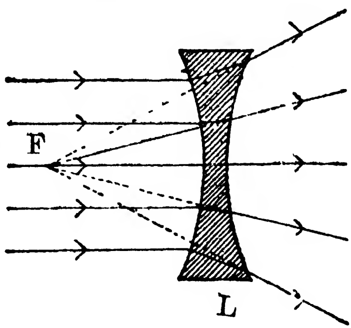


FIG. 35 (*ii*)

157. Optical Centre of a lens. Optical centre of a lens is a fixed point, inside or outside it, such that the rays passing through it are not deviated.

Let C_1 and C_2, Fig. 36 be the centres of curvature of the two faces of the lens. Draw C_1A and C_2B two radii, parallel to each other. Join

AB to intersect the principal axis at O, then O is the optical centre of the lens.

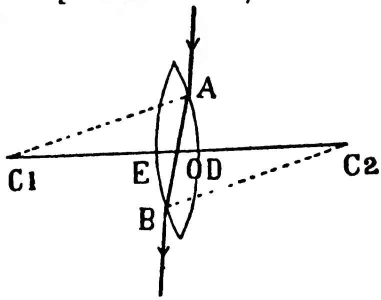


FIG. 36 (*i*)

A ray, which has its path in the lens denoted by AB, makes equal angles with the normals at those places; therefore the incident and emergent rays corresponding to AB, the refracted ray, must also make equal angles with the normals. Hence they must be parallel and thus suffer no deviation; though the emergent ray will be slightly displaced laterally. But this lateral displacement will be negligible in the case of thin lenses.

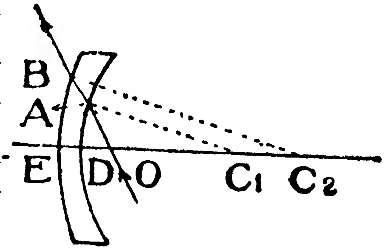


FIG. 36 (*ii*)

Now $\triangle$s OAC_1 and OBC_2 are similar

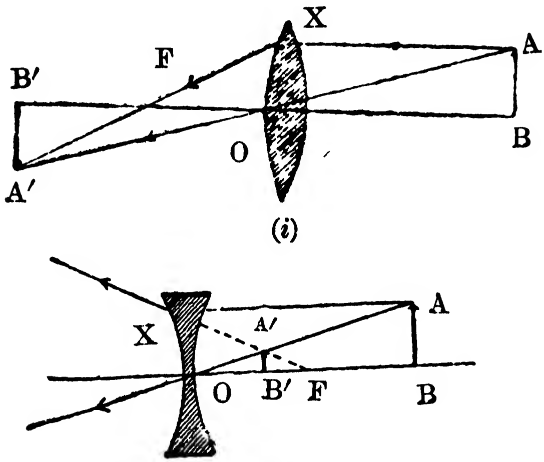
$$\therefore \frac{OC_1}{OC_2}=\frac{AC_1}{BC_2}=\frac{r_1}{r_2}$$

$$\text{or } \frac{r_1}{r_2}=\frac{r_1-OC_1}{r_2-OC_2}=\frac{OD}{OE}.$$

Thus the distance of the optical centre, which may lie within or without a lens, from any face is proportional to the radius of that face.

158. Focal length of Lenses. To get the position of the image of an object, two rays are generally taken from one extremity A of the object: one parallel to the principal axis, which (in convex lenses) converges to or (in concave lenses) appears to diverge from the *principal focus*; and a second ray, passing through the optical centre, follows its course without any deviation. Where these two rays intersect, that is the position of the image

of A. It is real, when the rays actually pass through the point of intersection and virtual, when the rays only appear to intersect. Thus the image $A'B'$ in fig. 37 (*i*)



(*ii*)

FIG. 37

is real and image $A'B'$ in fig. 37 (*ii*), formed by a concave lens, is virtual.

Now the triangles $OA'B'$ and OAB, having their apices at O are similar,

$$\therefore \quad \frac{A'B'}{AB} = \frac{\text{Image}}{\text{Object}} = \frac{OB'}{OB} \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

And the triangles $FA'B'$ and FXO, having their apices at F, are similar,

$$\therefore \quad \frac{A'B'}{XO} = \frac{FB'}{OF}.$$

But $XO = AB$, by construction;

$$\therefore \quad \frac{A'B'}{AB} = \frac{FB'}{OF} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

From equations (*i*) and (*ii*), it is clear that the left-hand expressions are equal, therefore their right-hand expressions must also be equal.

$$\therefore \quad \frac{OB'}{OB}=\frac{FB'}{OF},$$

Now, bearing in mind that distances measured towards the incident ray are regarded as positive and those in the opposite direction as negative and *that the lenses are thin and of small apertures,* we have

(*i*) For a **convex lens** :—

$$\frac{OB'}{OB}=\frac{FB'}{FO} \text{ or } \frac{OB'}{OB}=\frac{OB'-OF}{OF},$$

or $\dfrac{-v}{u}=\dfrac{-v-(-f)}{-f}$, where v=distance of the image, u=distance of the object, and f=distance of the principal focus, from the optical centre of the lens.

$\therefore \quad vf=-uv+uf$

or $uv=uf-vf$; dividing by uvf, we have

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \qquad \text{.......... .. . ..}(iii)$$

(*ii*) For a **concave lens** :—

$$\frac{OB'}{OB}=\frac{FB'}{FO} \text{ or } \frac{OB'}{OB}=\frac{FO-B'O}{FO},$$

or $\dfrac{v}{u}=\dfrac{f-v}{f}$, where the symbols have the above meanings,

$\therefore \quad vf=uf-uv,$

or $\quad uv=uf-vf$, dividing by uvf, we have

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u} \qquad \text{............ ....}(iv)$$

Thus having regard to the signs, the focal length of both the lenses is given by the formula

$$\frac{1}{f}=\frac{1}{v}-\frac{1}{u}.$$

Again **magnification**, which is defined as the ratio of the linear dimensions of the image to those of the object, is given by equation (*i*),

$$\text{as } \frac{I}{O} = \frac{v}{u},$$

$$\text{and by equation } (ii), \text{ as } \frac{I}{O} = \frac{f-v}{f}.$$

159. The size, nature and position of the image formed by lenses.—

In concave lenses, the image is always *virtual*, *erect*, *smaller* and to the *same side* as the object.

In convex lenses, the *nature*, *position* and *size* of the image is determined by the *relative position* of the object. They are obtained mathematically, by giving different values to u varying from ∞ to 0 and finding the corresponding values of v. They can also be obtained by drawing the course of rays. It will be observed:—

(*i*) That as the object travels from ∞ to a distance equal to $2f$, the image travels from f to $2f$. It is *real*, *inverted* and *smaller*.

(*ii*) As the object travels from $2f$ to f, the image travels from $2f$ to ∞. It is *real*, *inverted* and *magnified*

(*iii*) As the object travels from f to 0, image travels from ∞ to 0. It is *virtual*, *erect* and *magnified*.

The student should draw diagrams as well as prove mathematically, the above relations for a convex lens.

160. Focal length of a combination of lenses.

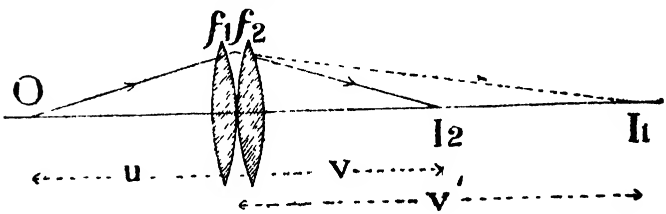


Fig. 38

Let an object O, fig 38, give rise to the image I_1 at a distance v', by refraction through the first lens of focal length f_1.

Then $\dfrac{1}{v'} - \dfrac{1}{u} = \dfrac{1}{f_1}$ ....(*i*)

Place the second lens of focal length f_2 in contact with the first lens and let it, in combination with the first lens, produce the image I_2 at a distance v. For the second lens the distance of the object is v', because I_1 serves as an object for it.

$$\therefore \quad \frac{1}{v} - \frac{1}{v'} = \frac{1}{f_2} \quad \ldots\ldots\ldots\ldots (ii)$$

Adding equations (*i*) and (*ii*), we get

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2}.$$

If the two lenses be considered as one lens of focal length F, we must have

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{F}.$$

$$\therefore \frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

Thus two thin lenses of focal lengths f_1 and f_2 are together equivalent to a single lens of focal length F given by the equation (iii) above.

161. Methods of finding the focal lengths of lenses.

(A) **Convex lens :—**

Experiment. (*i*) Arrange a convex lens, a wire-gauze and a screen in three uprights as shown in fig. 39. Place the lens between the wire-gauze and the screen. Illuminate the wire-gauze by a candle flame and get its image on the screen. Let the distance of wire-gauze O (which

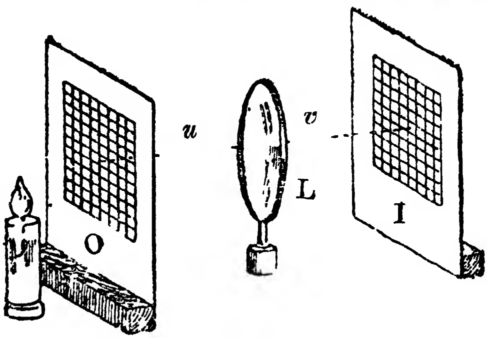


FIG. 39

serves as an object) from the lens be u, and that of the screen be v,

$$\text{Then } \frac{1}{v} - \frac{1}{u} = \frac{1}{f}.^*$$

(*ii*) Arrange a plane mirror behind a convex lens and a needle in front of it. (fig. 40). Remove the parallax between the needle and its image; then the distance between the needle and the lens is equal to f, the focal length of the lens. The rays diverging from the principal focus give rise to a parallel beam, after passing through the lens. This beam is reflected back along its original path and gives rise to an image on the principal focus, after passing through the lens.

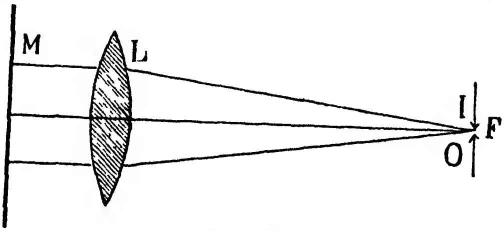


FIG. 40

(*iii*) Arrange two needles N_1 and N_2 far apart, the distance between them should be roughly more than $4f$. Place the lens first at L_1 so as to remove the parallax between N_1 and the image of N_2; and then at L_2 to do likewise.

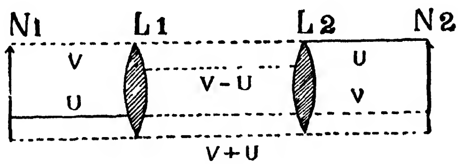


FIG. 41

Then $f = \frac{D^2 - d^2}{4D}$, where D = distance between N_1 and N_2 and d=distance between L_1 and L_2,

$$\text{For} \quad D = v + u$$
$$d = v - u$$
$$\text{but} \quad f = \frac{uv}{u+v} \text{ or } \tfrac{1}{4}\,\frac{(u+v)^2 - (u-v)^2}{u+v}$$

* Instead of the wire-gauze and screen, we may have two needles and remove parallax between them, still the above relation holds good.

$$=\frac{D^2-d^2}{4D}.$$

(B) **Concave lens** :—

By Combination.—Measure the focal length f_1 of a convex lens. Combine it with the given concave lens and find the focal length F of the combination, then f_2 the focal length of the concave lens is given by

$$\frac{1}{F}=\frac{1}{f_1}+\frac{1}{f_2}.$$

162. Power of a lens. The reciprocal of the focal length of a lens is termed its **power** or **dioptric strength.** To find the power of a given lens in **dioptres**, express the focal length in metres, obtain its reciprocal and change its sign. The last step is necessitated by the fact that the power of a convex lens is positive, while that of a concave lens is negative.

SUMMARY

1. **Laws of Refraction.** (*i*) The incident ray, the reflected ray and the normal lie in the same plane.

(*ii*) The rays bend *towards the normal* when they enter a denser medium and *away from* the normal when they enter a lighter medium.

(*iii*) The *ratio* of the sine of the angle of incidence to that of the angle of refraction is called refractive index and is constant for the two given media.

2. **Critical angle** is the particular angle of incidence for which the corresponding angle of refraction is 90°, when a ray is travelling from a denser to a rarer medium. If the angle exceeds this, total internal reflection takes place.

3. When a prism is in the position of minimum deviation, $\mu=\sin\frac{A+D}{2}\Big/\sin\frac{A}{2}$, where A is the angle of the prism and D is the $\angle$ of minimum deviation.

4. For thin prisms, $D=(\mu-1)A$.

5. In the case of lenses, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, where symbols have the meanings assigned to them.

6. **The power** of a lens is the reciprocal of its focal length.

7. To express the power of a lens in **Dioptres**, express the focal length in metres. Take its reciprocal and change its sign.

EXAMPLES

1. μ for water is $\frac{4}{3}$. Find the critical angle for water and air.

2. What is the smallest index of refraction of the material of a right-angled prism with equal sides, for which a ray entering one of the sides normally, will be totally reflected. (P U. 1919)

3. A concave lens and a concave mirror having a common axis are 6 ins. apart. Rays starting from a point on the axis 12 inches from the lens, fall on it and then on the mirror, and converge to a point $4\frac{2}{7}$ inches from the mirror. If the radius of curvature of the mirror be 6 inches, what is the focal length of the lens? (P.U. 1913)

4. An object 4 inches in height is placed at a distance of 6 feet from a lens and a real image is formed at a distance of 3 feet from it. Where and of what height will the image be?

5. A pencil of rays converging to a point 10 inches behind a lens, produces an image 20 inches behind it. Find the nature and focal length of the lens.

6. Two convex lenses, each of focal length 20 cms., are situated 10 cms. apart and have a common axis. An object 2 cms in height is placed at a distance of 15 cms. from the first lens. Find the size and position of the final image. Draw a diagram.

7. The distance between two walls is 20 feet. It is desired to throw a real image, twice as big as the object, on the opposite wall. Find the position and focal length of the lens.

8. A convex lens of focal length 10 cms. is used to form an erect image of an object, the image being twice as large as the object. Find the position of the object.

9. Light is converging to a point P and a convex lens of focal length 20 cms. is placed at A in the path of the beam, where $AP=30$ cms The beam now converges to Q. Calculate the distance AQ (P.U. 1931).

CHAPTER IV

DISPERSION

163. Newton observed that if a beam of white

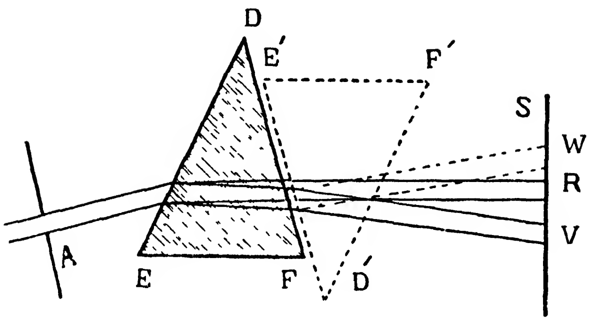


Fig. 42

light, admitted through a small hole in a darkened room, be incident on a prism; the emergent ray was not only deviated, but also showed a variety of seven colours, spread out on a screen: red at one end, violet at the other; and orange, yellow, green, blue and indigo in between them, from red to the violet end. This coloured band of light is called **spectrum** and the phenomena of splitting up of white light into its several constituents, due to its passage through a prism is called **Dispersion.**

The reason, why a spectrum is seen, when white light is refracted through a prism, is that white light is not homogeneous light; but is composed of several constituents, which are capable of exciting sensations of different colours on our retina. On passing through the prism, lights of different colours undergo different deviations and thus appear at different places on the

screen. Violet suffers the greatest deviation and red the least. *The deviation suffered by a ray of particular colour is called its* **refrangibility**. Thus the refrangibility of violet is greatest, while that of red is the least.

A further proof of the composite character of white light is furnished by the fact that if another prism such as $D'E'F'$ fig. 42, be placed as shown in the path of the coloured band, a white patch of light is produced at W, by the recombination of light of different colours into a white beam.

The fact that lights of different colours are deviated to different extents, when passing through a prism, shows that the refractive index of the same substance is different for lights of different colours For a prism of small angle, the deviation is given by the formula $D=(\mu-1)A$. Thus the deviation for the blue ray is $D_b=(\mu_b-1)A$ and for the red ray $D_r=(\mu_r-1)\,A$. Therefore we have

$$D_b-D_r=(\mu_b-\mu_r)\,A \quad \ldots\ldots\ldots (i)$$

The quantity D_b-D_r, which is the angle between the blue and the red rays, on emergence from the prism, measures the dispersion produced. If μ be the average of refractive indices for blue and red rays, then the deviation for the mean ray will be

$$D=(\mu-1)\,A \quad \ldots\ldots\ldots\ldots\ldots (ii)$$

By multiplying and dividing equation (i) by $(\mu-1)$,

we get $D_b-D_r=\dfrac{\mu_b-\mu_r}{\mu-1}\cdot(\mu-1)\,A \quad ..(iii)$

$\therefore \quad D_b-D_r=\dfrac{\mu_b-\mu_r}{\mu-1}\cdot D$, by substituting the value of $(\mu-1)\,A=D$, from equation (ii).

The quantity $\dfrac{\mu_b-\mu_r}{\mu-1}$, is called the *dispersive power* of the material of which the prism is made.

164. Pure spectrum. The spectrum obtained by placing a prism in the path of a white beam of light is *impure*, in so far as the various colours produced overlap.

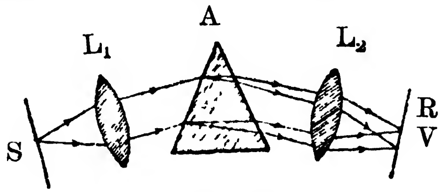
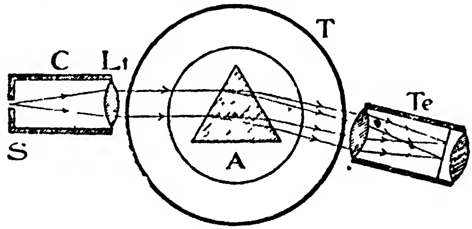


FIG. 43

To get a *pure*

spectrum in which the various colours do not overlap but are exhibited distinctly, the following arrangement is used:—(*i*) *A very narrow slit S* fig. 43 is placed at the principal focus of a lens L_1, which makes a parallel beam of light fall on the prism A; (*ii*) The prism A must be placed in the position of minimum deviation and (*iii*) a lens L_2 is used to focus the rays of different colours on the screen.

To study the spectrum carefully, an instrument known as the spectrometer is used. The arrangement is just as above indicated except that the spectrum instead of being obtained on the screen is viewed by means of the eyepiece of a telescope. The prism A rests upon a table provided with levelling screws. A tube C called the collimator carries an adjustable narrow slit S and a convex lens L_1. A telescope T_e is used to see the spectrum and can be rotated about the axis. Scales and verniers are attached to measure the angles.



Plane of the spectrometer
FIG. 44

If the slit of the spectrometer be illuminated by Sunlight and a prism of high dispersive-power substance, such as quartz or carbon bisulphide, be placed on the turn-table and the resulting spectrum be viewed with a high-power telescope, a very large number of fine black lines is observed, distributed throughout the spectrum. These lines are called **Fraunhofer lines** and denote certain missing light waves.

165. Invisible spectrum. The visible portion of the Solar spectrum is comprised between the violet and the red. The wave-length of the violet is ·0000393 cm. and that of red is ·0000759 cm. Thus waves, having wave-lengths between the above two limits only, can

affect our retina. If the wave-length is beyond these, then the waves are incapable of affecting our retina. The Solar spectrum extends beyond its visible limits at both ends. The portion of the invisible spectrum extending beyond the red end is called *infra-red spectrum.*

Infra-red spectrum. It consists of waves longer than ·00008 cm. and these waves are detected by their heating effects. In order to study it carefully, a prism of some substance such as rock-salt or quartz, which is diathermanous to long waves, must be used. A sensitive ether thermoscope with a black bulb or a thermopile, placed just beyond the red end of the spectrum, will indicate a rise of temperature, showing that radiations of long wave-lengths, which have heating effects, extend beyond the red end.

Ultra-violet spectrum. The portion of the invisible spectrum, extending beyond the violet, is called *ultra-violet spectrum.* It consists of waves smaller than ·00004 cm and is detected by its chemical effects on a photographic plate. These rays are sometimes called *Actinic rays.*

166. Forms of Spectra. There are three classes of spectra :—

(*i*) *Continuous.* It consists of an unbroken luminous band extending from one side to the other, varying in colour from point to point. It is produced, when the temperature of the radiating body is very high.

(*ii*) *Line.* It consists of sharp, well-defined lines. It is generally produced by glowing simple gases.

(*iii*) *Fluted.* It consists of broad luminous bands and is generally produced by gaseous compounds.

Light which consists only of one wave-length is called *monochromatic light.* Thus if light coming from a sodium flame (*i. e.* light coming from a flame in which sodium salt is strongly heated) be examined, it gives a yellow prominent line; such light is called monochromatic light.

Absorption spectrum. When white light passes through certain substances, they absorb light of par-

ticular wave-length and allow the rest to pass. If a spectrum of this transmitted light be observed, black lines corresponding to the absorbed wave-lengths appear. *Such a spectrum, crossed with black lines, is called absorption spectrum.*

Kirchoff formulated the law that a *substance, which emits light of a particular wave-length when hot, absorbs light of that very wave-length when cold.*

Thus glowing sodium vapour gives a prominent line in the yellow colour of the spectrum, but a sodium flame placed in the path of light coming from an arc lamp, gives a black line in the yellow spectrum. Fraunhofer lines, which are seen in the solar spectrum are actually produced by the absorption of waves of particular wave-lengths. The visible portion of the Sun, called the **photosphere**, which emits white light, consists of glowing gases at extremely high temperature. Surrounding the photosphere, is an atmosphere of vapours of various elements, at a comparatively lower temperature, called the **chromosphere.** Light corresponding to the substances in the chromosphere is absorbed and gives rise to Fraunhofer lines. The presence of dark lines is indicative of the existence of those elements in the chromosphere, which in the hot state give out rays, corresponding to black lines. In this way several elements have been located in the chromosphere and certain elements such as helium have in consequence been discovered.

In support of the correctness of the above explanation, it may be noted that during a Solar Eclipse, the photosphere is covered by the Moon and the spectrum obtained from the radiations, coming from the chromosphere, is reverse of the spectrum ordinarily observed.

167. Colours of bodies. The colours of *opaque* bodies are due to *selective reflection.* A body appears to be black when it absorbs indiscriminately all the rays which fall on it. A body appears to be red, when it is capable of reflecting only the red rays and absorbs all the rest. Thus if a deep-red cloth be placed in

green light, it appears to be black; for the incident beam consists only of green light, which the red cloth totally absorbs. Certain colours are not pure, *i.e.* they do not consist of only one colour but a mixture of two or more colours. Thus a blue dye will not become black in any part of the spectrum and is thus not a pure colour. An impure colour will have the colour of the part of the spectrum in which it is placed.

The colours of *transparent* bodies are due to *selective transmission*. Thus a piece of glass held before the eye appears to be red, green or blue according as it transmits the red, green or blue portion of the light incident on it. The rest of the colours are absorbed by it.

Primary colours. According to Young-Helmholtz' Theory, red, green and blue are known as primary colours and all the rest are called secondary colours; for they can be obtained by mixing the primary colours in suitable proportion.

Complementary colours Two colours are said to be complementary, when on adding them together white is produced.

Experiment. Get a spectrum of white light by a prism as explained already, intercept a portion of the spectrum and place a prism in the opposite direction to recombine the transmitted portion of the spectrum. See that white light is not produced. Now allow the intercepted portion also to be transmitted. See that on recombining through the second prism, white light is at once produced In this case the intercepted portion and the transmitted portion are complementary colours.

It may be clearly noted that the term complementary colours applies only to spectrum colours and not to pigment colours at all Thus if yellow and blue pigments be mixed, the resultant colour is green and not white; though yellow and blue are complementary spectrum colours. The reason is this, that the yellow pigment absorbs red and blue rays and the blue pigment absorbs red and yellow rays, both however, reflect the green and thus this is the only colour reflected by a mixture of the two pigments.

168. Achromatic combination of Prisms. When white light passes through a prism, the path of the ray is deviated and at the same time dispersion is produced. It is possible however, by using prisms of different substances to get deviation without any dispersion. A combination of prisms of the above nature is called an **achromatic combination.** Thus if a prism of Crown glass have an angle of 60° and that of flint glass of 29°—17′, the dispersion produced by each is the same; for the flint glass has a very great dispersive power. On placing the two above prisms in opposite directions, the total dispersion will be zero ; but the refractive index of flint glass is only slightly greater than that of crown glass, therefore the deviation produced by the crown-glass prism, will not be totally annulled by the flint-glass prism. There will be deviation without dispersion.

168 (*a*) **Chromatic aberration.**

Experiment. Place a lens *L* in the path of Sunlight and hold a screen near to the lens. See that its periphery is red and its central portion blue. Move the screen away from the lens Notice that the periphery becomes blue and the central edge reddish.

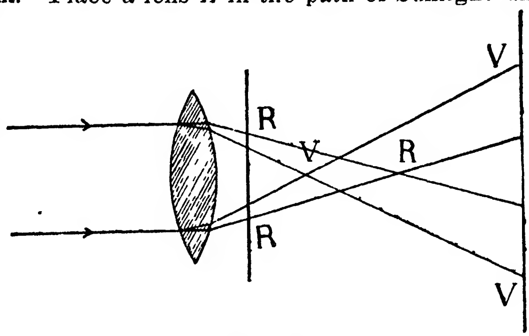


FIG 45

The reason is that glass has greater refractive index for violet rays than for red rays. Therefore violet rays come to the focus near the lens and red rays farther away from it, as shown in fig 45. Thus if the screen be in position *RR*, red will be on the outerside and if the screen is shifted to the position *VV*, then violet will be seen on the outerside.

Hence whenever a single lens is used to produce an image by white light, the image shows colours; this

defect is known as **chromatic aberration.**

Just as it is possible to get an achromatic combination of prisms; similarly by combining a converging crown-glass lens and a diverging flint-glass lens, so that the whole acts as a converging lens, it is possible to prevent dispersion altogether. Such a combination of lenses is called an **Achromatic combination of lenses.**

SUMMARY

1 The breaking up of white light into its *constituent colours due to its* passage through a prism is called **dispersion.** The deviation suffered by a ray is called its **refrangibility.**

2. **Dispersive power** is the angle between the blue and red rays; and is equal to $\frac{\mu_b - \mu_r}{\mu - 1} D$.

3. **A pure spectrum** is one, in which the various colours do not overlap. It is obtained by (*i*) having a narrow slit; (*ii*) placing the prism in minimum deviation position, and (*iii*) using lenses to focus different rays at different points.

4 Spectrum extending beyond the red end consists of long heat rays and is called **infra-red spectrum** ; while that extending beyond the violet end, consists of short actinic rays, called **ultra-violet rays**

5. **Forms of Spectra** —(*i*) Continuous, line and fluted. Dark lines in solar spectrum are called **Fraunhofer lines.**

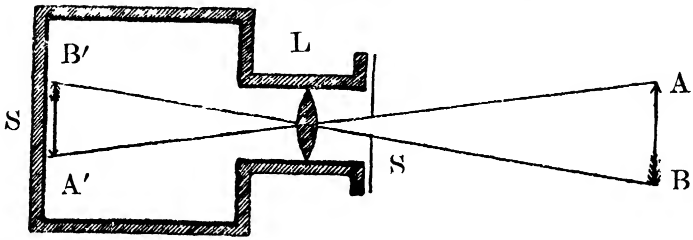
6. The colours of bodies are due to selective absorption. The red, green and blue are known as **primary colours** Two colours which when mixed together give white, are known as **complementary colours.**

7 The image produced by an ordinary lens is coloured at the periphery, this phenomena is known as **chromatic aberration** Two lenses joined together so as to produce no colouring, are known to form an **Achromatic combination.**

CHAPTER V

OPTICAL INSTRUMENTS

169. Photographic camera. It consists of a con-



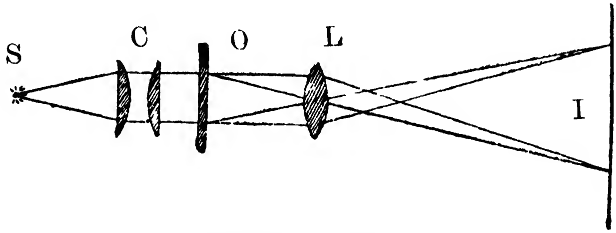
Photographic camera

Fig. 46

vex lens, fitted to one face of a box, lined black inside. The opposite face carries a ground-glass screen which can be replaced by a sensitive plate. The size of the box can be varied by a rack-arrangement so that the distance of the lens from the screen can be varied and a real inverted image of an external object produced on the screen. The lens carries an adjustable diaphragm, having a circular aperture to cut off light, which does not pass through the central parts of the lens.

170. Projection lantern. It consists as shown in fig. 47 of two convex lenses or a system of lenses The source of light, which should preferably be a point, is situated at the principal focus of a lens-system (called the condenser), consisting of two plano-convex lenses with their plane faces outwards. The parallel beam of emergent light illuminates the object O and a second lens L throws a real inverted image I of the object on the screen. As the distance of the object from the lens L is smaller than the distance of the screen, a magnified image is

seen and to get an erect image, the object which usually consists of a slide, is inverted.



Projection lantern
FIG. 47

170. (*a*) **Kinematograph.** The function of the Kinematograph is to reproduce pictorially upon a screen, the movements of objects. In order to achieve this end a series of instantaneous photographs of the moving object is taken with a *special kinematograph camera.* In this camera, turning of the handle automatically swings the shutter across the camera lens, moves the film after short exposure and coils up the exposed film. Each picture is an isolated snap-shot, taken in the fraction of a second upon a sensitised transparent celluloid film

When such films are projected upon the screen and are made to follow one after the other so rapidly that each remains in sight for so brief a period that the successive views dissolve into one another and the missing parts of the motion (*i.e.* the parts of motions lost between the taking of two consecutive pictures) are not detected. the eye imagines that it sees the whole of the process of displacement in the moving objects, although really it sees only one half, *i.e.* the half of which the successive pictures have been taken. What occurs between the interval of taking successive pictures, when the lens is shut, is not recorded at all. The appearance of natural movements is due to a physiological phenomenon which is termed " persistence of vision " or "optical illusion."

No doubt, eye is the quickest of all the other senses

and is about a million times more sensitive than the most sensitive photographic plate yet prepared; still it is deceived very successfully That the effect is due to persistence of vision is conclusively shown by the fact that no animation is produced at all if the light be not totally cut off during the interval of exposure of two successive pictures. The reason is that after the disappearance of the picture from the screen, the white flash is sufficient to wipe out the image of the picture, which in its absence would have lingered on in the brain.

The apparatus The apparatus consists as shown in fig. 47 (a) of a combination of a magic lantern and an automatic arrangement for rapidly bringing consecutive pictures of the film (at the rate of 16 to 20 pictures per sec) very exactly into position before the gate-aperture G, through which the light passes, to keep the picture stationary in this position for about $\frac{1}{20}$ of a second and to interrupt the image falling upon the screen, during the time a picture is being replaced by its succeeding one. In order to feed the machine regularly with the film, its sides are perforated. In the diagram, the film is drawn from the upper reel-box B, by a uniformly driven cylinder C, the teeth of which engage the film projections. From this it goes to the aperture F, through which the light from the condenser passes. The film is now again rolled round the lower reel-box J. Loops of the film are provided at K and L to prevent the film from being broken The

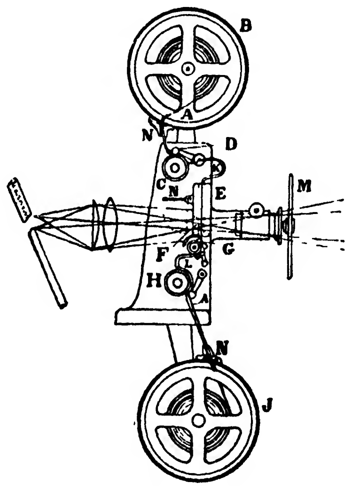
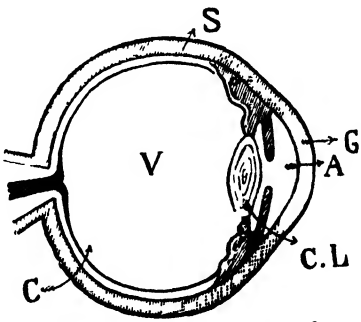


FIG. 47 (a)

shutter M has the form of a rotating disc and is placed just outside the objective. It cuts off the light during the period of transference of the pictures.

For successful working of the apparatus, it is necessary that the speed of movement of the operation is the same as that during the taking of the series of photographs with the camera. It is of great scientific value for the analysis of very quick motions, the pictures being taken at a rapid rate in the camera and projected upon the screen at the rate of about sixteen per sec It is used in schools for educational purposes and in cinema-houses for purposes of entertainment

171. The eye. It consists of an outer tough coating called the *sclerotic*, which distends in front. The front portion of the sclerotic is transparent white and is called the *cornea* Inside the sclerotic is a black coating called the *choroid*; which in front, just behind the cornea, assumes the form of a diaphragm, and is called *iris*. The aperture in the iris is called the *pupil*.



Horizontal section of the Eye

FIG. 48

Behind the pupil is a *crystalline lens* suspended by means of ciliary muscles. The chamber between the cornea and the lens is called the *anterior chamber* and is filled with a watery fluid called *aqueous humour*.

The choroid is lined in the posterior portion with a membrane of nerve-tissue, which is sensitive to light and is called *the retina* The space between the lens and the retina, called the *posterior chamber*, is filled with a fluid called *vitreous humour*. From the physical point of view, eye is analogous to a photographic camera, having a very wonderful adjustable lens. In the camera, the focussing is done by changing the position

of the lens ; while in the eye, the focal length of the lens is changed by the ciliary muscles. The power of adjustment is known as *Accommodation.* The image formed on the retina is inverted, but the brain receives the impression of an erect object. How this happens, cannot be guessed clearly.

172. Defects of the vision. The power of the normal eye is approximately 59 dioptres. By the process of *Accommodation*, the power can be varied at will so as to bring objects at various distances into focus. This power of accommodation is greatest in children and diminishes regularly with age. The following are the defects of vision, which are remedied by lenses :—

1. **Myopia or short-sight.** In this, the observer can see the objects, situated near to him, but cannot see those very far off. This defect is due either to the elongation of the eye-ball or to the decrease of the focal length of the crystalline lens. The images of near objects are formed on the retina, while those of distant objects are formed in front of the retina.

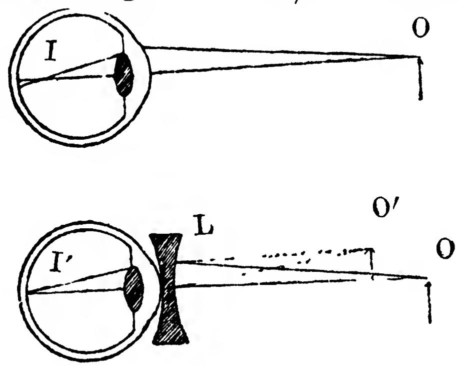


FIG. 49

In order to find the proper focal length of the lens L, which ought to be prescribed to remedy this defect, let an object be slowly moved away from the eye, till the object ceases to be clearly seen. This point is called the **far-point** for the given eye and let its distance be denoted by D. In a normal eye, the far-point is at infinity and the distance v of the retina from the lens would be equal to f, the focal length of the crystalline lens. For a short-sighted person, when the object

is at the far-point, we have

$$\frac{1}{v}-\frac{1}{D}=\frac{1}{f} \qquad (i)$$

Let us now combine a lens of focal length f_1 with the crystalline lens, in order to make the *far-point* infinity, so that v the distance between the retina and the lens equals the focal length of the combination of the two lenses, then

$$\frac{1}{v}-\frac{1}{\infty}=\frac{1}{f}+\frac{1}{f_1},$$

$$\text{or } \frac{1}{v}=\frac{1}{f}+\frac{1}{f_1} \qquad (ii), \text{ for } \frac{1}{\infty}=0.$$

Subtracting equation (i) from (ii), we have

$$\frac{1}{D}=\frac{1}{f_1},$$

or f_1 the required focal length $= D$, the distance of the far-point. The positive sign shows that the lens must be a diverging lens.

2. **Hypermetropia or Long-sight.** In this the

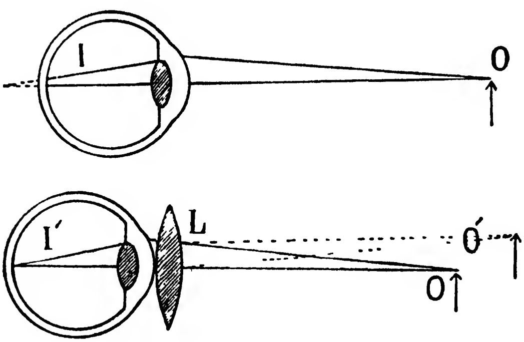


FIG. 50

observer can see the objects, situated far away from him, but cannot see objects situated very near to him. This defect is due either to the shortening of the eye-ball or to the increase of the focal length of the crys-

talline lens. The image of objects situated far off is formed on the retina, while those of near objects is formed behind the retina.

In order to find the focal length of a lens L, which ought to be prescribed to remedy the defect, move an object from a far-off point towards the eye; till it ceases to be clearly seen without straining the eye. This point is called the **near point** and let its distance be D. In a normal eye this point is nearly 25 cms. from the eye and is called the *least distance of distinct vision*; while in a defective eye this is more than 25 cms. For a long-sighted person when the object is at the *near point*, we have

$$\frac{1}{v}-\frac{1}{D}=\frac{1}{f} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

Let us now combine a lens of focal length f_1 with the crystalline lens in order to make the *near point* at 25 cms.

$$\therefore \quad \frac{1}{v}-\frac{1}{25}=\frac{1}{f}+\frac{1}{f_1} \quad \ldots\ldots\ldots\ldots\ldots(ii)$$

Subtracting equation (i) from (ii), we have

$$\frac{1}{D}-\frac{1}{25}=\frac{1}{f_1}.$$

Now D being greater than 25, $\frac{1}{D}$ will be less than $\frac{1}{25}$, therefore the focal length f_1 will be negative quantity. Thus the lens required is a convergent lens.

3. **Astigmatism** means variation of power in different meridians. This defect is due to irregularity in the curvature of the cornea and such an eye can see objects in one plane but cannot see those in another plane. The image of a point-source appears no longer as a circular disc but as an ellipse. This defect is remedied by the use of cylindrical lenses of suitable power.

173. The simple microscope. It is a single convex lens placed close to the eye to magnify an object

near it. The image obtained is a virtual magnified image. In order to see the image at the distance of distinct vision and to increase the field of view, the eye should be placed close to the lens. Thus in fig. 51, AB is the object and its image $A'B'$ is formed at the distance of distinct vision.

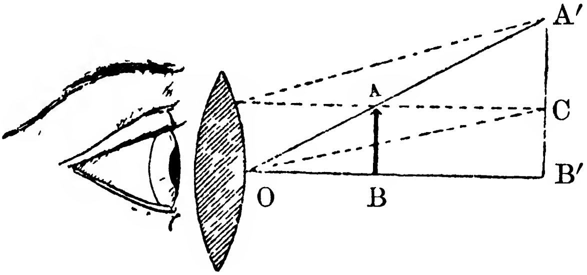


FIG. 51

174. Magnification. *In the case of optical instruments, it is the ratio of the angle subtended by the image to the angle subtended by the object on the retina.*

Thus in the above case, magnification is $\angle A'OB'/\angle COB'$, for $\angle A'OB'$ is the angle subtended by the image and $\angle COB'$ is the angle, *which the object would subtend, if it were in the position CB' i.e. at the distance of distinct vision.* It may be clearly noted that the angle subtended by the object is *not* AOB; for if the lens were not used the object will not be seen by the eye without exertion and to see the object without the lens, the object ought to be shifted to the position CB' at the distance of distinct vision.

Thus magnification in the above case is $\angle A'OB'/\angle COB'$

$$=\frac{\angle AOB}{\angle COB'} \text{ for } \angle AOB=\angle A'OB' \text{ by construction.}$$

Measuring angles by their tangents, we have

$$M=\frac{\angle AOB}{\angle COB'}=\frac{\frac{AB}{u}}{\frac{CB'}{D}}, \text{ where } BO=u \text{ and } B'O=D.$$

Or $M=\frac{D}{u}$, for $CB'=AB$ .. ..(i)

Again $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$; or in the present case,

$$\frac{1}{D}-\frac{1}{u}=\frac{1}{f},$$

i. e. $$\frac{1}{D}-\frac{1}{f}=\frac{1}{u},$$

or $$\frac{f-D}{Df}=\frac{1}{u} \quad \text{or} \quad \frac{D}{u}=\frac{f-D}{f} \text{.........}(ii)$$

But $\frac{D}{u}$ is the magnification as proved in equation (i).

$\therefore$ magnification is equal to $\frac{f-D}{f}=1-\frac{D}{f}$.

The focal length f of a convex lens is always a negative quantity, therefore the expression for magnifying power of a simple microscope becomes $1+\frac{D}{f}$, where D is the distance of distinct vision.

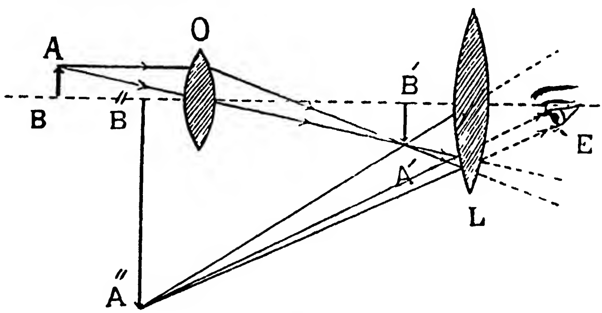


FIG. 52

175. Compound Microscope. It consists as shown in fig. 52 of two convex lenses O and L. The object AB to be magnified lies in front of the lens O, called the *object-glass*, at a distance greater than f but less than $2f$; so as to produce a real, magnified, inverted image $A'B'$. This

image serves as an object for the second lens L, called the *eye-piece* and is situated at a distance slightly less than its focal length. Thus it gives rise to a magnified virtual image $A''B''$. The course of rays is shown in fig. 52. The magnification depends upon the focal lengths of the two lenses and the relative distance between them.

176. Astronomical telescope It consists as shown in fig. 53 of two convex lenses: O, the object-glass of very long focal length and E, the eye-piece of very short focal length. The object to be viewed is situated very far off and gives rise to a real inverted image $A'B'$, which lies in front of the eye-piece at a distance less than f from it and serves as an object for it. The eye-piece forms a virtual magnified image of it (*i. e.* $A'B'$) at $A''B''$. The course of rays is shown in fig. 53.

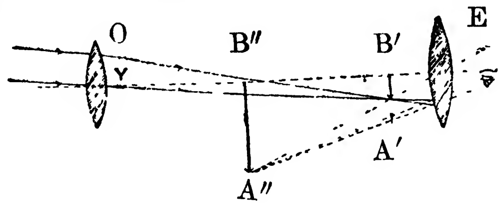


FIG. 53

176 (*a*) **Magnification.** Supposing the object to be very far off as compared to the length of the telescope, the angle subtended by the object on the retina, is equal very approximately to $\angle B'YA'$. This being vertically opposite angle to that subtended by the distant object on the objective O while that subtended by the image is $\angle A''EB'' = \angle A'EB'$ by construction.

$\therefore$ measuring the $\angle$s by their tangents, we have

$$\text{magnification} = \frac{\text{Tan } \angle A'EB'}{\text{Tan } \angle B'YA'} = \frac{\dfrac{A'B'}{B'E}}{\dfrac{A'B'}{B'Y}} = \frac{B'Y}{B'E}$$

Now $B'Y$ will be very nearly equal to F the focal length of the objective, if the object is at infinity and $B'E$ is nearly equal to f, the focal length of the eye-piece by construction.

∴ *magnification is equal to F/f i. e. focal length* of the objective divided by the focal length of the eye-piece.

177. Galileo's Telescope. The image seen in astronomical telescope is inverted, it does not make any difference when viewing heavenly bodies, which are spherical in shape; but when used to see terrestrial objects, the inversion of the image becomes a serious defect. To remedy it a concave lens is used as an eye-piece in the astronomical telescope. The telescope thus formed by an *objective, a convex lens* and an *eye-piece*, a *concave* lens is called Galileo's telescope. It gives erect images and its length is much less than that

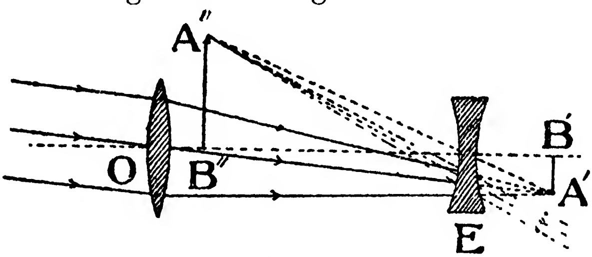


Fig. 54

of the astronomical telescope, because the eye-piece is placed between the object-glass and the position of the real inverted image $A'B'$ formed by the object-glass and at a distance equal to the focal length of the eye-piece from the position of the image $A'B'$. The course of rays is shown in fig. 54.

$A''B''$ is the final, virtual, erect image seen. As before

$$\text{the magnification} = \frac{F}{f} = \frac{\text{focal length of the objective}}{\text{focal length of the eye-piece}}.$$

177. (a) Prismatic Binocular. In a telescope, to get good magnification, the object-glass should be of long focal length, moreover in an astronomical telescope, the distance between the object-glass and the eye-piece must be greater than the focal length of the former. This causes the telescope length to become inconveniently long; further the image seen is always

inverted. To overcome these difficulties two totally reflecting prisms are arranged as shown in the figure. In this way the length is reduced to nearly $^1/_3$ of an ordinary telescope and an erect image is seen. The first prism produces lateral inversion, while the second inverts the image. The instruments are usually made in pairs, one for each eye. The prisms enable the distance between the object-glasses to be more than that between the eye-pieces. This arrangement makes it possible to focus a large number of objects simultaneously.

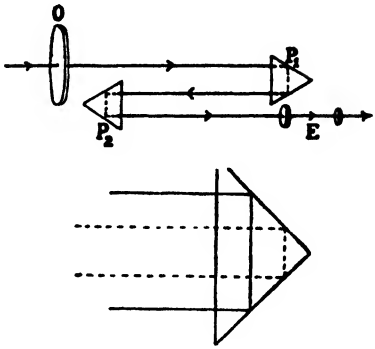


FIG. 54 (*a*)

(*b*) **Ordinary Binoculars.** They are nothing but a pair of Galilean telescopes with comparatively short focal-length objectives. The magnification produced is small.

178. Reflecting telescope. Due to the difficulty of constructing lenses of large apertures, which may not exhibit chromatic aberration, the objective of a telescope may be replaced by a concave mirror. Such a telescope as shown in fig. 55, is called a reflecting telescope. It consists of a concave mirror *C* of large aperture, which gives rise to a real inverted image of an object situated very far off. As the real image is formed in the path of the incident light, special devices are necessary in order to see it. In the path of the reflected rays, is placed a plane mirror

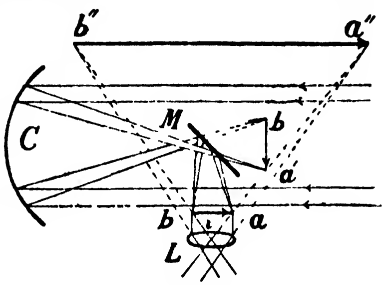


FIG. 55

M inclined to the principal axis at an angle of 45°. Due to the reflection from the mirror M, a real inverted image is formed at i, instead of at ba. This image is seen by an eye-piece L and a virtual magnified image is seen in the position $A''B''$.

The image formed by such a telescope is generally bright and devoid of colours, due to the absence of chromatic aberration, but is always distorted due to spherical aberration.

Newton devised the first reflecting telescope to avoid defects due to chromatic aberration; but after the discovery of achromatic combination of lenses, reflecting telescopes have gone into disfavour.

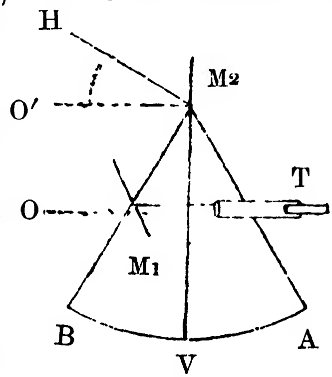


FIG. 56

179. Hadley's Sextant. This instrument is used for finding the angle subtended by two objects or by two points on the same object at the place of observation. The principle of the instrument is that when a mirror is rotated through any angle, the reflected ray is turned through double that angle.

It consists as shown in fig. 56, of two plane mirrors M_1 and M_2. The mirror M_1 is fixed to the radius BM_2 of the frame, which is an arc of the circle with two fixed radii. This mirror M_1 is partially silvered, so that a distant object O can be directly seen through the upper unsilvered portion of the mirror. The other mirror M_2 is mounted on a movable arm M_2V, which carries a vernier along the graduated arc AB. When V is at A, the zero of the scale, the two mirrors M_1 and M_2 are parallel to each other. In this position the observer will see two images of the same object, one directly through the unsilvered portion of M_1 and the other by the path $O' M_2M_1T$; *i.e.* the ray $O' M_2$ after reflection from M_2 goes along M_2M_1 and then after reflection from M_1 goes along M_1T.

If now the arm M_2 V carrying the mirror M_2 be moved through an $\angle\theta = \angle AM_2V$; then an incident ray HM_2 coming from another object H will, after reflection from M_2 and M_1, coincide with OM_1, the ray coming directly from the object O. The angle between the object O and another object $H = \angle HM_2O'$. This angle is equal to $2\angle AM_2V$ *i.e.* 2θ.

For if the ray TM_1 be imagined to be reversed; then if V is coinciding with A, the ray will go along M_2O' after reflection from M_1 and M_2, and the ray will go along M_2H, after reflection from M_1 and M_2, when V is shifted through an $\angle\theta = \angle AM_2V$. The angle between the two reflected rays $O'M_2$ and M_2H will in accordance with the laws of reflection and the principle enunciated above will be 2θ. Usually the scale is divided into degrees and each degree is numbered as two degrees, to give the reading directly, as the angle between the two objects.

EXAMPLES

1. If the focal length of a lens be 2 inches and the minimum distance of distinct vision be 10″. What is the magnifying power? (P. U. 1914.)

2. A convex lens, 5 cms. focal length, is placed at a distance of 2 5 cms from the eye of a person whose least distance of distinct vision is 20 cms. How far from the eye must a small object be placed so as to be seen distinctly. (P. U. 1917).

3. The focal length of the objective of a simple astronomical telescope is 300 cms. and that of the eye-piece is 3 cms. Find the distance between the lenses, when the instrument is focussed on a star. (P. U. 1921).

4. What lens would be required to enable an eye, that cannot focus objects nearer than six feet, to read a book at 10 inches distance (L. U).

5. A person, whose nearest distance of distinct vision is 15 cms., uses a lens of 5 cms. focal length to magnify a small object. What is the distance of the object when in focus, and what magnification is obtained? (L. U.)

6. A source of light is at first 10 ft. from a convex mirror, and is then moved up to a distance of 2 ft. from it. How much does the image move, if the radius of curvature of the mirror be 4 8 ins.? (P. U. 1928.)

CHAPTER VI

VELOCITY OF LIGHT

180. The velocity of light is so great, that the time required by it to travel any terrestrial distance is very small and thus special devices have to be used to measure velocity with any accuracy.

(*i*) **Romer's method**. In the Solar system, the Sun is supposed to have no motion of translation. It may simply rotate on its axis. Several planets revolve round the Sun in elliptic orbits. Earth is one of those planets and is nearer to the Sun than Jupiter, which describes a bigger orbit than the Earth. Jupiter has several satellites revolving round it. Romer directed his telescope towards one of them and found the period of its revolution round Jupiter $42\frac{1}{2}$ hours, when the Earth and Jupiter were in positions E_1 and J, Fig 57. He observed that after six months, when the Earth is in position E_2 and the Jupiter in position J' (because Jupiter takes $11\frac{1}{2}$ years in once going round the Sun), the observed time was 42 hours and $46\frac{1}{2}$ minutes. Romer explained this increased interval of $16\frac{1}{2}$ minutes by the time taken by light to cross the Earth's orbit. Taking the distance of the Sun from the Earth to be about 93000000 miles, the velocity of light is given by $\frac{2 \times 93000000}{990}$, which is equal to 193000 miles per second.

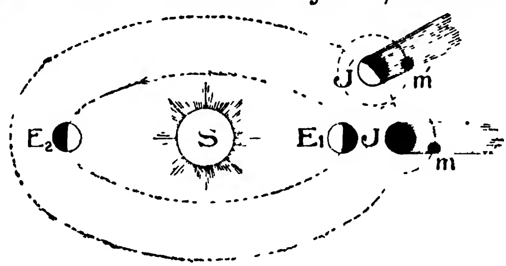


FIG. 57

(*ii*) **Fizeau's method.** The first terrestrial measure-

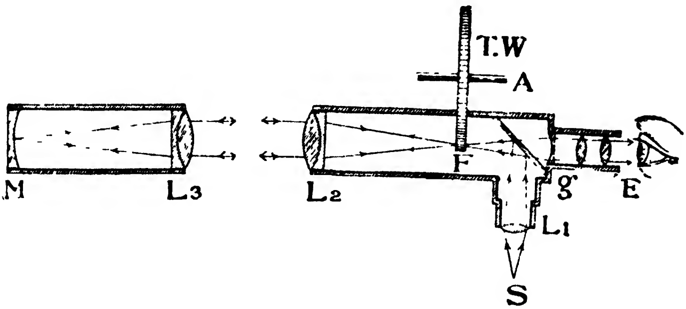


FIG. 58

ment of velocity of light was made by Fizeau in 1849.

The plan of his method is shown in Fig. 58. Light from the point-source S falls on a lens L_1 and the emergent beam is reflected by a ground-glass screen placed at an angle of 45° to the incident beam. It is then brought to focus at the point F on the toothed-wheel. This point F is at the principal focus of the lens L_2 and thus the emergent beam consists of parallel rays. These parallel rays after traversing a very great distance (about 1 kilometer) are incident on a lens L_3, which converges the beam to a point, where a concave mirror having radius of curvature equal to L_3M lies. The rays are reflected along their original paths, which they retrace, and reach the eye E.

The toothed-wheel at F is rotated so that successive teeth pass through the point F. Thus light at F is alternately intercepted by a tooth and allowed to pass from the space between the two teeth. If the wheel rotate slowly, then the light passing through a space has sufficient time to return through that very space; and thus an image of the source S will be seen by an Eye placed at E. As a space is followed by a tooth, which cuts off the light altogether, a flickering image will be the result, if the rate of motion of the wheel is slow. On increasing the speed, the flicker will cease, due to persistence of vision. On steadily

increasing the speed still more, a stage will be reached when the light, which has passed through the space between any two teeth, finds a tooth on its return journey to obstruct its path. At this stage the image of S will become invisible to the eye at E.

Now if m be the number of teeth on the circumference and n the number of revolutions made by the wheel per second; then the time of moving through *one tooth-space* will be $\frac{1}{2mn}$, supposing the width of tooth and space to be exactly equal; and during this time the light has travelled from F to M and back again. If this distance be denoted by d, then the velocity of light is equal to $\frac{2d}{\frac{1}{2mn}} = 4dmn.$

If the speed of revolution of the wheel be doubled, the image will re-appear, for a ray passing through a space will find its way through the next space, on its return journey. This method has the merit of being simple in theory and the image seen is well-defined; but it suffers from the drawbacks, that intensity of light is greatly diminished by successive refractions and the field of view is illuminated by the light intercepted on the tooth.

EXAMINATION QUESTIONS VIII

1. Define intensity of illumination and illuminating power of a source of light. How are the illuminating powers of 2 sources compared? Two sides of a photometer look alike when the distances of the sources are 3 and 4 feet respectively. What should be the distance of the fainter source, when the brighter is 6 feet?

2. A circular uniform source of light is 2 inches in diameter and a sphere of the same size is placed 3 feet away. Find the size of the umbra and penumbra 3 feet away from the sphere.

3. Prove $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, where the symbols have their usual meanings. An object placed in front of a mirror gives rise to an image twice as large. The object is moved 10

cms. towards the mirror and the magnification becomes 3 times. Find the focal length.

4. Describe and prove the laws of refraction. Give a method of finding μ for water. What do you understand by total reflection and critical angle ?

5. Prove $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$, where the symbols have the usual meanings A convex lens casts an image equal in size to the object, which is placed 20 cms. away from the lens. If another lens be placed in contact with the first, the image is found to be reduced to one-quarter of its previous linear dimensions. What are the focal lengths of the two lenses ?

6. Describe carefully, with the course of rays, the various optical instruments. What are the defects of the eye and how are they remedied ? Find an expression for magnification of a simple microscope.

7. What do you understand by *dispersion*. Give a method of getting a pure spectrum. How do you account for the colour of bodies ? What are spherical and chromatic aberrations and how are they remedied ?

8. Give Fizeau's method of finding the velocity of light.

SOUND

CHAPTER I

SOUND IS ALWAYS THE RESULT OF VIBRATORY MOVEMENTS

181. Sound is a form of energy, which is produced by the rapid vibrations of a body and propagated by waves travelling through a material medium. It affects the organ of hearing.

Experiment. Take a tuning-fork and set it into vibration by striking it with a wooden hammer Note that sound is produced. Bring a pith-ball pendulum near it; notice that the ball is driven away, as if it had received a number of blows, and this continues till the tuning-fork continues to give sound.

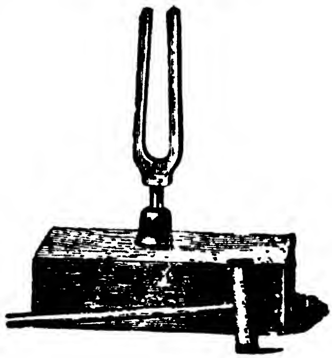


FIG. 1

The vibratory movement giving sound may be produced in a solid, liquid or even gas, as in the case of organ pipes.

Sound cannot travel through vacuum. Within the receiver of an air-pump arrange an electric bell, which can be rung by a bell-push from outside. Work the pump to exhaust the receiver and press the bell-push Notice that the loudness decreases, as the density of the air inside the receiver decreases, *showing that the loudness of the sound depends upon the density of*

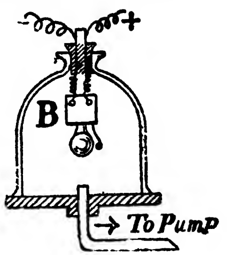


FIG. 2

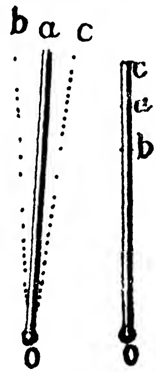
the medium, in which the body vibrates. If the pump be worked on to produce high degree of exhaustion, the sound may become too feeble to be easily audible. *This shows that some material medium is necessary for the propagation of sound from one place to another.*

Vibration. A point is said to vibrate, when it executes the same series of movements at regular or nearly regular intervals ; thus the bob of a pendulum is said to vibrate

Period of Vibration The time required to complete a vibration is known as the period of vibration. It is the time, which elapses between the moment when the vibrating body passes through any point to the moment, when it passes the same point in the same direction to repeat the same movements , and is denoted by t.

Frequency. The number of complete vibrations made in one second is called the frequency and is generally denoted by n Thus $n = \frac{1}{t}$.

182. Longitudinal and Transverse vibrations. When displacements take place in a direction, perpendicular to the length of a body as shown in fig. 2 (*i*), the vibrations are called transverse vibrations. Here one end of the rod is fixed in a vice and the other end is moved slightly to one side (*c*), and then released. It tries to recover its normal position (*a*), but as it comes to it, it overshoots the mark due to its kinetic energy at (*a*) and thus goes on to (*b*) on the other side. This process is repeated, till it is slowly brought to rest by the friction of the air.



(*i*) (*ii*)
FIG 3

When however, the vibrations are backwards and forwards along its length, they are called *Rectilinear*, as shown in fig 3 (*ii*). Here the end (*a*) is elongated to (*c*) by a pull and then released. It tries to go back to its normal position, but

again overshoots its mark and goes on to *b*. Such movements are repeated.

The fixed end of the rod in each case is called the *Node*; this is the point of maximum strain and no displacement. The free end is called the *Antinode*, this is the point of no strain and maximum displacement.

Amplitude The distance from (*a*) to (*c*) *i e* from the mean position of rest of a vibrating body to either of its extreme positions is called amplitude.

183. Simple Harmonic Motion or S H M. In most cases a vibrating body executes a motion called simple harmonic. The motion of a particle is said to be S.H M, when it moves along a line straight or curved in such a way, that its acceleration is directed always towards a fixed point and is proportional to its displacement at any instant. Thus if a body *A* describe a circle with uniform velocity about a point *O* as centre, then the

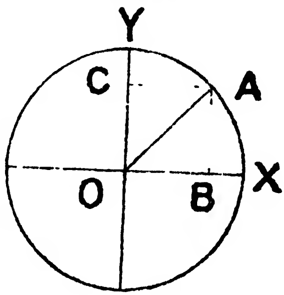


FIG 4 (*i*)

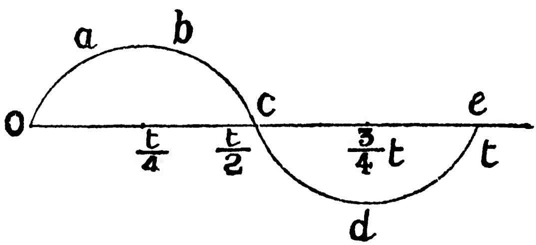


FIG. 4 (*ii*)

projection of *A* (such as *B* on any fixed line *OX*) represents a simple harmonic motion.

The acceleration of $A = \frac{v^2}{r}$, along *AO* Its component along *XO* will be

$$\frac{v^2}{r}\cos\theta = \frac{v^2}{r} \times \frac{OB}{r} = \frac{v^2}{r^2}.\ OB = w^2 \times OB,$$

where w=angular velocity and is equal to $\frac{v}{r}$. Thus the acceleration of B is proportional to displacement and is directed towards O.

Thus the acceleration $a=w^2 . OB$

or $\frac{a}{OB}=w^2$ or $\sqrt{\frac{a}{OB}}=w$.

But T, the time of one complete vibration is $\frac{2\pi}{w}$ or $\frac{2\pi}{\sqrt{\frac{a}{OB}}}=2\pi\sqrt{\frac{OB}{a}}$. Thus the time of vibration in S.H.M. is equal to $2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$.

A simple harmonic motion is sometimes represented by a curve *abcde*, where the abscissæ denote *times*, with the period of a complete vibration as unit, and the *displacements* of B fig. 4 (*i*), at the corresponding times as the ordinates. Such a graph is called a *Displacement curve.* Its form is that of a Sine curve.

Phase. In fig. 4 (*i*), suppose A starts from X and is at A after a time t. Let the time of one complete vibration of A be denoted by T; then the ratio $\frac{t}{T}$, is the phase of A at the given position. It is defined as the *ratio* of the time, which has passed since the body was *last* at its mean position of rest, to its period of vibration.

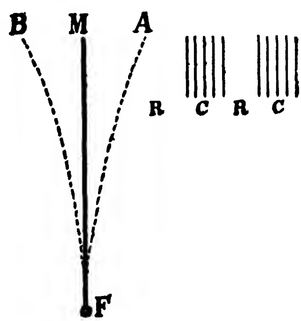


Fig. 5

184. Mode of propagation of sound. The way in which sound travels can be best understood by reference to fig. 5. When the prong of a tuning-fork

starts from M towards A, the layer of air just in contact with it is compressed; this compression is imparted to the next layer and thus it is passed on from layer to layer. On reaching A, the prong retraces its path and moves over to B, thus producing a partial vacuum or rarefaction. This rarefaction is passed from layer to layer in the wake of the compression. On reaching B, the prong begins to move towards A again and to send off a compression. Thus compressions and rarefactions are sent in quick succession and constitute sound waves.

It should be clearly understood, that no portion of the medium leaves its place altogether; the motion of the molecules takes place in their own orbits, while the disturbance travels further on. In the above example, the particles move in the same direction as the disturbance. It is spoken of *as* **Longitudinal wave.**

An illustration of this is furnished by a number of marble balls resting in close contact with each other in a groove. If a ball A be struck against one end of the row, then the last ball B of the row flies off. The reason is that A presses the first ball against which it strikes, the first presses the second and so on, till the last is driven off.

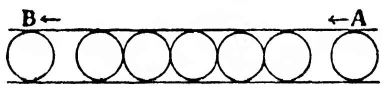


FIG. 5 (a)

The way in which sound waves travel through the atmosphere is further beautifully illustrated by Crova's disc, fig. 6. A number of circles with uniformly increasing radii are drawn with centres equidistant round a small circle on a circular piece of white cardboard. A rectangular piece of cardboard having a rectangular slit is placed over it and the circular piece, with the circles on it, is rotated. Compressions and rarefactions are then seen to travel along the rectangular slit.

A graph may be drawn showing the displacements of particles in the direction of motion as ordinates

and the positions of the particles as abscissæ. Such a graph is known as the displacement curve for the longitudinal wave. It is similar to a sine curve, when the body

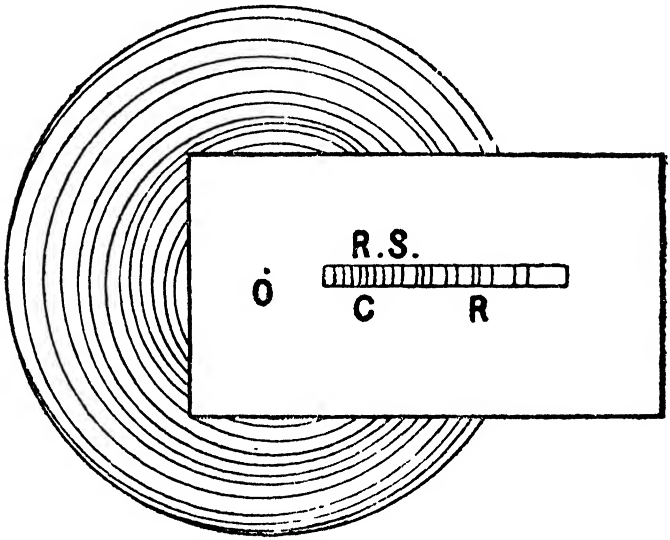


FIG. 6

vibrates harmonically.

185. Transverse Waves. The waves, which are produced when a small pebble is thrown in the centre of a pond, are different from the longitudinal waves, we have considered above. In this case the particles of water travel at right angles to the direction in which the disturbance travels. Such a wave is called a *Transverse Wave.*

Transverse waves are seen, when one end of a rope is jerked. The particles of the rope move up and down in a sine curve as shown in fig. 7, while the disturbance travels forward.

The distance, which the wave or disturbance travels during the period of vibration of the vibrating body, is called the wave-length and is generally denoted by λ.

It is evidently the distance between two consecutive parts of the medium, whose vibrations are in the same phase.

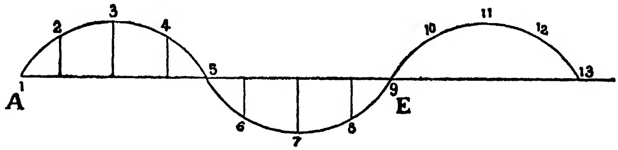


FIG 7

Thus in fig. 7 the distance A to E is a wave-length.

The velocity of a wave is the distance it travels in one second If t be the period of vibration, then the number of vibrations made by a particle in one second is $\frac{1}{t}$, which is denoted by n; and the distance the wave travels in one second is $n\lambda$, where λ is the wave-length.

Thus we have $v=n\lambda$.

The quantity n is called the frequency and denotes the number of vibrations made by a particle per second, or the number of waves passing a given point in one second.

Whenever waves are produced, as when a pebble is thrown into water, it is noticed that successive particles are disturbed in turn

The locus of all those points, which are just on the point of being disturbed, is called a **wave-front.**

Stationary waves. Fix one end of a rubber tubing and hold the other in your hand; move it to and fro slowly for some time. Notice that some parts of the tube (marked A in fig 8) have considerable displacement, while the others marked N remain almost stationary The tube seems to be divided into vibrating loops or segments. Such waves, produced by the combination of incident and reflected waves are called *stationary waves.* The two sets of waves are travelling along the tube in opposite

directions. Where the *crest* of the *incident* wave coincides with the *trough* of the *reflected* wave, the one will neutralize the other and the point will remain fixed The stationary point is called a **node** and the point *A* where the displacement is maximum is called an **antinode.** The distance between two consecutive nodes or antinodes is one-half of a wave-length.

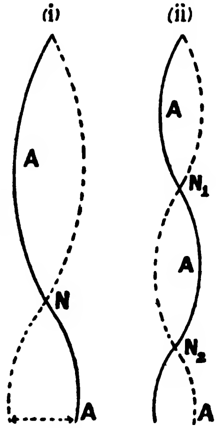


FIG. 8

186. General characteristics of wave-motion.

1. The medium as a whole does not move, but the particles oscillate in their own positions.
2. The velocity of the disturbance or wave is not the same as the velocity of the particles.
3. Wave propagation takes time.
4. Waves can be reflected and refracted.

SUMMARY

1. A point is said to vibrate when it executes the same series of movements at regular intervals.
2. The time required to complete one vibration is known as **period of vibration**
3. The number of vibrations made per second is called **frequency.**
4. When the displacement takes place in a direction perpendicular to the length of a body, the vibration is known as **transverse.** When however, the displacement is along the length of the body, the vibration is known as **longitudinal**
5 The fixed end of a rod, where no motion takes place, but strain is maximum is called a **node.**
6. The free end of a rod where maximum motion takes place but no strain is experienced, is called an **antinode.**
7 The distance between the mean position of rest of a vibrating body and its extreme position is called an **amplitude.**
8. Sound cannot travel through vacuum.

9. **Phase.** It is the *ratio* of the time, which has elapsed since the body *last* left its position of rest, *to* its period of vibration.

10. S. H. Motion is the motion of a particle, whose acceleration is always proportional to its displacement and is directed towards a fixed point. The period of vibration of a body executing S. H. M. is equal to

$$2\pi\sqrt{\frac{\text{Displacement}}{\text{Acceleration}}}.$$

11. The distance travelled by the disturbance during the period of vibration is called the wave-length and is denoted by λ.

12. **Longitudinal waves** are those, in which the particles oscillate in the same direction in which the disturbance travels.

13. **Transverse waves** are those, in which the particles oscillate in a direction perpendicular to that in which the disturbance travels.

14. The velocity of wave-propagation is equal to $n\lambda$.

CHAPTER II

VELOCITY OF SOUND

That Sound *takes time* to go from one place to another is obvious from the fact that *some time does elapse* between the flash of the lightning and the sound of the thunder.

187. Velocity of sound by Gun-method. Two stations A and B are selected and the distance d between them is carefully noted. A gun is fired at A and the observer at B notes down the time t_1, which elapses between the flash and the report of the gun. The experiment is repeated by taking what is known as the *reciprocal observation*; *i.e.* the gun is fired at the station B and the observer at A notes down the time t_2, which elapses between the flash of the gun and the hearing of the report. To get the velocity of sound, the distance d is divided by the average time $\frac{t_1+t_2}{2}$.

The average of the two times is taken in order to avoid errors due to wind.

Further corrections become necessary to eliminate errors due to the effects of temperature and moisture. Moreover allowance is to be made for personal equation *i.e.* the errors, which are introduced on account of the fact that sight-impression of the flash and the sound-impression due to the report of the gun take different times to react upon the observer and to be recorded by him.

The laboratory method of finding the velocity of sound by a resonance-tube will be described later at its proper place.

Velocity of sound from theoretical considerations. Newton showed that the velocity of sound in

any medium is given by the formula, $V=\sqrt{\frac{E}{D}}$, where E is the elasticity and D the density of the medium.

In the case of gases, under isothermal conditions elasticity is equal to the pressure and therefore the above formula becomes

$$V=\sqrt{\frac{P}{D}}.$$

From the formula, the value obtained for air is 280 metres, while that actually observed is 332 metres. Newton tried to explain this by saying that sound did not take any time in going through the substance of the molecules and thus 280 metres represented the velocity in intermolecular spaces. Laplace, Newton's contemporary, showed that Newton's reasoning to explain the difference between the deduced and observed values was incorrect in so far as sound could not actually travel through vacuum. He said that Newton's primary formula, $V=\sqrt{\frac{E}{D}}$, is perfectly true but his deduction viz. $V=\sqrt{\frac{P}{D}}$, is erroneous in the case of gases; because $E=P$, under *isothermal* conditions *i.e.* when the temperature remains constant. In the case of sound waves, we are *not* justified in taking $E=P$; for the compressions and rarefactions take place in so quick a succession, that heat has no time to escape and therefore the temperature does not remain constant, while on the contrary the quantity of heat remains the same. The changes are *adiabatic* and therefore elasticity is equal to γP, where γ is the ratio of the two specific heats of the gas. Substituting this value for elasticity, we get $v=\sqrt{\frac{\gamma P}{D}}$; and we find that the value obtained by this formula and the actual observed value agree within reasonable limits. Therefore Laplace's correction must be correct.

188. Effect of Pressure, Temperature and Humidity on the velocity of Sound.

(*a*) **Effect of Pressure.** By Boyle's Law, $PV=P'V'$;

$$\text{or } \frac{P}{P'}=\frac{V'}{V}=\frac{D}{D'},$$

i.e. the density and the pressure vary directly, therefore the ratio $\frac{P}{D}$ does not change. In other words, velocity of sound, $v=\sqrt{\frac{\gamma P}{D}}$, does not change with variations of pressure.

(*b*) **Effect of Temperature.** When the temperature increases, density decreases, because the volume increases.

$$\text{We have } V_0=\sqrt{\frac{\gamma P}{D_0}}$$

$$\text{and } V_t=\sqrt{\frac{\gamma P}{D_t}}.$$

Where V_0=velocity at 0° D_0=density at 0°C.
V_t=velocity at t° and D_t=density at t°C.

But $\frac{D_0}{D_t}=\frac{T}{T_0}$, *i.e.* the density varies inversely as the absolute temperature. By Charles' law, we have

$$D_0=D_t\left(1+\frac{t}{273}\right) \text{ or } \frac{D_0}{D_t}=\frac{273+t}{273}=\frac{T}{T_0}$$

$$\text{Therefore } \frac{V_t}{V_0}=\sqrt{\frac{T}{T_0}}.$$

i.e. the velocity of sound in air is proportional to the square root of the absolute temperature.

Velocity of sound increases at the rate of 2 feet per sec. or 61 cms. per sec. per degree centigrade.

(*c*) **Effect of Water-vapour.** The water-vapour in air reduces the density of air and thus increases the velocity of sound.

189. Velocity of Sound in water. Velocity of sound in water was determined in 1826 in Lake Geneva.

Two boats were stationed far apart. A bell was struck under the surface of water by a lever, which produced a flash by igniting gun-powder at the same time. The observer on the other boat noted the time *t* between the flash and the report of the sound as heard by an ear-trumpet dipped in the water and then the velocity of sound in water was determined by dividing the distance by the time. Velocity in water was thus found to be 1435 metres per sec.

190. Reflection of Sound. When sound waves are incident on an extended and somewhat smooth obstacle, they are reflected according to the laws of reflection applicable to heat and light waves. When sound is reflected from a distant obstacle, the reflected sound is called an **echo.** For an echo to be heard distinctly, it is necessary that the obstacle should be very far off.

To speak or hear distinctly, it is necessary that 1/5 of a second must elapse between successive syllables. Hence if an observer were to speak *a*, *b*, *c*, *d*, etc.; then assuming the velocity of sound to be 1120 feet per second and the distance of the obstacle to be 112 feet, it is evident that when he is about to utter the letter *b*, the reflected sound of *a* reaches him and thus the direct sound of *b* and the reflected sound of *a* will be blended. Similarly the direct sound of *c* and the reflected sound of *b* will be blended and so on for the remaining syllables; but when he stops speaking, he will hear distinctly the echo of the last syllable only. *Such an echo in which only one syllable is heard distinctly is called a monosyllabic echo.* If however, the distance is twice, thrice or four times 112; then two, three or 4 syllables will be heard distinctly and the echoes are called *disyllabic, trisyllabic or polysyllabic.*

190. (*a*) **Refraction of Sound.** Like heat and light waves, sound waves are refracted when they pass from one medium to another. Thus refracted sound waves are brought to focus on passing through a lens

filled with carbon-dioxide. A solid lens cannot be used,

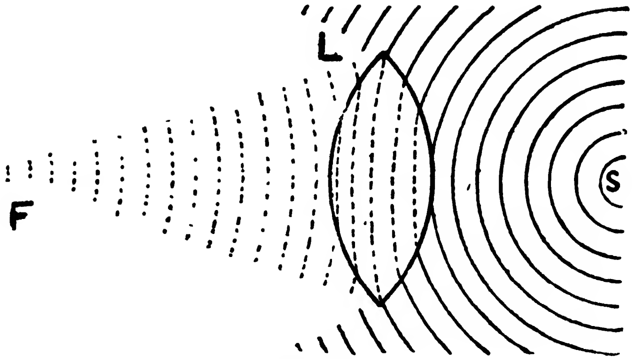


FIG. 8

because most of the energy is reflected from its front surface.

SUMMARY

1. Velocity of sound in air has been obtained by Gun-method and is equal to 1090 feet per second at 0°C.

2. Newton proved that velocity of sound $=\sqrt{\frac{E}{D}}$, where E is the elasticity and D the density of the medium through which the waves travel.

3. In the case of gases, E=Pressure under *isothermal* conditions.

4. Laplace modified Newton's formula and the corrected expression for the velocity of sound in gases is

$$v=\sqrt{\frac{\gamma P}{D}}, \text{ where } \gamma=1{\cdot}41.$$

5. Velocity of sound in water is nearly four times its velocity in air.

6. Sound can be reflected and refracted like light.

EXAMPLES

1. Find at what temperature the velocity of sound in air is double the velocity of sound in air at 0°C. (P.U. 1914)

$$2=\sqrt{\frac{T'}{T}}$$

$$4=\frac{273+t}{273}$$

$$\therefore \quad t=819^{\circ}\text{C}.$$

2. An echo repeats two syllables at 0°C., find the distance of the reflecting surface.

For 2 syllables, the sound ought to take 2/5 of a second in going to and coming from the obstacle or 1/5 of a second in going *only*.

$\therefore$ the distance of the obstacle must be

$$\frac{1090}{5}=218.$$

3. A flash of lightning is observed and the thunder is heard 3 seconds afterwards. How far away did the flash occur?—temperature=15°C.

4. A man sets his watch by a gun, 1 mile distant. Will it be too fast or too slow and by how much? Temperature=15°C.

5. The whistle of an engine is heard after reflection from a cliff, after an interval of 8 seconds. Four minutes later the interval is found to be 6 seconds. How far is the engine from the cliff and at what rate is it proceeding?

CHAPTER III

LAWS OF VIBRATING STRINGS

191. Sonometer. The instrument used for studying

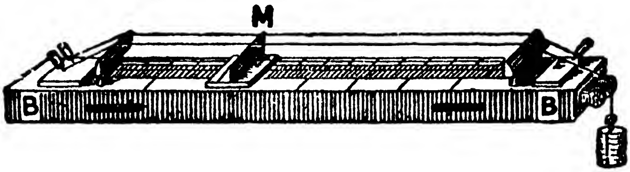


FIG. 9

the laws of vibrating strings is called a *sonometer or monochord.* It consists of a sounding-box *BB* about 120 cms. long. Two wires are stretched over it. One wire is attached, on both sides to wrest-pins; while the other is attached to a wrest-pin on one side, and on the other side it passes over a pulley and carries a scale-pan. Each string passes over two fixed bridges 100 cms. apart. A movable bridge *M* intervenes between the two fixed bridges, which can be shifted anywhere and thus any required length of either wire can be set into vibrations.

192. Laws of vibrating strings:—

1. The frequency varies inversely as the length. In the C.G S system, it is expressed in cms.; *i.e.* all other things remaining the same, if the length of a vibrating string be halved, the frequency is doubled.

Set the monochord into vibrations as a whole and then set half its length into vibrations by inserting the bridge *M*; notice that the frequency of the sound emitted is doubled in the latter case.

2. The frequency varies directly as the square root of the stretching force. In the C.G.S. system, it

is expressed in dynes. Thus other things remaining constant, to get a note of double the frequency in this case, the stretching force must be four times as much as in the first instance.

3. The frequency varies inversely as the square-root of the density of the material of the wire.

4. The frequency varies inversely as the square-root of the area of cross-section of the wire.

The last two laws can be combined by saying that frequency varies inversely as the square-root of the mass of 1 cm. of the wire; because mass of 1 cm. of a wire depends both upon the area of cross-section and the density of the material of the wire.

These laws are demonstrated practically by the help of the monochord, by varying the length, the tension and the mass per unit length of the wire in turn.

The frequency of a vibrating string is given by the formula

$$n=\frac{1}{2l}\sqrt{\frac{T}{m}}=\frac{1}{2l}\sqrt{\frac{T}{\pi r^2 d}}$$

$$=\frac{1}{2rl}\sqrt{\frac{T}{\pi d}}$$

where n=frequency, l=length in cms.
T=stretching force in dynes. m=mass in gms. of 1 cm. of the wire.

The sound given out by a tuning-fork is pure, *i.e.* it consists of one frequency only and is called a *tone*; while the sound given out by a vibrating string is not pure, but consists of a mixture of tones and is called a *note*.

In a note, tone of the lowest frequency is called a *Fundamental* and one of double the frequency is called an *Octave*; while tones of frequencies three times, four times etc. of the fundamental are called *Harmonics or Overtones*.

The points of a vibrating string, which pass over the bridges and are in contact with them, are called

the *Nodes*; while the middle point, where the amplitude is maximum, is called the *Antinode*.

Node is thus a point in a vibrating body, where the displacement is minimum and the changes of pressure maximum; while antinode is a point where the displacement is greatest and the variations of pressure least.

193. Harmonics in the vibration of a string. When a string is vibrating as a whole in one segment as shown in fig. 10 (*i*), the note emitted is its fundamental tone, the wave-length is twice the length of the string and the frequency of the note emitted is given by $n=\frac{V}{\lambda}=\frac{V}{2l}$, where V is the velocity of disturbance in the string.

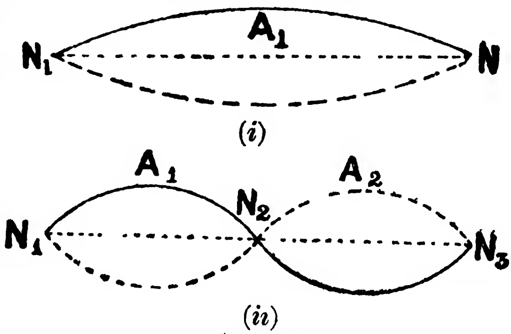


FIG. 10

If however, the same string is made to vibrate in two segments as shown in fig. 10 (*ii*), by bowing it at a point $^1/_4$ of the length of the wire from one end and damping it in the middle, by slightly touching it with a twisted piece of blotting paper; it will be noticed that there are nodes at N_1, N_2 and N_3 and antinodes at A_1 and A_2. The note emitted will be an *Octave* of the fundamental. The existence of nodes and antinodes is shown by placing V-shaped paper riders on the string. They are thrown off at the antinodes and remain at rest at the nodes.

When a string is plucked at random, its vibrations are made up of all sorts of vibrations and the note emitted is a mixture of various simple tones.

SUMMARY

1. The frequency of the note emitted by a sonometer is given by $n=\frac{1}{2l}\sqrt{\frac{T}{m}}$, where l is the length in cms., T the tension in dynes and m is the mass of wire per cm.

2. The *note* of lowest frequency is called the **fundamental.** a note of double the frequency is called the **octave** and notes of frequencies three, four times etc. of the fundamental are called harmonics or overtones.

3. The fixed end of a rod is called a **node** and the free end an **antinode.**

4. The note emitted by a vibrating string is not pure but contains **harmonics or over-tones.**

EXAMPLES

1. A copper wire (density 8 8 gms. per c c.), 100 cms. long and 1·88 mms. in diameter is stretched by a weight of 20 kilos. Calculate the pitch of the fundamental tone (C U. 1922.)

2 A steel wire 60 cms. long and ·5 mm. diameter gives a note of pitch 240 when stretched with a certain weight. A second steel wire bears the same weight as the first, but is 40 cms. long and 6 mm. in diameter. Find its frequency. (P. U. 1919)

3. A wire of length 140 cms. and mass 52 grams is stretched by means of a load of 16 kilograms. Calculate the frequency of the fundamental vibration, $g=981$ cms. per sec. per sec. (London University).

4. A copper wire one metre long, is vibrating in two segments, when stretched by a weight of $^1/_4$ kilogram. Find the frequency of the note, if the mass per centimetre be ·01 gm., $g=980$. (London University).

5. Two similar strings on a sonometer are tuned to unison, one is 36 inches long and stretched by 100 lbs. Find the weight on the other, which is 72 inches long.

CHAPTER IV

MUSICAL SOUND

194. Music and Noise. When sound is produced by the vibrations of a body having a definite period, it is called *music* and when it is produced by irregular movements, it is called *noise.* Musical sound such as that of a fork, a harmonium or a flute is pleasing to the ear; while noise, such as the sound of a thunder or the report of a cannon is displeasing to the ear. There is however, no very sharp distinction between the two.

195. Characteristics of musical Sound. Musical sounds differ from one another in respect of three qualities. *Loudness, pitch and quality.*

1. **Loudness.** It is the intensity of the sensation, with which sound affects the organ of hearing.

Loudness depends upon: (*i*) The *amplitude* of the vibrating body. It varies directly as the square of the amplitude. (*ii*) The *density* of the medium. It varies directly as the square-root of the density. (*iii*) The *distance* of the observer. It varies inversely as the square of the distance of the observer from the vibrating body. (*iv*) The *direction* of the wind. It is more intense when sound is going in the direction of the wind and less intense when it is going against the wind. (*v*) It depends also upon the surface area of the vibrating body, being more intense if the area is larger and less intense if it is smaller.

2. **Pitch or Frequency.** It is that characteristic by which an acute sound can be differentiated from a grave or flat one. Human ear can differentiate between two tones of different pitches. A person is said to possess a good musical ear, if he can form an accurate judgment of the pitch of a sound.

3. **Quality or Timbre.** It is the peculiarity which distinguishes two sounds of the same loudness and frequency, when they are produced by two different instruments. Thus we can easily distinguish the sound produced by a harmonium from that produced by a flute, though they may be of exactly the same loudness and pitch.

We have learnt already (Chapter III), that every tone, except the one produced by a tuning-fork, is accompanied by its harmonics, the nature and character of which differ in different instruments. The difference in quality is due to the difference in harmonics, which accompany the same fundamental tone, when produced by two different instruments. It is these harmonics, which add to the fulness of the note and give it a musical tone.

196. Methods of measuring Pitch or Frequency.

1. *Chronograph method.* A piece of smoked

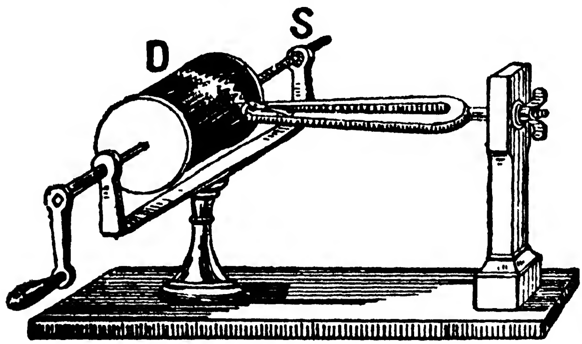


FIG. 11

paper is wrapped round a drum *D*, which rotates about the axle *S*. As the drum rotates, it advances slowly further on. The tuning-fork of which the frequency is to be measured, is provided with a style, so that it leaves a trace on the drum as it rotates. The frequency of the fork is determined by counting the wavy marks on the smoked paper in a given interval of time.

2. *Savart's toothed-wheel method.* A wheel having a large number of teeth on its circumference is rotated by a handle and a piece of steel disc or card-board is so adjusted as to be struck by the wheel in quick succession.

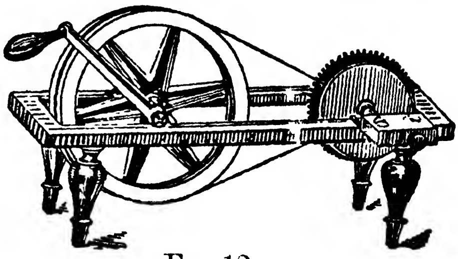


Fig. 12

The speed of revolution is regulated so that the pitch of the note produced is equal to the pitch of the note to be measured. Then the pitch of the note is given by $n \times m$, where n is equal to the number of teeth and m=the number of revolutions made by the wheel per second.

3. *Cagniard de la Tour's Siren Method.* A siren consists of a wind-chest *W.C* fig. 13, the upper plate of which has oblique holes. A disc *D*, mounted upon a vertical axis, fits over it. The disc has as many concentric holes as the plate; but their slope is in opposite direction to that of the plate-holes. The upper end of the vertical axis of the disc is connected to a clock-work, which records the number of revolutions made per second.

When air is forced in the wind-chest by bellows, it escapes in a slanting direction through the holes of the plate and strikes against the sides of the holes in the disc and thus causes it to rotate. The speed of revolution will vary with the pressure in the wind-chest.

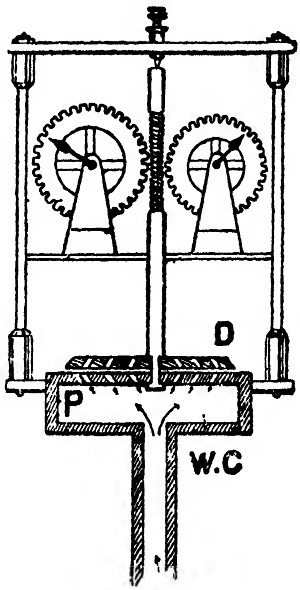


Fig. 13

Every time the upper and lower holes come together, puffs of air escape and give rise to a note.

To measure the frequency of a sound, the siren is adjusted so as to give a note of the same frequency. Then the frequency of the sound will be $n \times m$, *where n is the number of holes and m the number of revolutions made per second by the disc.*

4. *Sonometer Method.* The sonometer and laws of vibrating strings have already been described in Chapter III. The length of the sonometer wire is so adjusted by the movable bridge that it emits a note of the same pitch as that of the given sound; then its frequency is expressed by

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$, where the symbols have their usual meanings.

5. *Interference or Beats Method.* When two waves are traversing a medium simultaneously, the actual disturbance at any point is the resultant of the component disturbances; this fact is known as the principle of superposition and forms the basis of the phenomenon of Interference.

Suppose two tuning-forks of frequencies 256 and 252 are set into vibrations simultaneously. It is evident that after 1/8th. of a second, the first tuning-fork will have executed 32 vibrations, while the second only $31\frac{1}{2}$; *i. e.* they will be in opposite phases, so that condensations of the first are neutralised by the rarefactions of the second, and the intensity of sound will be minimum After 1/4th. of a second, the first tuning-fork will have completed 64 vibrations while the second only 63; *i.e.* they will be in the same phase and thus the resultant sound, being of maximum intensity, will be the sum of the two. After 3/8, 5/8 and 7/8ths. of a second, the condition will be similar to that, after 1/8th of a second, *i.e.* one of minimum intensity; and after 1/2, 3/4 and 1 second, the condition will be one of maximum intensity. Thus in one second, the sound will have

maximum intensity four times and minimum intensity four times. *These variations in intensity of the sound, when two notes of slightly differing frequencies are sounded together, are spoken of as* **Beats**. The number of beats heard per sec. is always equal to the difference in the frequencies of the two notes. Thus in the present case the number of beats heard is 4, which is the difference in the frequencies of the two tuning-forks.

This principle is made use of in finding the *frequency* of a given note in the following manner:—

A tuning fork of unknown frequency is vibrated in conjunction with one of known frequency (say n) and the number of beats (say x) heard per second is noted. Then the frequency of the unknown note must be $n \pm x$. To decide whether + or − sign is to be taken, a little piece of wax is attached to the tuning-fork of unknown frequency and the number of beats again noted; if the number of beats increases −sign is to be taken and if it decreases +sign is to be taken. The effect of attaching a piece of wax is always to lower the frequency. Thus if the frequency of the unknown tuning-fork is already lower, the difference between the frequencies of the two will increase still further and thus the negative sign gives the actual frequency. If however, the frequency of the unknown is higher, then the difference between its frequency and that of the known fork will decrease and thus +sign should be taken.

The resonance of a column of air, the principle of which will be described in Chapter V, is also used to find the frequency of a note.

197. Diatonic Scale. This is a universally-employed set of musical notes between the fundamental and its octave, used to keep company with human voice. The number of musical notes in a scale including the fundamental, called the *key-note*, and its octave is eight. The relative frequencies of the notes of the scale are represented in whole numbers as follows:—

C	D	E	F	G	A	B	C'
24	27	30	32	36	40	45	48

The ratio of the frequency of a note to that of another is called the *Interval* between them.

Thus the intervals between successive pairs of consecutive notes of the above scale are—

$$\frac{9}{8}, \quad \frac{10}{9}, \quad \frac{16}{15}, \quad \frac{9}{8}, \quad \frac{10}{9}, \quad \frac{9}{8}, \quad \frac{16}{15}$$

The interval $\frac{9}{8}$ is called the *Major-Tone*,

" " $\frac{10}{9}$ is called the *Minor-Tone*,

and " " $\frac{16}{15}$ is called the *Semi-Tone.*

The frequency usually assigned to the key-note C is 256. Thus the notes of the diatonic scale will have the following frequencies:—

C	D	E	F	G	A	B	C'	D'
256	288	320	341	384	427	480	512	576

Musical Temperament. In actual music, a variety of key-notes is necessary. Thus if instead of (C), frequency 256 being taken as the key-note, (D) 288 be taken as the key-note, the frequencies ought to be

D	E	F	G	A	B	C'	D'
288	324	360	384	432	480	540	576

Thus if D is to serve as the key-note; notes of frequencies 324, 360, 432 and 540 must be introduced. If the musical scale is to serve with any note as the key, a large number of notes will be required. This introduction of new notes for every new key makes the instrument unmanageable. A compromise is made in actual practice by a little alteration in the frequencies of the various notes of the Diatonic scale, so that they may serve with any note as the key and this adjustment of the notes is called *Temperament.*

Equal Temperament. In equal temperament, the interval between the fundamental and its octave has 11 notes, *i.e.* the total number including the fundamental and its octave is 13 and the frequencies are in the ratio:—

C	Ci	D	Di	E	F	Fi	G	Gi	A	Ai	B	C'
1	$2^{\frac{1}{12}}$	$2^{\frac{1}{6}}$	$2^{\frac{1}{4}}$	$2^{\frac{1}{3}}$	$2^{\frac{5}{12}}$	$2^{\frac{1}{2}}$	$2^{\frac{7}{12}}$	$2^{\frac{2}{3}}$	$2^{\frac{3}{4}}$	$2^{\frac{5}{6}}$	$2^{\frac{11}{12}}$	2

Thus in an equally-tempered scale, 5 additional notes denoted by the black-reeds in harmonium are added and by so doing the scale is used with any note as the key. The interval is the same throughout and is equal to $2^{\frac{1}{12}}$ or 1·06.

197. (*a*) **The gramophone.** It is a machine to record and reproduce speech. It was first invented by **Edison** in 1877. It consists of two separate instruments "the Recorder" and "the Reproducer." It is generally the latter, which is called the gramophone.

The Phonograph Recorder. It consists as shown in fig. 13 (*a*) of a large horn *LN* suspended by a string *M* at the mouth and is connected at the small end by a tube to the recording box *R*. This box is counterpoised by a weight *P*. The box can be carried by an arm, which can be traversed along a slide *C* by a screw *B*. A disc of wax upon which the record is to be taken is mounted on a turn-table *V* and rotated uniformly by a clockwork. Thus the needle or 'stylus' cuts a spiral groove on the wax*, which should neither be too soft nor too brittle.

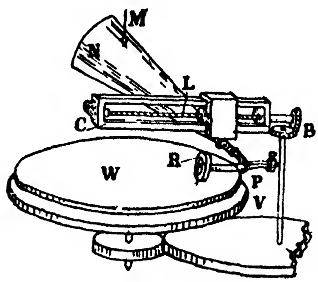


FIG. 13 (*a*)

This record is called the *mother-record.* From this an electrotyped copy of copper is obtained, which is called the *father-record.* This father-record is used to form *daughter-records,* by squeezing black discs on to it in a hydraulic press. These are sold in bazaars. The composition of the material of daughter-records is a trade secret, but they generally consist of shellac, lamp-black, cotton fluff and sulphate of barium etc.

* It is a mixture of paraffin-wax, ozokerite, soap and Japan-wax.

The records are in the form of circular discs, usually 10 to 12 inches in diameter with a spiral trace having about 4 turns to the mm.

The Recording box. The most important part of the recorder is the recording box to which the steel needle is attached. The needle is clamped into the lower end of a metal lever, having its fulcrum near to the needle end; while the other end of the lever is attached to the centre of a circular disc of thin mica, mounted between two solid rings of rubber, which

(i) prevent it from rattling,
(ii) damp its vibrations,
and (iii) allow it to move freely.

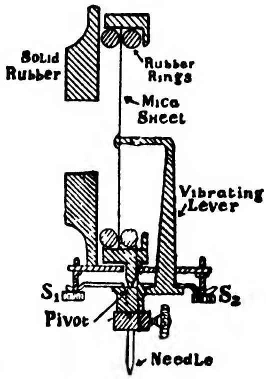


Fig. 13 (*b*)

The vibrations of the mica disc are faithfully reproduced by the stylus, which cuts a sinuous curve on the mother-record.

The Reproducer. In the reproducer, the reverse process to that of the recorder takes place. Here the movements of the needle are converted into the vibrations of the disc of the *sounding box*, which is just similar to the recording box described above, with this difference that a rounded stylus is used, instead of a sharp needle. The sounding box has a tube opposite to the diaphragm.

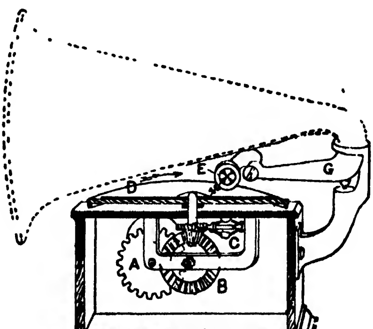


Fig. 13 (*c*)

This tube is inserted in the end of a pivoted conical

tube called the *tone-arm* and this terminates into the base of the horn.

The machine requires some energy to make it go and this energy is supplied by winding up the clock spring in the base of the instrument. In uncoiling itself the spring turns the cylinder A, which conveys this motion to a large wheel B. By means of the pinion motion, this revolves the turn-table to give necessary motion to the record. As the record turns round, the needle or the stylus fits into the groove on the record and is guided by this groove from the edge of the record towards its centre. At the same time it reproduces in the diaphragm vibrations, which are recorded on the record.

It is very important that the angular velocity of the turn-table, both in making the record and in reproducing it, shall be perfectly uniform; otherwise the pitch will vary. The linear velocity will change as the diameter of the spiral. In order that the needle may follow the groove easily, it is essential that it (the needle) should always lie in a plane tangential to the groove.

The following defects are noticed in the reproduced sound:—

1. A scratching noise is heard specially in the more delicate form of instruments.
2. Due to the metallic horn, an unpleasant nasal sound is heard.
3. Some of the consonants are not reproduced at all.
4. The voice of the singer, singing with musical instruments, is merged in that of the instruments.

198. Concord and Discord. When two notes are sounded together and produce a pleasing effect, they are said to be *in concord* or *in consonance*; if however, they produce a displeasing sensation, they are said to be in **Discord** or in **Dissonance.**

Helmholtz has shown that if the frequencies of the two notes bear a simple ratio to each other, concord

will be the result; and if they do not bear a simple ratio, discord will be the result. Helmholtz has further shown that concord can only be the result, if some of the near harmonics of the two notes are common.

SUMMARY

1. **Music.** When sound is produced by the vibrations of a body having a definite period, it is called *music* and if the vibrating body has no definite period, the sound produced is called *noise*.

2. The three characteristics of a musical sound are (*i*) *Intensity or loudness*, (*ii*) *Pitch or frequency and* (*iii*) *Quality or Timbre*

3. There are six methods of measuring frequency:— (*i*) Chronograph, (*ii*) Savart's toothed-wheel, (*iii*) Siren, (*iv*) Monochord, (*v*) Beats and (*vi*) Resonance of an air-column.

4. **Beats.** When two notes of slightly different frequencies are sounded together, the intensity of the resultant sound increases sometimes but at other times it decreases; these variations in intensity are known as Beats. The number of beats heard per sec. is equal to the difference in the frequencies of the two notes.

5. **Diatonic Scale** is a set of musical notes between the fundamental and its octave, universally employed to keep company with human voice. The number of notes including the fundamental and its octave is 8.

6. The *ratio* of the frequency of a note in a scale to that of the one just preceding it, is called an **interval.** There are three chief intervals known as the **major tone**, the **minor-tone** and the **semi-tone.**

7. In order that the scale may be used with any note as the key, a tempered scale is used, which has 13 notes including the key-note and its octave. The interval is the same throughout and is equal to $2^{\frac{1}{12}}$ or 1·06.

8. When two notes sounded together produce a pleasing sensation, **concord** is said to be the result and if they produce a displeasing sensation, **discord** is said to be the result.

EXAMPLES

1. Two tuning-forks produce 30 beats in 6 seconds. The frequency of one is 256, when the other is loaded

with a little wax, the beats are reduced to 20 in 5 seconds. Find the frequency of the second fork.

2. Build up a scale of frequencies of the notes in the diatonic scale, on the key-note 256.

3. Explain why the velocity of sound in air increases with the temperature, but is independent of the pressure.

The velocity of sound in air at 14°C. is 340 metres per sec. What will it be, when the pressure of the gas is doubled, and its temperature raised to 157·5°C ? (P. U. 1928).

4. You are given a glass rod of 70 cms. length ; how will you find the velocity of sound in it?

Calculate the velocity of sound in a gas in which two waves of lengths 100 and 101 centimetres produce 20 beats in 6 secs. (P. U. 1928).

CHAPTER V

RESONANCE AND VIBRATION OF COLUMNS OF AIR

199. Forced Vibrations. Whenever a periodic force is applied to a body capable of vibration, it begins to vibrate in the same period as that of the applied force. Such vibrations are known as forced vibrations.

Experiment. Suspend four simple pendulums A, B, C and D from a stretched India-rubber tubing. A and B are pendulums of the same length, D is a pendulum of length slightly shorter than A, while C has a slightly longer length. Set A (which has a rather heavy bob) into vibration. Notice that B is quickly set into vibrations and soon acquires the same amplitude as A has.

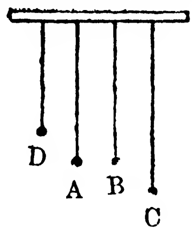


FIG. 14

Pendulums D and C swing through small arcs and come again to rest. They repeat this process several times; but ultimately the pendulums begin to vibrate in the same period as that of A, with this difference however, that *D will be in the same phase as A, while the phase of C will be opposite to that of A.*

The explanation of the above is that the vibrating pendulum A gives periodic pushes to the stretched rubber-tubing, at its point of attachment. The latter transfers the same to the other pendulums. which in turn begin to vibrate as described above. Similarly it is due to the forced vibrations of the wood that a vibrating tuning-fork gives more intense sound, when held over a table, than when held in the hand. In the latter case it does not give energy quickly to the air, while in the former case, *i.e.* when it is held over

the table, it sets same into forced vibrations, which having a large surface-area, gives out energy quickly and increases the intensity of the emitted sound.

200. Resonance. When the period of vibration of two bodies is the same and one of them is set into vibration; the other body begins to vibrate quickly in sympathy with the first, this phenomenon is spoken of as *Resonance*. In the previous experiment of pendulums, the vibrations of *B* furnish an example of resonance.

Resonance, in fact, is a *particular case* of forced vibrations, the essential condition being *equal periodicity of the two vibrating systems.*

Experiment. Take two tuning-forks of exactly the same frequency and hold them on a sounding-board. Set one into vibration by bowing it and after a little while, touch it to damp its vibrations; hold your ear near the other tuning-fork and notice that it is vibrating This is a case of *resonance.*

201. Longitudinal vibrations of columns of air:—

(*i*) It should be noted that whenever reflection takes place from the surface of a denser medium, there is no change of type of the incident and the reflected waves, *i.e.* a compression is reflected as a compression and a rarefaction is reflected as a rarefaction. When however, reflection takes place from the surface of a rarer medium, the incident wave is of a different type to that of the reflected wave, *i.e.* a compression is reflected as a rarefaction and a rarefaction as a compression.

(*ii*) The open end of a pipe is always an Antinode, while the closed end is a Node.

(*iii*) Nodes and Antinodes must alternate.

(*iv*) The distance between a Node and the next Antinode is a quarter of a wave-length.

Experiment. Take an apparatus as shown in fig. 15. Hold a vibrating tuning-fork *T* over the open end of the tube and adjust the level of the liquid, till the air in the tube begins to resound in sympathy with the tuning-fork. The reason why the column of air begins

to resound is that, as the prong of the tuning-fork moves from b to a, a compression is sent down the tube, which is reflected back as a compression from the closed end B When this compression reaches the open end A, it is again reflected downwards, *not as a compression but as a rarefaction*; and if at that time the prong has reached the extreme downward position a and is on the point of moving upwards to b, the prong also sends a rarefaction downwards. Thus the two trains of rarefaction augment each other.

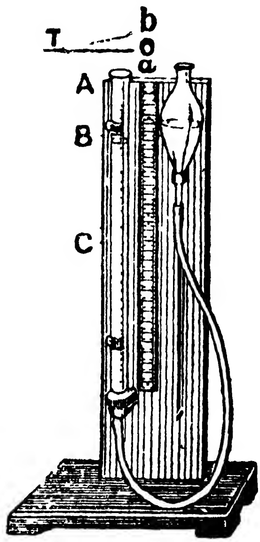


FIG. 15

This augmented rarefaction travels down and is reflected back as a rarefaction. When this rarefaction reaches the open end, the prong reaches the position b and is on the point of moving downwards to send a compression. The compression due to the reflection of the rarefaction from the open end thus coincides with the compression of the fork. The motion of the fork and of the air in the pipe thus synchronize and resonance is produced.

It is clear that resonance can only occur, if the time taken by the pulse to travel twice the length of the tube is equal to half the period of vibration of the fork, *i.e.* the wave-length of the note emitted by the tuning-fork is four times the length of the air-column.

$$i.e. \quad \lambda = 4l$$

$$\text{But } V = n\lambda \quad \text{or} \quad V = n.4.l$$

End-Correction. In actual practice, it is observed that the column of air extends a little beyond the open end and to obtain the true length L, a distance equal to 0·6 times the radius r of the tube is added to the observed length l,

$$i.e. \ L = l + \cdot 6r$$

From the above result, if the frequency of the

fork be known, V the velocity of sound becomes known. Conversely if the velocity of sound is known, the frequency of the fork becomes known.

The end-correction is eliminated by finding a second length of the tube AC, which resounds to the fork. This length will be nearly three times the first length; because resonance in the latter case can take place, only when the pulse first started, reaches the open end after the fork has made $1\frac{1}{2}$ vibration, instead of $\frac{1}{2}$ vibration. The difference BC (Fig. 15) between the first length l_1 and the second length l_2 is equal to half the wave-length,

$$i.e.\ l_2 - l_1 = \frac{\lambda}{2}$$

$$\text{and } V = n\lambda = n.2(l_2 - l_1)$$

202. Overtones in closed Pipes. Bearing in mind the points noted in the last article, it is obvious that the possible modes of vibration of a column of air in a closed pipe are as shown in figs. 16. (1, 2 and 3)

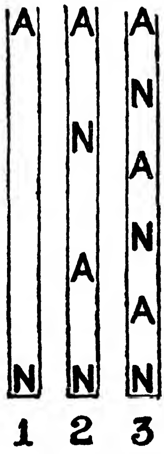


FIG. 16

In fig. 16 (1) the pipe is giving its fundamental

$$\text{and } l = \frac{\lambda}{4}$$

∴ the frequency of the note emitted will be $n_1 = \dfrac{V}{\lambda} = \dfrac{V}{4l}$.

The second possible mode of vibration is that shown in fig. 16 (2); the column may vibrate in three parts, an intermediate node and antinode being formed;

$$\text{and } l = \frac{3}{4}\lambda \text{ or } \lambda = \frac{4l}{3}$$

∴ the frequency of the note emitted will be

$$n_2 = \frac{3V}{4l} = 3n_1$$

The third possible mode of vibration is that shown in fig. 16 (3); the column may vibrate in five parts, there being two intermediate nodes and antinodes;

$$l=\frac{5}{4}\lambda \quad \text{or} \quad \lambda=\frac{4l}{5}$$

$\therefore$ the frequency will be

$$n_3=\frac{V}{\lambda}=\frac{5V}{4l}=5n_1$$

It is obvious that in a closed pipe, the possible harmonics are odd, *i.e.* having frequencies, three, five times etc. of the fundamental. The even harmonics are missing and therefore the sound produced is not of rich quality.

202. (*a*) **Overtones in open Pipes.** The possible modes of vibration of an open pipe are shown in Figs. 17 (1, 2 and 3). In fig. 17 (1) the pipe is giving its fundamental and the length of the pipe is given by

$$l=\frac{\lambda}{2} \text{ or } \lambda=2l.$$

The frequency of the note emitted will be $n_1=\dfrac{V}{\lambda}=\dfrac{V}{2l}$.

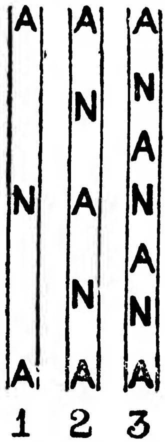


FIG. 17

The second possible mode of vibration is that shown in fig. 17 (2), the length of the pipe is given by

$$l=\lambda$$

The frequency of the note will be

$$n_2=\frac{V}{\lambda}=\frac{V}{l}=\frac{2V}{2l}=2n_1$$

The third possible mode of vibration is that shown in fig. 17 (3), the length of the pipe is given by

$$l=\frac{3}{2}\lambda \quad \text{or} \quad \lambda=\frac{2l}{3}$$

and the frequency of the note will be

$$n_3 = \frac{V}{\lambda} = \frac{3V}{2l} = 3n_1$$

In the open pipes, all the harmonics are present and therefore the sound is of rich quality.

203. Organ Pipes. An organ pipe is a wind-instrument. It consists of a tube, which is always long as compared with its diameter. One end of the tube is provided with a mouth-piece to cause the air to enter in an intermittent manner, while the other end is closed or open.

There are two kinds of organ-pipes: (*i*) *The flute pipe* and (*ii*) *The reed pipe.*

In the flute pipe Fig. 18, the air enters at A, issues from a narrow slit S, impinges against the thin lip L and thus sets it into vibrations. The vibrations of the lip, which synchronize with the period of the pipe, are taken up by it and resonance is produced. If the wind enters A with great force, harmonics may be produced.

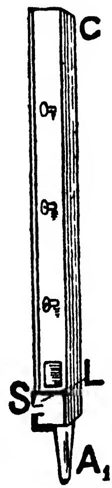


FIG. 18.

FIG. 19

(*ii*) **The reed pipe.** A reed is a flexible strip of metal, which covers wholly or partly the aperture through which the air passes to the pipe.

Fig 19 shows a reed pipe; when air is blown through the wind-channel, the reed is set into vibrations, which cause the air-column to resound.

204. Longitudinal vibrations of rods. Clamp a metallic rod in the middle and stroke it with a resined piece of flannel. It will begin to vibrate. The clamped middle-point acts as a node and the free ends as antinodes. Thus the frequency of the note will be

$$n = \frac{V}{2l},$$ where l is the length of the rod, and V the velocity of sound in it.

But $V=\sqrt{\frac{E}{D}}$ (Newton's formula)

Thus $n=\frac{1}{2l}\sqrt{\frac{E}{D}}$.

SUMMARY

1. Whenever periodic force is applied to a body capable of vibration and the body begins to vibrate in the same period as that of the applied force, the vibrations are known as *forced vibrations.*

2. If two vibrating bodies have the same period and when one of them is set swinging, the other also begins to vibrate; this phenomenon is called **resonance.**

3. The **harmonics** produced in the case of closed pipes are odd, while in the case of open pipes all the harmonics are present; and thus an open pipe is more musical than a closed one.

4. The frequency of a rod vibrating longitudinally is given by $n=\frac{1}{2l}\sqrt{\frac{E}{D}}$.

EXAMPLES

1. A tuning-fork of frequency 256 resounds in sympathy with a closed tube, calculate the length of the latter. ($v=1120$ ft per second)

2 If the length of a closed pipe, which resounds with a fork of frequency 1500, be 16 cms. at *N.T.P.*; find the length of the open pipe, which will vibrate with a frequency of 1500 at 576° and at the same pressure.

3. A tuning-fork is held over a tall glass-cylinder into which water is gradually poured, until maximum resonance is produced and the length is then found to be 64 8 cms. What is the frequency of the fork? ($v=330$ metres per sec.)

4. A closed tube 15 cms. long resounds, when full of oxygen, to a given fork. Give the length of an open tube, full of hydrogen, which will resound to the same fork.

5. The disc of a siren possesses 32 holes and makes 720 revolutions per minute. Find the length of the closed organ pipe, which will resound when emitting its fundamental.

EXAMINATION QUESTIONS IX

1. Define:—Pitch, wave-length, wave-front, phase,

amplitude, period of vibration, octave, harmonics, beats, node, antinode, consonance, dissonance, longitudinal and transverse waves.

2. Describe experiments to show that sound takes time to travel distances. What is Newton's formula for the velocity of longitudinal waves? State clearly Laplace's correction to the same, when sound is travelling through gases.

3. Describe a sonometer and explain the laws of vibrating strings.

4. What are the three characteristics of musical sound, and on what do they depend? What is a diatonic scale and what do you understand by temperament?

5. Give in detail the various methods of finding the pitch of a note. State how the pitch of a note emitted by a vibrating column of air is affected by variations of temperature, pressure and density of air.

6. What is forced vibration? Describe harmonics developed in closed and open pipes. Show how it is possible to find the velocity of sound by a closed pipe.

7. What are polysyllabic echoes? At what distance must an obstacle be situated, so that three syllables be clearly heard?

8. Prove that $v=n\lambda$. Find the density of a wire 5 sq. mms in area of cross-section, 100 cms. long, stretched by a 20 kilograms weight, which gives 6 beats with a tuning-fork of 256 frequency and 2 beats when the fork is loaded with a little wax.

STATICAL ELECTRICITY

CHAPTER I

FUNDAMENTAL PHENOMENA

205. Introductory. As far back as 600 B.C., it was known to the Greeks, that when amber was rubbed with wool, it acquired the property of attracting light bodies towards itself. In 1600 A. D. however, Dr. Gilbert Physician to Queen Elizabeth, showed that the property of attracting light bodies, was not possessed by amber alone, but by some other substances as well, such as ebonite, glass, resin etc. These substances were called *electrics* by him, after the Greek name electron for Amber; and a body is said to be *electrified* or *charged* with *electricity*, when it acquires the property of attracting small bodies towards itself.

206. Attraction of objects by electrified bodies. All bodies which are not charged with electricity, are attracted by electrified objects.

(*a*) Rub a glass rod with silk and hold it over small pieces of paper. Notice that they are attracted towards the rod.

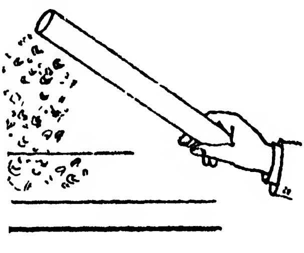


Fig. 1

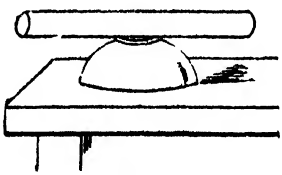


Fig. 2

(*b*) Take a metre rod and place it on a smooth inverted china-clay dish with spherical bottom. Hold

a charged ebonite rod near one end of the metre rod, the latter would rotate so as to approach the former. Instead of placing the rod on the disc, it may be suspended in a stirrup.

207. Two kinds of Electricity. Rub an ebonite rod with dry flannel and suspend it in a stirrup, made of silk thread. Rub another ebonite rod with flannel and bring it near the electrified end of the suspended rod. It is **repelled**. Now rub a glass rod with silk and bring it similarly near the electrified end of the ebonite rod It is **attracted.** Repeat the experiment by placing a glass rod rubbed with silk in the stirrup, it is seen, that now the glass rod is repelled by a glass rod, while it is attracted by an ebonite rod. If we take a rod of any other substance on which electricity may be developed, it will repel one and attract the other; thus its electricity will resemble either that of glass or that of ebonite. From this experiment we conclude, that two and only two distinct kinds of electricity are produced, one by rubbing ebonite rod with flannel and the second by rubbing glass rod with silk.* Electricity induced on ebonite rod, happens to be known as **negative** and the one produced on glass rod, when it is rubbed with silk, is known as **positive.**

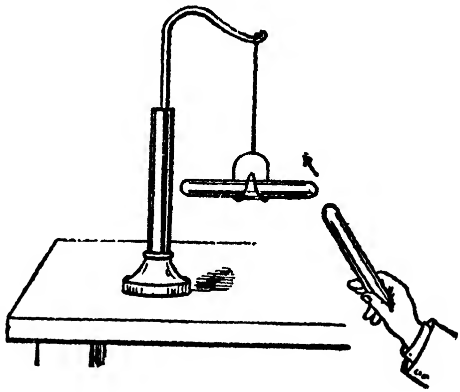


FIG. 3

208. Laws of Electric Attraction or Repulsion The above experiment also shows that :—

(i) Negative electricity repels Negative.

* Great care should be taken in using glass rods, for without any evident reason, the glass will sometimes become negatively charged

(*ii*) Negative electricity attracts Positive.
(*iii*) Positive electricity repels Positive.
(*iv*) Positive electricity attracts Negative.

or briefly we can say that *like kinds of electricity repel each other, while dissimilar kinds of electricity attract each other.* This statement is known as the **first law of electrostatics.**

The above experiment can be shown conveniently by a pith-ball pendulum, which consists of a pith-ball suspended by a silk fibre from glass rod, by bringing an excited ebonite rod near the pith-ball. It is attracted at first; but after coming in contact with the rod, it is repelled; and is again attracted by a glass rod, which has been rubbed with dry silk. The explanation is the same as given above. When the ebonite rod is first presented to the pith-ball, it is attracted, because it is neutral; but when it comes in contact with the charged rod, it shares its charge and acquires a charge of the same kind as that on the rod and is therefore repelled. It is attracted when an electrified glass rod is presented, on account of the opposite kind of electricity, residing on it.

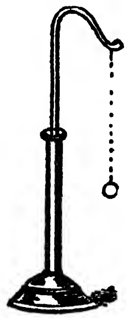


FIG. 4

209. Conductors and Insulators. (A) Take a rod of brass and rub it quickly with dry silk, the rod shows no sign of electrification. Hold this rod firmly into a glass tube and repeat the process while holding the glass end in hand; now the brass rod at once acquires the property of attracting light bodies towards itself. The explanation is, that the brass rod acquires electric charge by rubbing, in both the cases. In the first case, the rod being in contact with the hand, allows the charge to flow; while in the second case, it is in contact with glass, which does not give any passage to electricity to flow. Glass is known as an *Insulator*, while brass and hand are called *conductors*. The terms are of course relative, because in Nature there is neither a perfect conductor nor a perfect insulator.

(B) Substances like metals which allow a free passage to the flow of electricity are called *conductors*; while those, which do not allow electricity to flow from one point to another, are called *Insulators*.

The following is a list of Conductors :—

1. Silver.
2. Copper.
3. Aluminium.
4. Other metals.
5. Acids.
6. Human body.

The following is a list of good Insulators :—

1. Fused Quartz.
2. Dry air.
3. Dry glass.
4. Paraffin.
5. Shellac.
6. Sulphur.
7. Wool.
8. Oils.

Cotton, wool, paper, wood, stone, etc. are partial conductors. It is a noteworthy fact, that moist air and air at a sufficiently high temperature are good conductors ; hence an ebonite rod can be discharged, by passing it through a Bunsen's flame.

210. Conduction. Charge an ebonite rod and bring it in thorough contact with an insulated conductor. Bring the conductor near to a pith-ball pendulum, see that it is attracted, which shows that the conductor has acquired a charge. This method of imparting electricity by actual contact of the two bodies is known as *Conduction.*

211. Electroscopes.—Any appliance, by means of which it is possible to detect small charges of electricity, is known as an electroscope.

A pith-ball pendulum, such as that described above, may be used as an electroscope; but the more usual form of the instrument is the Gold-leaf electroscope, a simple form of which is shown in fig. 5.

It consists essentially of two rectangular strips of thin gold leaf, attached by their upper ends to the end of a brass rod, which passes through the neck of a glass bottle and carries a small circular brass disc at its upper end. This disc is known as the *cap*. The glass bottle protects the leaves from draught and insulates the metallic rod. To ensure perfect insulation, the cork and the outer and the inner surfaces of the neck of the glass bottle are varnished with shellac or paraffin-wax. The interior is kept dry by calcium chloride, soaked in sulphuric acid, which is contained in a small vessel. The sides of the case are generally lined on the inside with strips of metal, which are connected to the earth.

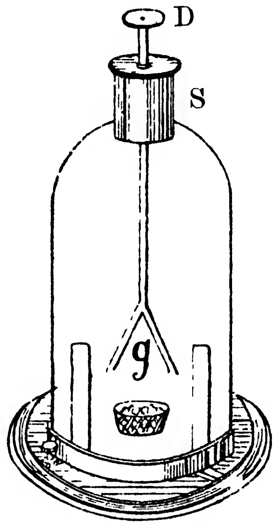


FIG. 5

I. These strips prevent the leaves from being broken by coming in contact with the sides of the vessel.

II. They do not allow the leaves to diverge too much.

III. Leaves induce electricity of opposite kind on these strips and thus make the instrument very sensitive.

The principle of action of the instrument depends upon the fundamental principle, that same kinds of electricity repel while dissimilar kinds of electricity attract each other. Thus if a charge be given to the disc, it communicates a portion to the gold leaves, which become similarly charged. They then repel each other and diverge.

The magnitude of the divergence would be roughly equal to the amount of the charge on the leaves.

Experiment. Charge the gold-leaf electroscope by touching the upper plate with an excited ebonite rod.

The electroscope will become negatively charged. If another negatively-charged rod is now brought near the disc of the electroscope, the leaves will diverge still further. If however, a positively-charged body is brought near the cap, the leaves will collapse. Thus if the electroscope is charged negatively, an increase in the divergence of the leaves shows the presence of a negative charge. if it is positively charged, increased divergence shows the presence of a positive charge.

A collapse of the leaves indicates the presence of a charge of opposite sign to that on the electroscope; *but it is not a conclusive proof*, for the leaves would collapse, even if an earth-connected conductor be held near to the cap of a charged electroscope. Thus we deduce the following rules from the above observations:—

(*i*) *Divergence is increased by a similar neighbouring charge.*

(*ii*) Leaves collapse by a neighbouring dissimilar charge or by an earth-connected conductor. **Thus divergence is the only sure test of the sign of a charge.**

Caution.* *The leaves diverge only when a potential difference exists between them and the tinfoil strips.* Place the electroscope on an insulating stand and connect the disc to the tinfoil strips by means of a thin wire. Hold a strongly charged body near to the disc and note that the leaves do not diverge.

Further experiments readily indicate that the greater the potential difference between the *leaves* and the *case*, the greater is the divergence of the leaves. Generally, the case of the instrument is earth-connected (and therefore at zero potential), so that the *actual potential* of the leaves will be the *potential difference* between the leaves and the case.

When a body is highly charged or is too big to be brought near the gold-leaf electroscope, with convenience, use is made of an instrument known as the *Proof-plane*, for detecting the charge of that body. The *instrument consists of a conducting metal-disc,*

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* See Chapter II, P. 424.

mounted on an insulating handle. The disc, when it is placed in contact with an electrified body, shares some of its charge and the proof-plane is then brought near the electroscope and the sign of the charge determined in the usual way. This will be the sign of the charge on the electrified body.

212. Law of electrical force and Unit of charge. The force of *attraction* between two *oppositely* charged bodies or the force of *repulsion* between two *similarly* charged bodies depends upon three factors: (*i*) *upon the quantity of electricity with which each of the two bodies is charged*, (*ii*) *upon the distance between the bodies and* (*iii*) *upon the nature of the intervening medium.*

Suppose two small positively-charged bodies are separated by a definite distance; then if the charge on one be doubled, the force of repulsion would also be doubled. Similarly if the charge on the other be doubled, the force would again be doubled: *i. e.* by doubling the charge on each of the two bodies, the force is increased four times and this is expressed by saying that the force exerted varies as the product of the charges.

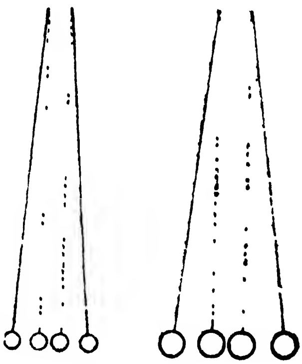


Fig. 6

It is found by experiment, that if the distance between two charged bodies be doubled, the force is reduced to one-quarter: *that is*, the force exerted between two charged bodies varies *inversely* as the square of the distance between them. These statements are known *as the* **Second law of Electrostatics** and may be defined thus:—*The force of attraction or repulsion between two charged bodies varies directly as the product of the two charges and inversely as the square of the distance between them.*

From experiments, it can be proved that the above two laws hold good for all media; but the *magnitude*

of the force varies with the nature of the medium.

Thus, if two bodies A and B have charges equal to q and q' and are d cms. apart; then the force exerted between them would be given by the equation

$$f \propto \frac{qq'}{d^2} \text{ or } f=\frac{qq'}{d^2}\times\frac{1}{K},$$

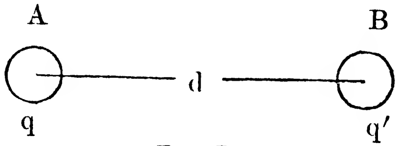


Fig. 7

where K is a constant depending upon the nature of the intervening medium. For air, the value of K is supposed to be unity and therefore the above equation reduces to:—

$$f=\frac{qq'}{d^2}. \text{ And if } q=q', \text{ we have } f=\frac{q^2}{d^2}.$$

Moreover, if the bodies are one cm. apart and the force f equal to 1 dyne, we have $q=1$: or the **Unit quantity of electricity** *is that quantity, which when placed in air at a unit distance from a similar and equal quantity, would repel it with a force of one dyne*

213 Electric field. Suppose there is a charged conductor A and if a body B, which is similarly charged, be brought near it, the force of repulsion would be perceptible; then the whole space surrounding a charged body, in which these electrical forces are perceptible, is called a *field of force and the* **intensity of the field** *at any point is the force, on a unit positive charge, when placed at that point.*

Thus if the intensity of a field be denoted by I, then the force F acting on a charge q when placed there, would be qI dynes.

The field at a given point is said to be of **unit strength,** *if a unit charge placed at that point experiences a force equal to one dyne, due to the field.*

SUMMARY

1. A body is said to be electrified, when it attracts small particles towards itself.

2. There are two kinds of electricity: (*i*) **Positive** and (*ii*) **Negative.**

3. *Like* kinds of electricity repel, while *dissimilar* kinds attract each other.

4. Bodies which allow electricity to flow from one point to the other are known as **conductors**, while those which do not allow electricity to flow from one point to the other are called **Insulators.**

5. *Conduction* is the process of charging a conductor by bringing it in *actual contact* with some electrified body.

6. Gold-leaf electroscope is an instrument for *detecting* charges of electricity

7. The force of attraction or repulsion varies directly as the product of the charges and inversely as the square of the distance between their centres.

8. **Unit Charge** is that, which when placed in air at a unit distance from an equal and similar charge, would repel it with a force of one dyne.

9. Space around a charged body is called a *Field of Force.*

10. The force acting on a unit positive charge at any point is called the **intensity** at that point.

EXAMPLES

1. What are the laws of electrostatic attraction and repulsion?

2. What do you understand by an insulator, a conductor, unit charge and intensity?

3. Describe the use and construction of a gold-leaf electroscope.

4. A piece of soft iron will attract either end of a compass-needle and so will an electrified glass rod. What experiments would you make to show that the observed fact is due to different causes in the two cases?

(*Oxf.* Loc. June, '04)

5. Two small balls are charged with +12 and −4 units of electricity respectively. With what force will they attract one another, when placed at a distance of 6 cms. ?

CHAPTER II

POTENTIAL

Introductory. If an insulated conductor A be positively charged and be then brought in contact with another insulated conductor B,

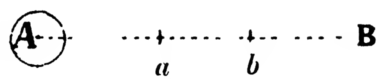


FIG. 8

three conditions are possible (*i*) a portion of A's charge may be transferred to B, (*ii*) a portion of B's charge may be transferred to A, or (*iii*) there may be no transference It can be easily shown that it is not necessarily the ball having the bigger charge, which parts with electricity. Just as a red-hot needle, being at a higher temperature, will give out heat to a beaker of water, although the quantity of heat in the latter is far greater than that in the needle, similarly a ball, having a smaller charge, may give up electricity to one having a greater charge. The *condition, which determines the transference of charges between conductors, is termed* **Electric Potential.**

214. Electric Potential. Next, let us suppose that in the field of a charged conductor A, a body B having a unit positive charge were to be moved from b to a; it is evident that work must be done in so doing, against the electric force of repulsion. *The measure of this work (in ergs), done in moving a unit positive charge from b to a, is spoken of as the difference of potential between a and b, due to the charge on A.* The potentials of two points differ by unity when one erg of work is done in transferring unit quantity of electricity from one point to the other. Now suppose the point b were at an infinite distance from the charged conductor. *Then the total work done in moving a unit positive charge from an infinite distance to the point a, is called the*

potential or the **absolute potential** *at the point a*; while the difference of potential between two points is defined as the work done in moving a unit $+ve$ charge from one point to the other.

Just as difference of temperature is a necessary condition for the flow of heat, or difference of liquid-level, a necessary condition for the flow of liquid; so a difference of potential is a necessary condition for the flow of electricity; *i.e* electricity will not flow from one place to another at the same potential, no matter what the charges may be. Thus if two bodies A and B which are at different potentials be brought in contact with each other, electricity flows from a body at a higher potential to that at a lower potential, until *potential equilibrium* is reached.

It may be noted here, that for all purposes *earth* is assumed to be at zero potential. This is because the size of the earth is so great, that the charges with which we are concerned, cannot produce any appreciable electrical change in it. Mathematically it can be shown, that the potential at a distance r from a body having q units of charge is $\frac{q}{r}$.

To prove this, let us suppose that the body A has

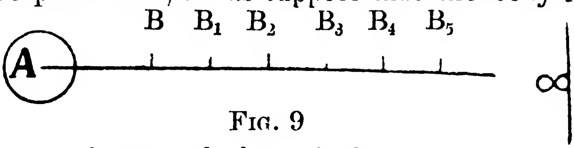


FIG. 9

q units of charge and that a body having a unit $+ve$ charge is situated at B, at a distance r from A. Join AB and produce it to infinity, mark out small equal distances B_1, B_2, B_3, etc. from B and let their distances from A be denoted by r_1, r_2, r_3, etc. Then the force of repulsion at B would be $\frac{q \times 1}{r^2}$ and at B_1, $\frac{q \times 1}{{r_1}^2}$. The mean force in the small space between B and B_1 would be the arithmetic mean of $\frac{q \times 1}{r^2}$ and $\frac{q \times 1}{{r_1}^2}$; but if B and B_1 are very

near together, then the difference between the arithmetic and the geometric means would be negligibly small. Hence instead of the arithmetic mean, we can take the geometric mean for the sake of convenience and the geometric mean of the above two quantities $= \frac{q \times 1}{r_1 r}$*. This is the proper value of the force acting between B and B_1.

Now if a unit $+ve$ charge is moved from B_1 to B, the work done is equal to the force multiplied by the distance through which it is moved. Hence the work done in moving a unit $+ve$ charge from B_1 to B is equal to

$$\frac{q \times 1}{r_1 \times r}(r_1 - r) = \frac{q}{r} - \frac{q}{r_1}$$

Similarly the work done in moving a unit $+ve$ charge from B_2 to B_1 would be

$$\frac{q \times 1}{r_2 \times r_1}(r_2 - r_1) = \frac{q}{r_1} - \frac{q}{r_2}$$

and so on

and finally from ∞ to $\infty - 1$, it would be

$$\frac{q}{r_\infty \times r_{\infty - 1}}\left\{r_\infty - r_{\infty - 1}\right\} = \frac{q}{r_{\infty - 1}} - \frac{q}{r_\infty}.$$

By adding all these quantities, we get the work done

*The average force between B and B_1 would be

$$\frac{1}{2}\left\{\frac{Q}{r^2} + \frac{Q}{r_1^2}\right\} = \frac{1}{2}\left\{\frac{Qr_1^2 + Qr^2}{r_1^2 r^2}\right\} = \frac{Q}{2}\left\{\frac{r_1^2 + r^2}{r_1^2 r^2}\right\}.$$

Suppose the distance between B and $B_1 = \delta$, where δ is very small, then $r_1 = r + \delta$.

Substituting this value of r_1 in the above equation, we have

$$\frac{Q}{2}\left\{\frac{(r+\delta)^2 + r^2}{r_1^2 r^2}\right\} = \frac{Q}{2}\left\{\frac{r^2 + \delta^2 + 2r\delta + r^2}{r_1^2 r^2}\right\}$$

$= \frac{Q}{2}\left\{\frac{2r^2 + 2r\delta}{r_1^2 r^2}\right\}$, where δ^2 is neglected owing to its small magnitude as compared with r.

$$= Q\left\{\frac{r^2 + r\delta}{r^2 r_1^2}\right\}$$

$= Q\left\{\frac{r+\delta}{r r_1^2}\right\}$, dividing both by r.

$= \frac{Q}{r r_1}$ for $r + \delta = r_1$.

in moving a unit + charge from infinity to the point B. Its value is clearly $= \frac{q}{r} - \frac{q}{\infty} = \frac{q}{r}$, because $\frac{q}{\infty} = 0$.

Hence the potential at any point due to a charge of q units $= \frac{q}{r}$.

From this it is evident, that the potential varies directly as the charge and inversely as the distance.

Caution. *The amount of work done is independent of the path along which a charge is moved.*

For let A and B represent two points, the potentials of which are V_a and V_b respectively. Suppose that a unit $+ve$ charge is conveyed from B to A along the path a and W ergs of work are done during the transference; and that $(W+w)$ ergs of work are done, if the path taken were along b. Hence if the charge traverses the path b, and is conveyed back to B along the path a; then $(W+w)$ ergs are done by the electric forces and W ergs are done against the electric forces. Thus the charge returns to B with an increase of energy, represented by w ergs; but we see that no permanent change has taken place. This is contrary to the principle of conservation of energy. Hence the work done must be the same and independent of the path traversed.

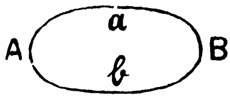


FIG. 10

Free and Induced Potential. Consider an isolated positively charged conductor in the middle of a big room. The conductor is at positive potential: and since this is due to its own charge, it is spoken of as *free potential.* Similarly a negatively charged conductor, when no other charge is near it, is at *free negative potential* due to its own negative charge.

Next consider an uncharged metal knob (fig. 11), joined by a wire to the disc of an electroscope. The leaves do not diverge; since the knob, the disc, the leaves and the case are all at zero potential. Now bring a charged conductor C near to the knob. It is seen that the leaves diverge more and more as the charged con-

ductor approaches the knob. Thus the knob, though possessing no charge, yet it acquires a potential due to the presence of the charged conductor. Such a potential is spoken of as an *induced potential* and is always due to the inductive influence of the charged body. It should be carefully noted that the *induced potential* is always of the *same* kind as the *potential of the inducing charge*. Thus if the conductor is positively charged; the induced potential of the knob must also be positive, though it has no charge. If the knob be earthed, its potential becomes zero; but it acquires a negative charge. If the conductor be negatively charged, the knob acquires a negative potential without having any charge. When the knob is earthed, its potential becomes zero; but it acquires a positive charge.

215. Potential gradient.—The expression $\frac{q}{r}$ for potential shows, that the potential goes on decreasing as the distance continues to increase; and if a graph be drawn showing the relation between the potential and the distance, we would get an idea as to how the potential falls per unit length. This is called the *potential gradient*. This fall of potential can be easily shown with the help of a charged conductor and an electroscope placed at a great distance. A thin long wire is attached

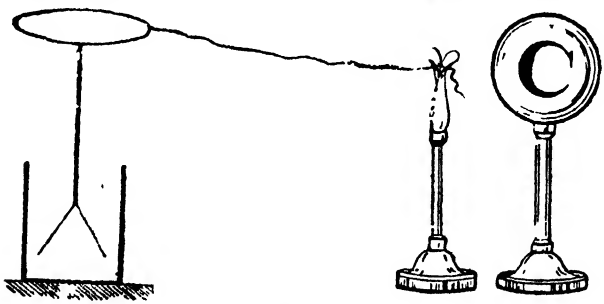


FIG. 11

at one end to the electroscope. At the other, it ends in a very small knob, which is held by a vulcanite stand, and is moved away from the charged conductor to the

electroscope. It is seen that the divergence of the leaves goes on decreasing, showing that potential decreases as the distance of the knob increases from the charged conductor. The rate at which the potential diminishes per unit distance, is equal to the intensity; and if the rate of fall of potential in any field be uniform, then the intensity must also be uniform at all points.

216. Energy of electrification. Every charged body is capable of attracting other bodies, or that it is capable of doing work ; electrification therefore must necessarily consist in the gain of energy, but electricity by itself, whatever its nature, is not energy.

Mechanical work, spent in friction, is not converted into energy of electrification ; but whenever electricity is produced, both kinds are generated in equal quantities. The *rubber* and the *object rubbed* attract each other; and *the work, that has to be done in separating these two, appears as the energy of electrification.*

Experiment. Take a flannel cap, which fits exactly an ebonite rod. Rub the two together and bring them both near the gold-leaf electroscope. Notice there is no divergence. Now separate the two and bring each in turn near the disc of the gold-leaf electroscope Notice the divergence of the leaves, which is to the same extent in both the cases This shows that equal quantities of both kinds of electricity are produced simultaneously.

SUMMARY

1. The **Potential at any point** is the work done in bringing a unit positive charge from infinity to that point.

2. The *potential at any point* due to a charge q is given by $\frac{q}{r}$, where r is the distance of that point from the charged body.

3. A *curve* showing the fall of potential with increase of distance is called a **potential gradient.**

4. The *energy of a charged body* is derived from the work done, in separating it from the *rubber*, which always becomes oppositely charged.

5. Earth is assumed to be at zero potential.

EXAMPLES

1. Explain what you do understand by potential.
2. Wherefrom is the energy of a charged body derived?
3. Prove that the potential at a distance r from a charge Q equals $\frac{Q}{r}$.

CHAPTER III

THEORIES OF ELECTRICITY

217. Theories of electrification. Up to this time, we have made no attempt to ascertain the nature of electrical phenomena; but various theories were propounded at several stages of the development of this Science. The earliest attempt at theorising was that of Symmer. According to him, there are two electric fluids of opposite kinds, present in all substances; and a body becomes electrified, whenever one or the other of the two fluids is added or subtracted. The two fluids were supposed to be indestructible, perfectly weightless, self-repellent and mutually attractive. This theory has been called the *two-fluids theory*. It is very easy to explain many of the well-established facts of Electrostatics by this theory.

After this, in the year 1747, Benjamin Franklin suggested the *one-fluid theory*; according to which there is only one fluid called the *positive*. To him we owe the terms *plus* and *minus* or *positive* and *negative*. In this theory, matter takes the place of the negative fluid. The particles of matter and that of the fluid are supposed to be self-repellent and mutually attractive. According to this, a body becomes electrified, when it contains more or less than the normal amount of fluid. A body becomes positively charged, if it contains more than the normal amount; and it is said to be negatively charged, when it contains less than the normal amount. The modern theory is the *electron theory*, according to which there exist particles, much smaller than the Chemists' atom; and these particles are always supposed to carry a definite amount of negative charge, *viz.* $(4{\cdot}7 \times 10^{-10})$ C.G.S. units. Such particles have been termed *electrons* or *corpuscles*. These electrons are readily expelled from

matter by electrical forces. According to this theory, the transference of electricity consists in the movement of negative elections, from a point where there is a gain of positive to where there is a gain of negative electricity. A positively-charged body is one, which has been deprived of some corpuscles; while a negatively charged body is supposed to have an excess of these. A glance over the one-fluid theory shows, how amazingly strong is the similarity between the two: *if only* the words positive and negative be interchanged in Franklin's theory.

218. Experiment.—Charge an insulated conductor '*C*' positively and bring it near one end *A* of an insulated conductor *AB*, which is absolutely uncharged before. Put a proof-plane at the end *B* of the conductor and then remove it and bring it near the gold-leaf electroscope. The leaves diverge, showing that the proof-plane has acquired a charge. Test the sign of the charge. See that it is positive.

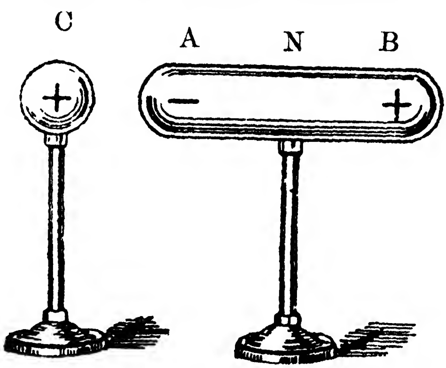


FIG 12

This phenomenon of electrification is known as **Induction**. An explanation of this can be given in Faraday's language of stress and strain in the medium; but as it is difficult for a beginner to follow it clearly, we will restrict ourselves to its explanation on the two-fluids' theory. According to this, positive and negative electricity exist beforehand; and the action of the positively-charged conductor *C*, consists simply in urging negative electricity towards the end *A* and positive electricity to the other. In *the above* case, negative electricity is attracted by the positive charge and hence it accumulates at the end *A*; while positive electricity is repelled and is found at the end *B*.

Further, we know that there is a positive charge on C, and that there will be a large positive potential in its neighbourhood, which goes on decreasing as we move away from C. In the neighbourhood of A there will thus be a higher positive potential than in the neighbourhood of B. But all points on the surface of a conductor must be at the same potential. This can be secured only by a negative charge at A and a positive charge at B, for the negative charge at A would tend to decrease the positive potential there and the positive charge on B would increase the positive potential at B. Thus the potential of the surface AB would become uniform, and we see that this explanation on the potential basis leads us to the very same conclusion that the *+ve* charge on C attracts negative charge towards A and repels positive charge towards B.

Now in the last figure, if the point B be touched and then the positively-charged conductor C be removed, the conductor AB is found to be negatively charged The explanation is the same as given above, *i.e.* the positive charge on 'C' attracts the negative and repels the positive, which on touching with the hand, is transferred to the earth and the negative charge remains at A. When 'C' is removed, the negative charge distributes itself on the whole surface. Thus we see that when an insulated conductor is held near a charged body, electrical separation results and a charge of dissimilar character is attracted by the charged body to the nearer portions of the insulated conductor, while a charge of similar character is repelled to the more distant portions. This phenomenon is termed as **Induction.**

Hence by the process of induction, a body may be charged by the influence of a charged body without in any way affecting or impairing the charge on the latter. *In this process, the charge is not shared but is brought into existence by electric separation, without in any way affecting the charge of the charged body.*

219. Induced and Free Potential. When a body is given a charge, its potential rises; such a poten-

tial is called *free potential*, as it is due to the charge on the body and is independent of its position in space. If however, a conductor is moved in a field of force due to an electric charge, then its potential changes as the strength of the field varies. The potential is in fact due to the presence of neighbouring charges, which act on the given conductor inductively. Such a potential is called Induced Potential.

Thus in fig. 12, when C is held near to AB, its potential due to Induction becomes $+ve$ When B is touched with hand, its potential becomes zero. On removing the hand and then the charge C, the potential of AB becomes $-$ *ve*. Therefore we see that induced potential depends upon the position of other charges or conductors in its vicinity.

To show that equal amounts of two kinds of electricity are produced by Induction.

Experiment Put two insulated spheres A and B, in contact with each other and hold a positively-charged conductor C near A. Separate B from A and then remove the conductor C. Bring B near the gold-leaf electroscope and notice the divergence of the leaves. Remove B and bring A near the electroscope. The leaves diverge to the same extent again. Now connect A and B, and *bring* them together near the gold-leaf electroscope. Notice there is no divergence.

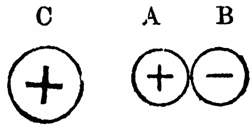


FIG. 13

This proves conclusively that the induced charges are equal in magnitude and of opposite ***kinds***.

219. (*a*) **To charge a gold-leaf electroscope by Induction**.

In the gold-leaf electroscope already described, we see that if a charged body be brought near it, the leaves begin to diverge, *even if*, the charge be not communicated to it; and they again collapse, when the charged body is removed farther away. This is due to the *Inductive action*. The charged body brings about separation of the two kinds of electricity in the disc of the gold-leaf electroscope. It attracts the dis-similar

kind and repels the same kind of electricity to the leaves, which diverge. If now the disc be touched with hand, the leaves collapse; because the free charge on the leaves flows to the earth. Remove the hand, the leaves remain collapsed; because the charge of dissimilar kind is kept bound on the disc by the charged body. Remove the charged body; the leaves again diverge on account of the opposite kind of charge, which was present on the disc, but which now distributes itself over the disc and the leaves. Thus the electroscope becomes charged with opposite kind of electricity to that of the charged body, without wasting or in any way sharing the charge of the body with the gold-leaf electroscope.

220. To test the sign of the charge by a gold-leaf electroscope.

Experiment Charge the gold-leaf electroscope positively and bring the charge, the sign of which is required, near the disc of the gold-leaf electroscope. If the leaves diverge more, then the charge is of the same sign as that on the electroscope, for *only a positively-charged body produces more positive* electricity on leaves by induction, *hence* the charge must be *positive.* If however, the leaves collapse, then the charge is either negative or the body is not charged at all. Now charge the gold-leaf electroscope negatively and bring the charged body near its disc. If the leaves diverge more, then the second charge must be similar and therefore the charge on the body should be negative, but if the leaves collapse, then either the body is + *vely* charged or is neutral From this we conclude, *that divergence alone is a sure test of the sign of the charge.*

221. Charge resides always on the outer surface of a conductor, provided *there is no insulated charge inside.*

Faraday's Butterfly-net experiment. A conical net of linen-gauze is mounted on an insulated stand. A string of silk is attached to it, such that it can turn it inside out. The net is charged, If the inside of the net be touched by a proof-plane and tested the proof-plane shows no sign of electrification. Now touch the proof-plane with the outer

surface of the net and test by means of a gold-leaf electroscope, the leaves diverge. This simple experiment shows that electricity remains on the outer surface. Now turn the net inside out and repeat testing with the proof-plane; the charge is still found to be on the outer surface, showing that the charge has changed surface.

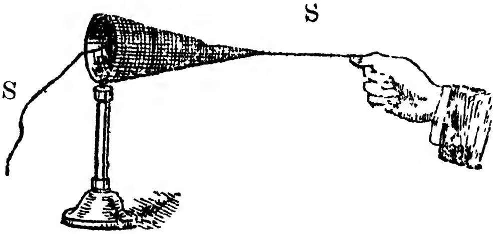


Fig. 14

The phenomenon, that the charge resides on the outer surface of a conductor, was demonstrated by **Biot** with the help of the instrument, shown in fig. 15. The insulated sphere is charged highly and the two metallic hemispheres with insulated handles are brought near it, so as to thoroughly envelop the sphere. These hemispheres touch the sphere at the lower point. Now remove the hemispheres and on testing with the gold-leaf electroscope, it is found that the sphere is totally deprived of the charge, which now accumulates on the hemispheres. The same phenomenon can be easily shown, by lowering a small charged sphere in a hollow insulated spherical conductor, on an insulating stand; so long as the charged body does not touch the spherical conductor, the body retains its charge and as soon as it touches the conductor, the whole of the charge is transferred to the conductor and none remains on the sphere. From this it is evident, that *whenever*

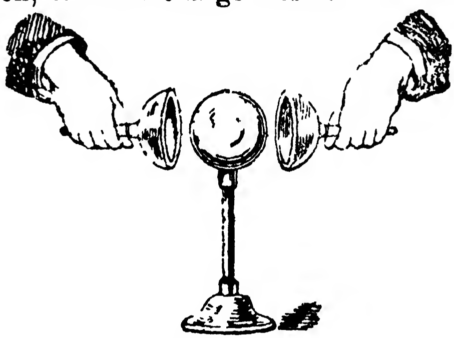


Fig. 15

it is required to transfer the whole of the charge from one body to another, the charged body must be put inside the body, to which the charge is to be transferred.

222. Faraday's Ice-Pail experiment. Place a calorimeter on the cap of a gold-leaf electroscope, Fig. 16. Attach a metallic ball to a silk thread and charge it positively. Lower it slowly into the calorimeter. The gold leaves begin to diverge, because the charge on the sphere acts inductively It attracts the negative on the inner side of the calorimeter and repels the $+ve$ to the outer surface. The divergence goes on increasing as the ball is lowered into the calorimeter, but when the ball reaches well within the calorimeter, no more increase in the divergence takes place. This will happen when the total induced charge has become equal to the inducing charge If the ball be now touched with the innerside, the divergence remains the same. Remove the ball and test with another gold-leaf electroscope, it is found to have lost all its charge. This experiment is known as Faraday's Ice-Pail experiment and it shows: (*i*) *the total induced charge is equal to the inducing charge and* (*ii*) *the free charge resides on the outer surface of a conductor*

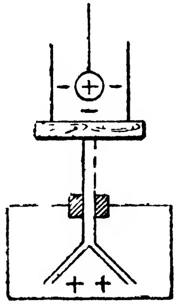


FIG. 16

222. (*a*) **Successive inductances and multiplicity of charge**:—Arrange two vessels A and B, one inside the other on ebonite stands, as shown in the figure. Charge a ball C positively and lower it inside A. Negative charge will be induced on its inner side and an equal amount of positive charge will appear on its outer side. This will act inductively on B, producing negative on its inner surface and repelling positive to its outer surface. This furnishes us with an example of successive inductances. Now connect A and B, the positive on the outer surface of

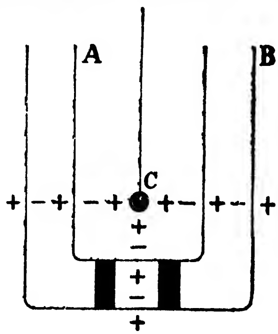


FIG. 16 (*a*)

A combines with the negative on the inner surface of *B*, leaving *B* positively charged. Now remove the ball *C* and then the inner vessel *A*, neutralize *A* by touching it. Place *A* in its proper place and again lower the positively-charged ball *C* Successive inductance will take place, as before. Join *A* and *B*, the positive charge on the outer surface of *A* will neutralize the negative on the inner surface of *B* which will now have double quantity of positive charge. Proceeding in this manner, *theoretically* we can multiply the positive charge on *B*. *In actual practice* there is a limit and the charge cannot be multiplied indefinitely.

223. To show that the Potential at every point of a charged conductor is the same. Charge the insulated cylinder, Fig. 17. Connect the disc of a proof-plane to the cap of a gold-leaf electroscope by a thin metallic wire. Hold the proof-plane by the insulated handle over the surface of *P* and move the disc of the proof-plane from one end of the conductor to the other: the divergence remains the same. This shows that the potential at all points of a charged conductor is the same; for the divergence of the leaves is a measure of the potential of the point, with which the proof-plane is in contact.

224. Distribution of the charge on the outer surface of a conductor. We have shown that the potential at all points of a conductor is the same; but it does not follow from this, that the quantity of electricity is uniformly distributed over its surface. In fact the distribution depends upon: (1) the shape of the conductor itself and (2) the proximity of neighbouring conductors, whether charged or not. For the present,

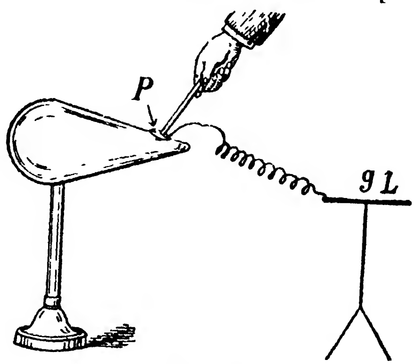
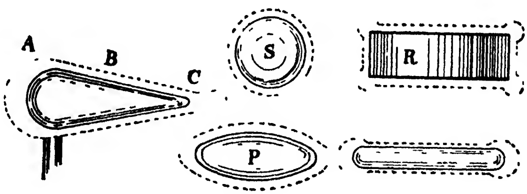


FIG. 17

we shall not take into account the effect of neighbouring conductors; and shall only study the distribution of the charge on the surface of an isolated insulated conductor, with reference to its shape.

Experiment. Take a conical conductor of the form shown in fig. 18. Put a proof-plane first at *A*, remove it and bring it near a gold-leaf electroscope. Note the divergence of the leaves Similarly repeat your observations by putting the proof-plane at *B* and *C*. In the last case note that the leaves diverge to a greater extent, *showing that the charge accumulates at the pointed ends, leaving the round portions less heavily charged* Similar experiments may be performed with insulated conductors of different shapes, such as a sphere, a pear-shaped conductor and a metal plate as shown in figs 18.

Draw the outlines of the conductors and show by means of dotted lines, outside these conductors, as to how the charge is distributed. The above fact would be easily shown in all of them. This is expressed by saying that the **surface density,** or sometimes also called the **superficial density,** at points and edges is greater than at other points of a conductor. and the *surface density at any point is defined, as the quantity of charge per unit area of the surface containing that point.* This



Figs. 18

definition is not logical: because we suppose the distribution to be uniform in that unit area, which is seldom the case. Therefore, it would be more correct to say that *surface density at any point, is the ratio of the quantity of electricity on a very small area containing the point to that area*; because when the area is extremely small, the distribution may be presumed to

be uniform. In the case of a sphere the distribution is uniform, because the curvature is uniform; but in all other conductors the *surface density would be maximum at the points, edges or corners of a conductor.*

225. Action of Points—We have seen that a charge accumulates on the points, corners and edges of a conductor. The surface density may, if the conductor be highly charged, become so high at the points, as to charge the dust and air particles around it, which being similarly charged would be repelled and their place, taken by others. Thus a kind of electric wind may be set up. which may be sufficient to blow a candle flame aside. Each particle, which is repelled, carries with it a small charge and thus the conductor, to which the point is attached, loses electricity by this means. This principle is made use of in electric machines, for transferring electricity from the charged plate to the prime conductor.

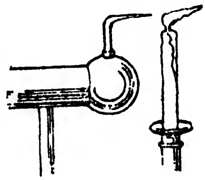


FIG 19

The action of points may be beautifully illustrated by an electric whirl, which is capable of rotation on a central axis. The reaction of the repelled particles, when the machine is worked, drives it round in a direction opposite to that in which the pointed ends of the whirl are. The action of the points can also be illustrated by attaching a pin to the disc of a charged gold-leaf electroscope, the charge escapes and the leaves collapse.

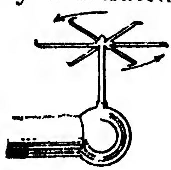


FIG 20

If a person were to stand on an insulated chair and to grasp the prime conductor of an electric machine, he would become charged. If the machine were worked and the person were to place the knuckle near a gas jet, a spark would pass and the gas lit: but if the machine be stopped for a few minutes and the same thing repeated, no spark would pass, because the charge escapes through every hair-point.

It is essential that there should be no speck of dust over such conductors, as are meant to retain charges of electricity, because the specks of dust act as points and thus help the charge to escape.

226. Lightning conductors. During a thunder-storm, clouds may be charged to a high potential due to the coalescence of small drops into bigger ones (Section 231, *a*). Such charged clouds induce charges of opposite sign on the earth's surface and if the difference of electrical potential is sufficiently great, a spark may pass between the cloud and the nearest earth-connected conductor. The discharge may do damage to the building, which is usually made up of bad conducting materials. But a metal conductor *L*, at the top of the house, which has a good contact with earth, effectively prevents the building from any damage; because the upper end of the rod becomes charged, by induction from the cloud above, and this gives off an electric wind, which discharges the cloud above and no spark is produced. *Even if* a spark be produced at all, the discharge is carried by the metallic conductor without doing any damage to the building. *These rods*, held over the roofs of high buildings and connected by wires to metal plates, buried in a stratum of earth, *which is always wet*, so as to ensure good earth-contact, protect the buildings from being struck by lightning and are called **lightning conductors**. The theory of lightning conductors, given above, is very brief; but for absolute protection, instead of one rod, there should be a network of rods and wires, enclosing the building.

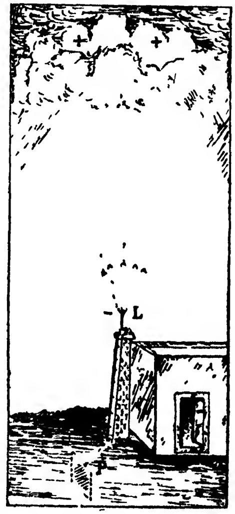


FIG. 21

The discharge is extremely rapid and of an oscillatory character; and to prevent any damage, it is desirable to retard the rapidity of the discharge. The capacity of the conductor should be as great as possible. For this reason, a flat metal band is preferable to a circular wire. Moreover, to minimize the effect of *self-induction*, the band should be as direct as possible.

SUMMARY

1. (*a*) **Symmer's** two-fluids theory of electrostatics assumes the existence in every body, in equal amounts, of two weightless fluids, which are supposed to be self-repellent and mutually attractive.

(*b*) In **Franklin's** theory it is assumed that everybody contains normal amount of a weightless, self-repellent fluid.

(*c*) The **electron** theory presumes the existence of corpuscles carrying a small amount of negative charge.

2. (*a*) The process of charging a conductor by the influence of another charged conductor, without in any way impairing the charge of the latter, is called *charging by induction.*

(*b*) When a charged body is held near to another uncharged conductor, both kinds of electricity are developed on the latter and this phenomenon is called **induction.**

3. Whenever electricity is produced by either method, equal amounts of both kinds are simultaneously produced.

4. Charge resides on the outer surface of a conductor. This is shown by *Faraday's* Butterfly-net experiment and by *Biot's* apparatus.

5. **Faraday's Ice-Pail experiment** shows that the charge resides on the outer surface of a conductor and that equal amounts of charges are simultaneously produced.

6. **Surface density** is the quantity of charge residing on a unit area on the surface of a conductor; or it is the *ratio* of charge to the extremely small area, on which it is distributed.

7. If the conductor is not a perfect sphere, then the surface density varies from point to point, but the potential on its surface does not vary from point to point.

8. Surface density at points, edges and corners is very high.

9. **Lightning conductors** are metal-points attached to the highest points of buildings and are connected to the earth

by metal wires. They protect the buildings from being struck by lightning.

EXAMPLES

1. Describe how you would charge a gold-leaf electroscope positively with an ebonite rod.

2. What do you mean by electrostatic induction? Explain the action of a sharp point in discharging a conductor.

3. Describe Faraday's Ice-Pail experiment; and show how it proves that the charge resides on the outer surface of a conductor.

4. Define electric density. A charge of 124 units of electricity is imparted to a sphere of 8 cms. radius. What is the density of the charge?

5. Define a unit of electricity. Find out at what distance must a small sphere, charged with ten units of electricity, be placed from a second sphere, charged with twenty units, in order to repel it with a force of ten dynes.

6. Describe in detail, the function of a lightning conductor.

7. The surface density of a sphere of radius 14 cms. is 5; calculate the total charge on it.

8. How would you communicate the whole charge of a conductor to another hollow conductor?

CHAPTER IV
ELECTRIC MACHINES

We have seen, that a body can be electrified either by friction or by induction. Any appliance by which large quantities of electricity can be produced is known as an **electric machine.**

227. Frictional machines. A frictional machine is an arrangement for generating electricity by the friction of two substances and for collecting the electricity so produced. The forms of such machines are many; but all of them consist essentially of three parts: (*i*) *the Generator*, (*ii*) *the Collecting combs* and (*iii*) *the Prime conductor.* A simple form of such a machine is shown in figure 22.

The generator consists of a glass cylinder capable of rotation about the horizontal axis by a handle. The cylinder when rotated is electrified by rubbing against leather-covered pads coated with a mixture of an amalgam of zinc and mercury. As the cylinder rotates, the electrified portions leave the rubber behind and pass close to the row of sharp-pointed wires called the collecting combs. These collect the charge from the surface of the glass in the manner described under the action of points and this charge is carried to and collected in the prime conductor. The negative charge induced on the pads is carried away to the earth by their being connected to it by a metallic wire. Another form of this machine is known as the

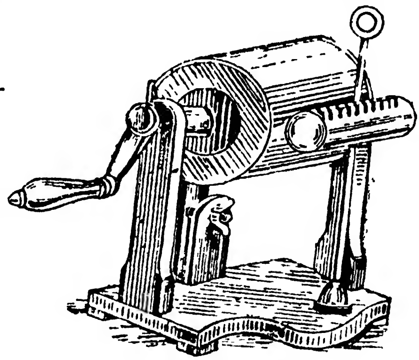


FIG. 22

plate electrical machine, in which a circular plate takes the place of a cylinder; otherwise the principle is exactly similar to the one just described.

228. The Electrophorus. This consists of a smooth plate of ebonite or some other insulating material resting on a metal plate called the **sole** and a flat metal disc with an insulating handle. Negative electricity is developed on the ebonite plate by rubbing it with a piece of cat-skin The metal disc with insulating handle is placed over it, then touched and removed. It becomes positively charged In this manner by repeating the process, a large amount of electricity can be obtained.

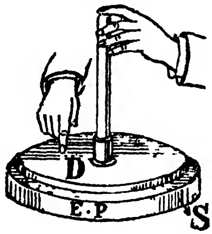


FIG. 23

The process which takes place is as follows:—When the metal disc with insulated handle is placed over the negatively-charged ebonite plate, the metal disc does not come in actual contact with the ebonite plate except at a few points; but is separated from it by a thin film of air. Consequently a separation of two kinds of electricity takes place in the metal disc by induction. Positive is attracted by the negative of the ebonite plate and negative is repelled On touching the disc with hand, the negative flows to the earth and on removing the plate by the insulated handle, the positive charge redistributes itself over the whole surface of the disc.

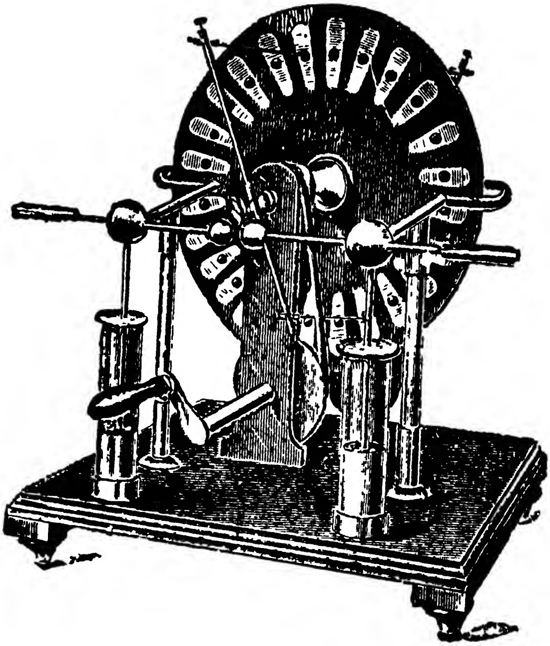
Action of sole. The sole prevents the charge on the ebonite plate from leaking away in the *following manner*:—When the metal disc is not lying on the ebonite plate and there is no sole attached, the negative electricity of the ebonite charges the air particles and thus loses its charge. When however, the sole is present, the negative electricity of the ebonite plate attracts $+ve$ from the sole and the negative from the sole flows to the earth. Thus the negative of the plate is kept bound by the $+ve$ of the sole. When the metal disc is kept over the plate and touched, both the

sole and the upper disc are connected to earth. But as the distance of the upper disc from the ebonite rod is much less than that of the sole, positive electricity now appears mostly on the disc only. On removing the disc +*ve* electricity again appears on the sole. Thus we see that, when the metal disc is touched, positive electricity flows out of the sole; and when the disc is removed, negative electricity flows out of it. These facts can be easily shown by putting the plate of the electrophorus on the cap of a gold-leaf electroscope and then proceeding in the manner described above.

From what has been said about electrophorus, it appears that an unlimited charge is obtainable from a single charge on the ebonite plate. The question naturally arises: Wherefrom is the energy of the charged disc derived? The answer is that in removing the positively charged disc from the surface of the negatively charged plate, work is to be done against the forces of attraction and *the energy possessed by the charged disc is equivalent to the work done against the electrical forces.*

229. The Wimshurst Machine. It is a form of influence machine and is the best of its kind for producing large quantities of electricity. It is diagrammatically represented on page 447 and a rough sketch is shown in fig. 24 (*a*), P. 448. It consists of two circular plates of ebonite or varnished glass, placed near each other and so arranged as to rotate in opposite directions. On the outer surface of each plate are stripped an even number of thin metal sectors, which act as inductors and carriers. Two diagonal metal conductors inclined to each other at right angles terminate in metal brushes, and touch the sectors as they pass. These are attached one each on the outer surface of either plate and are so arranged that each sector, when it rotates through an acute angle, after being discharged by the comb meets a brush. There are collecting combs on both sides of the horizontal diameter; these collecting combs are connected to the inside of the two

Leyden jars and are also provided with discharging knobs.



Wimshurst Machine
FIG. 24

To understand the principle, suppose that one of the sectors has a small $+ve$ charge, which is generally the case, and if there be no charge present, a small amount can be imparted either by conduction or by induction. When this comes opposite the brush n_1, it acts inductively on the sector touching n_1 and gives it a slight negative charge. At the same time it gives a $+ve$ charge to the sector touching n_2. These sectors with their induced charges leave the brushes and come opposite n_3 and n_4. Sectors touching n_3 and n_4 are then inductively charged, n_3 positively and n_4 negatively, to an amount double that of initial charge; thus after a

few rotations all the sectors become charged. These give over their charges to the combs, are neutralized and are again in a position to be charged. This machine is very reliable for getting large quantities of electricity. If a number of pairs of plates are used, a very big electric effect may be produced. Lord Blythswood constructed in his private laboratory, an immense electrical machine, having 160 plates. This machine was capable of producing lightning flashes, with almost deafening report.

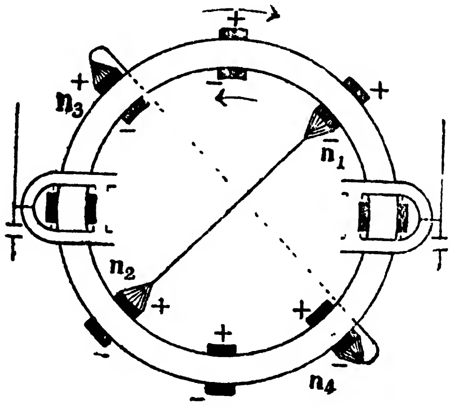


FIG. 24 (a)

230. Electrical discharge. If the knobs of an electrical machine be not far apart, sparks pass between them quickly and almost in straight lines; if however, the distance be increased, sparks become less frequent and follow an irregular path. The diminished frequency of the sparks, with increased distance between the knobs, shows that a greater difference of potential is required to break the resistance of air, and as a discharge is necessarily to follow the path of least resistance, small specks of dust are sufficient to divert the path of the spark and hence we get a zig zag spark. If a spark of this character be examined by a rapidly rotating mirror, it presents the appearance shown in fig 25, *i. e.* the tips of the branches point from positive to negative. This **is known as disruptive discharge**

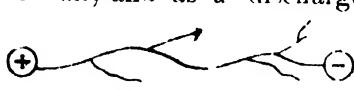


FIG. 25

The length of spark depends upon: (i) the potential difference between the terminals, (ii) the dielectric

(*iii*) the shape of the terminals and (*iv*) upon the pressure, if the dielectric is a gas. It has been proved experimentally that before a spark can pass, air between the knobs acquires slight conductivity. This conductivity of air is always due to ionisation and any agency, which accelerates the ionisation of air, facilitates the passing of sparks. Ultra-violet light and electric sparks possess this property.

Glow Discharge. If the knob of an electric machine be made pointed, the discharge instead of taking the shape as described above, assumes the form of a bright glow. This form of discharge is known as *Glow discharge*; and it takes place when a pointed earth-connected conductor is held near the prime conductor of an electric machine in a dark room.

Brush Discharge. When the discharging knobs of a large machine are well-polished and too far off to produce disruptive discharge, the discharge consists of a short bright line, originating from the positive terminal and ending in a radiating brush of comparatively slight luminosity.

230. (*a*) **Effects of electric spark.** An electric spark is accompanied by *mechanical, heating, lighting* and *magnetic effects.*

(1) *Mechanical effect.* Take a piece of paper, hold it between two knobs of an electric machine, and work it so that small sparks pass quickly between the knobs. Keep a piece of paper moving between them. Stop the machine and examine the paper. Notice that the paper is pierced with a large number of holes. If instead of paper, a thin piece of glass be interposed between the knobs, then the glass will also be pierced through In this case, the machine will have to be worked for a longer time before a spark can actually pass. If the glass piece is not thin, then the spark will pass with very great difficulty; and if it passes at all, then the glass plate will crack due to the high strain produced.

(2) *Heating effect.* Stand on an insulated stool, touch one of the knobs of a machine with one hand

and hold one of the knuckles of the other hand near the exit of a Bunsen burner, through which gas is escaping. Notice that the gas catches fire, as soon as the spark passes.

Take a small shallow metal can. Pour water into it and then a small amount of spirit, so that the latter floats over the former. As before take your stand on an insulated stool and touch the spirit with your finger. Notice that as soon as a spark passes between your finger and the spirit, the latter is set on fire due to the heating effect of the spark.

(3) *Lighting effect.* The very fact, that an electric spark is seen even in a darkened room, is proof of the statement, that the spark is self-luminous. The illuminating properties of lightning flash are self-evident proof of the lighting effect of an electric spark on an extensive scale.

(4) *The magnetic effect* of electric spark is beautifully shown by passing a spark above a magnetic needle, suspended so as to rotate freely in a horizontal plane. The needle is deflected, as if a wire carrying a current were held over it. (Sec. 289).

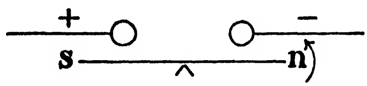


FIG. 25 (*a*)

SUMMARY

1. There are two kinds of machines (*i*) Frictional, and (*ii*) Influence.

2. When the knobs of an electric machine are brought near together, sparks are produced.

3. There are various forms of spark discharge :—
(*i*) Straight. (*ii*) Disruptive. (*iii*) Brush.

EXAMPLES

1. Describe the plate-form of frictional machine and state clearly the action of a pointed comb :—

2. Describe the working of an electrophorus. Wherefrom is the electric energy derived?

3. State the various forms of spark discharge How would you get a brush discharge?

4. Explain the action of sole in an electrophorus.

5. Describe the construction and working of a Wimshurst machine.

CHAPTER V
CONDENSERS

231. Take two insulated spheres of radii one and two inches respectively; connect and charge them by bringing them in contact with the prime conductor of an electric machine. The potential of the two spheres is the same, because they are in contact with each other. Now separate them and put a sphere of radius one inch into a calorimeter placed on the disc of a gold-leaf electroscope. As the charge resides on the outer surface of a conductor, the whole of the charge will be transferred to the calorimeter and the leaves of the electroscope will diverge. Note the divergence of the leaves. Now discharge the gold-leaf electroscope and introduce the sphere of radius two inches into the calorimeter. Note that the divergence in this case is greater than in the first. Thus we see that the charges carried by the two spheres are not equal, though their potentials were the same. This is expressed by saying that the **capacity** of the second sphere is greater than that of the first.

The capacity of a conductor is defined as the amount of charge required to raise its potential by one unit. Suppose we give Q units of electricity to a conductor and its potential rises through V units; then $\frac{Q}{V}$ units of electricity would be required to raise its potential through unity, and this is called its **capacity.** Or we may express that $\frac{Q}{V}=C$ or $V=\frac{Q}{C}$, where C is the capacity of the conductor.

We have shown that potential at a distance r from a charge Q is equal to $\frac{Q}{r}$. Potential at all points on

the surface of a conductor is the same: and in the case of a sphere, the whole of the charge acts on external points, as if it were concentrated at its centre. If we give a charge Q to a sphere of radius r, then the potential at its surface would be $\frac{Q}{r}$; therefore its capacity, *i.e.* the quantity of electricity required to raise its potential through unity would be Q, the charge divided by $\frac{Q}{r}$, the potential through which it is raised:

$$i.e. \quad C=\frac{Q}{\frac{Q}{r}}=r.$$

Thus the capacity of a sphere is equal to its radius.

231. (*a*) *Whenever a number of spherical drops, having electric charges, coalesce to form a big spherical drop, the potential always rises*

Let us suppose that there are n drops each of radius r and having a charge q. Before coalescence the common potential of all will be $\frac{q}{r}$, *i.e.* charge q divided by the capacity r.

When they coalesce to form one big drop of radius R, then $\frac{4}{3}\pi R^3=\frac{4}{3}\pi r^3\times n$ *i.e.* the volume of the big drop must be equal to the volume of n small drops.

$$\text{Thus} \quad R^3=r^3n$$

$$\text{or} \quad R=rn^{\frac{1}{3}}$$

The total charge must be $n\,q$

$\therefore$ the potential must be

$$\frac{nq}{R}=\frac{nq}{rn^{\frac{1}{3}}}=n^{\frac{2}{3}}\times\frac{q}{r}.$$

i.e. $n^{\frac{2}{3}}$ times the potential of one drop. This is always more than $\frac{q}{r}$, the potential of each individual drop, if n is 2 or more than 2.

Thus if the number of drops be 8, the potential

will be 4 times the potential of each drop This explains, how during a thunderstorm, due to the coalescence of small drops into bigger ones, the potential of clouds rises enormously.

232. Condenser.—*Any conductor, the capacity of which has been artificially raised, is called a condenser.* Before considering the methods by which the capacity of a condenser can be increased, it is worth while, to study the effect of the presence of neighbouring conductors on a charged conductor

(*i*) *Effect of a negatively-charged conductor on a positively-charged conductor* Suppose a body A is positively charged and another body B which is negatively charged is held near it. Then if a unit $+ve$ charge were to be brought from infinity to the surface of A, the work required to move it would be less than if B were absent; because the unit $+ve$ charge would be repelled by A and attracted by B. The potential however is measured by the amount of work necessary to move a unit $+ve$ charge from infinity to the surface of the charged conductor; therefore we see, that the effect of a negatively-charged conductor near a positively-charged one is to *decrease its potential.*

(*ii*) *Effect of an earth-connected conductor.*

Now in the last case, if instead of a negatively-charged conductor, we have an earth-connected conductor near the positively-charged conductor, the effect will still be the same: for the positively-charged conductor will induce $-ve$ electricity over the earth-connected conductor and repel the $+ve$ to the earth. That conductor will act as if it were $-$*vely* charged and the effect of its presence will clearly be to diminish the potential of the positively-charged conductor

(*iii*) *Influence of dielectric*—The insulating medium, surrounding a charged body, is called a *dielectric.* Thus air surrounding a charged sphere is a dielectric. According to the modern view, which is due to Faraday, the forces of electric attraction or repulsion are the result of a state of stress set up in the dielectric by the electrification of a conductor. If this view be accepted,

then it naturally follows that the dielectric must have some effect upon the force of attraction or repulsion. Thus, if the force of attraction between two charged bodies, having charges equal to q and q' and d cms. apart be F, when the intervening medium is air; and F_1 is the force between the same charges, when some substance other than air is the intervening medium; then the ratio of F to F_1 is called the **Dielectric constant** or the **Specific Inductive Capacity** of that dielectric. This ratio is denoted by K. Thus we have $\frac{F}{F_1}=K$, and as $F=\frac{q.q'}{d^2}$; F_1 would be equal to $\frac{q\,q'}{d^2} \cdot \frac{1}{K}$.

As K has a value greater than unity for all substances except air for which it is taken as unity, we see that the force must be less when a dielectric of S.I.C. higher than air is introduced. Consequently the work necessary to bring a unit positive charge would decrease and the potential would be lowered. In fact, the potential at a distance r due to a charge Q in a dielectric of S.I.C. K would be $\frac{Q}{r} \cdot \frac{1}{K}$, *i.e.* $\frac{1}{K}$ of the potential in air. Thus potential falls, when a dielectric of S.I.C. higher than air surrounds a conductor.

From the above considerations, we see that the potential of a charged body decreases even though its charge remains the same, if (*i*) its dimensions are increased, (*ii*) an oppositely-charged body is held near it, (*iii*) an earth-connected conductor is held near it and (*iv*) a dielectric of higher S.I.C. is substituted for one already surrounding the charged body. We have already seen that $C=\frac{Q}{V}$: *i.e.* if V decreases, capacity increases. Thus any of the above four methods, which lowers the potential, increases the capacity; because capacity varies inversely as the potential.

233. The Parallel Plate Condenser.

To construct a condenser, we want two conductors

separated by a layer of some dielectric ; one of these should be earth-connected, while the other ought to be insulated to receive the charge. Such an

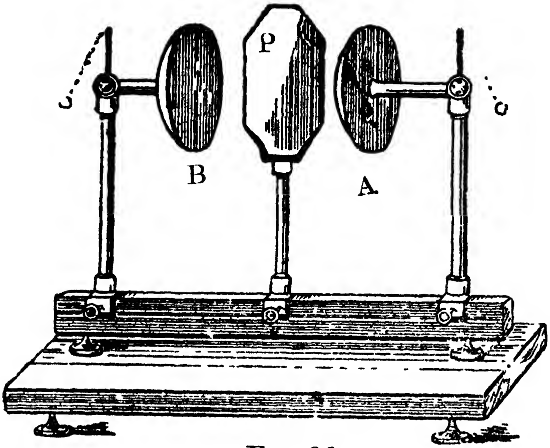


FIG. 26

arrangement would act as a condenser; because its capacity increases by the presence of the earth-connected conductor, as described above. One form of this kind of condenser is the plate condenser. This consists of two metal plates *A* and *B*, mounted on insulating stands, which can be moved by insulating handles These plates are separated by air, which can be replaced by glass or ebonite, etc. Suppose *A* is charged +*vely* by electrophorus. Now on bringing the two plates near together and touching the plate *B* with hand, the potential decreases This is shown either by the *pith balls* attached to *A* and *B* or by connecting a G. L. electroscope to the insulated plate by wire. Now if the two plates be separated, the pith balls are repelled, showing that the potential has increased. On bringing the two plates near together again, the potential falls and the pith balls hang, touching the stands. By substituting plates of ebonite, resin, shellac etc., the behaviour of different dielectrics can be studied with the aid of this instrument.

234. The Capacity of a Spherical Condenser. A spherical condenser consists of two concentric spheres of radii a and b cms. respectively. The inner sphere is insulated, the outer sphere is earth-connected and the dielectric is air. Suppose a charge Q is imparted to the inner sphere, this would raise its potential through $\frac{Q}{a}$. The charge Q on a would induce a charge $-Q$ on b and thus its potential will be $-\frac{Q}{b}$.

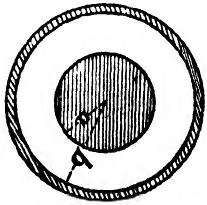


FIG. 26 (a)

Hence the resultant potential of the inner sphere

$$=\frac{Q}{a}-\frac{Q}{b}=Q\left(\frac{b-a}{ab}\right)$$

But Capacity $=\frac{Q}{V}=\frac{ab}{b-a}$.

If a and b differ by a very small amount: then $b=(a+x)$, where x is a very small length. The expression for capacity becomes $\frac{a(a+x)}{x}=\frac{a^2}{x}+a=\frac{a^2}{x}$ (very approximately), if x be very small as compared to a.

Hence $C=\frac{a^2}{x}=\frac{4\pi a^2}{4\pi x}=\frac{\text{Surface of the inner sphere}}{\text{thickness}}$.

When a becomes very large, the surface of the sphere may be assumed to become plane and thus the two spheres will act like parallel plate-condensers. Its capacity is evidently equal to, area of either plate divided by 4π times x, the distance between them; *i.e.* $C=\frac{area}{4\pi x}$, provided air is the intervening medium. If another dielectric of S.I.C. K is the medium, then

$$C=\frac{K.\ area}{4\pi x}.$$

235 Leyden jar*—A well-known form of condenser

* In principle, it is a form of parallel plate-condenser and its capacity is given by the expression $\frac{K.A}{4\pi d}$, where A is the area of the inner tin-foil, d the thickness of glass and K its S.I.C.

is the Leyden jar; it is so called, because it was first used by Van Mussch Cubrœk, a professor at Leyden. It consists, as shown in the figure, of a glass jar having an inner and an outer coating of tinfoil. These act as plates of the condenser. The mouth of the jar is closed by a lid of some good insulating material, through which passes a brass rod, having a knob outside and a loose piece of brass chain or spring at the lower end. This chain is in contact with the inner tinfoil. To charge the jar, we have simply to connect the knob to an electrified body and the outer coating to earth. In this case, the capacity is increased on account of two causes: (*i*) the outer coating acts as an earth-connected conductor and (*ii*) the glass intervening the two coatings has a higher S.I.C. than air.

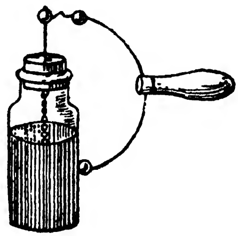


FIG. 27

When the inner coating is given a + *ve* charge, an equal amount of − *ve* charge is induced on the outer coating and the + *ve* from the outer coating flows to the earth. As these charges exert mutual attraction, they tend to be as near each other as possible; ***therefore, the charges come to reside on the inner and outer surfaces of the glass.*** That this is actually so, is shown by the following experiment:—

Take a Leyden jar with movable coatings, as shown in fig. 28. Charge the jar by connecting the inside with the prime conductor of a machine. Place it on an insulated plate. Lift the inner coating by an insulated handle and bring it near the gold-leaf electroscope; the leaves do not diverge, showing that there is no charge on the inner coating. Take out the glass jar and test the outer coating similarly; that too shows no sign of electrifi-

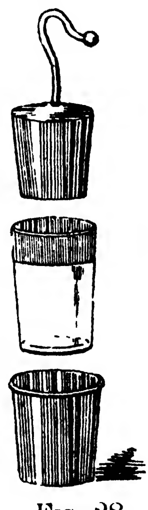


FIG. 28

cation. Finally bring the glass jar itself near the gold-leaf electroscope, notice that this also shows no sign of electrification. Put the jar again into the outer coating, replace the *inner coating* by an insulated handle and connect the two coatings by means of a discharger; a spark passes, which shows that the jar is charged as before, proving that the charge resides on the two surfaces of the glass jar. The reason why the jar shows no sign of electrification, when brought alone near the gold-leaf electroscope, is that equal amounts of opposite kinds of electricity reside on the two surfaces of the jar; and on account of their mutual attraction, they exert no influence on other conductors near them.

The question arises:—What then is the utility of the tinfoils, when the charge does not reside on them? The answer is, that tinfoils act simply as distributors and collectors of charge.

235. (*a*) **Residual Charge.** Charge a Leyden jar strongly and then discharge it. Wait for a *moment* and try to discharge it again: a small spark passes. The charge, which collects in the jar, after it is once discharged, and which gives rise to the second feeble spark, is called the *Residual charge*. This residual charge varies with the amount of the total charge and the nature and thickness of the dielectric. It is due to the gradual recovery of the dielectric from the strain, which is produced by the electric charges.

236. Potential energy of the charge. It has already been remarked that energy of electrification is due to the work, that has to be done in separating two oppositely-charged bodies. Now we have to find the magnitude of this energy or work. To do so, let us suppose, that a body A is charged with Q units of electricity, by bringing a very small amount of charge every time from infinity to that body; till the charge becomes Q, and let its potential then become V. Now in the beginning, the body is totally uncharged and hence its potential is zero; but at the end, its charge becomes Q and the potential V. Therefore during the

process of charging, the mean potential will be $\frac{V+zero}{2}=\frac{V}{2}$. Potential has been defined as the amount of work done in bringing a unit positive charge from infinity; thus in bringing one unit of charge, work equal to $\frac{V}{2}$ units must have been done, during the process of charging. To charge the body however, Q units of electricity have been brought. Therefore the work done must be

$$Q.\frac{V}{2}=\tfrac{1}{2}QV.$$

This then represents the potential energy of the charge. Now $Q=CV$, where C is the capacity of the body; hence the expression for energy may be written as

$$\tfrac{1}{2}CV^2 \text{ or } \tfrac{1}{2}\frac{Q^2}{C}.$$

237. Loss of energy on sharing charges. Let V_1 and V_2 be the potentials of two conductors of capacities C_1 and C_2 respectively. Their combined energies *before* sharing the charges will be

$$\tfrac{1}{2}C_1V_1^2+\tfrac{1}{2}C_2V_2^2 \quad \ldots\ldots \quad (i)$$

On sharing the charges, by connecting them with a long thin wire, their common potential becomes

$$\frac{C_1V_1+C_2V_2}{C_1+C_2}, \text{ i.e. } \frac{\text{Total charge}}{\text{Total capacity}}.$$

Their total energy, in the latter case, becomes

$$\frac{1}{2}(C_1+C_2)\left(\frac{C_1V_1+C_2V_2}{C_1+C_2}\right)^2 \quad (ii)$$

Therefore, loss of energy $=\frac{1}{2}C_1V_1^2+\frac{1}{2}C_2V_2^2$

$$-\frac{1}{2}\frac{(C_1V_1+C_2V_2)^2}{C_1+C_2}=\frac{\tfrac{1}{2}C_1C_2(V_2-V_1)^2}{C_1+C_2} \quad .(iii)$$

This is positive for all values of V_1 and V_2, except when $V_1=V_2$. Hence there is always a loss of energy, except when the potentials of the two conductors are the same, in which case there is no loss or gain.

Thus, whenever two bodies at different potentials are brought in electrical contact, a transference of electricity takes place; with the result that, some of the energy is dissipated away in the form of electric spark, though the total amount of charge remains the same.

238. Sometimes to increase the capacity, a large number of Leyden jars are connected *in parallel*: *i. e.* their inner coatings are connected together, and so also their outer coatings. The capacity of the combined condenser is equal to the sum of the capacities of the separate jars.

Another form of condenser, which is largely used in Induction coils, consists of a number of tinfoils, separated by paper, oiled with paraffin-wax. The alternate tinfoils are connected together, so as to form two plates of very large surface-area, as shown in fig. 29.

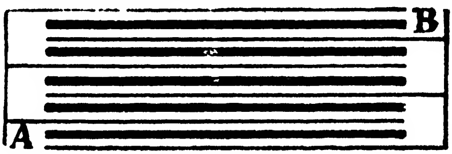


Fig. 29

SUMMARY

1. **Capacity** of a conductor is defined as the quantity of charge required to raise its potential through unity

2 Any conductor, the capacity of which is artificially increased, is known as a **condenser.**

3. The capacity of a sphere is equal to its radius, and of a simple air-condenser is equal to $\frac{A}{4\pi d}$.

4. **Dielectric constant** or the **Specific Inductive Capacity** of a substance is defined as the *ratio* of the force between two charged bodies, when air is the intervening medium, to the force which would be exerted, if the intervening medium were the given substance

5. **Leyden jar** is a form of condenser. In this case, the charge resides on the inner and outer surfaces of the glass jar and not on the inner and outer tinfoils.

6. If a strongly charged Leyden jar be once discharged and the inner and outer coatings be connected again by a

discharger, a feeble spark known as the **residual spark** is seen. This is due to what is known as residual charge.

7. Energy of charge $= \frac{1}{2}\ Q.V$.

EXAMPLES

1. A sphere of radius 4 cms. is charged with 20 *E.S.* units. Find the potential and the surface-density on the surface of the sphere.

$P=\frac{Q}{C}$, $\therefore$ the potential $=\frac{20}{4}=5$ units.

The area of the surface of a sphere $=4\pi r^2$ sq. cms. and the charge $=20$ units

$\therefore$ the surface density $=\frac{20}{4\pi r^2}=\frac{35}{352}$.

2. Define capacity of a condenser.

3. What do you understand by Specific Inductive Capacity or dielectric constant?

4. How would you show that in the case of a Leyden jar, the charge resides on the sides of the glass jar and not on tin-coverings?

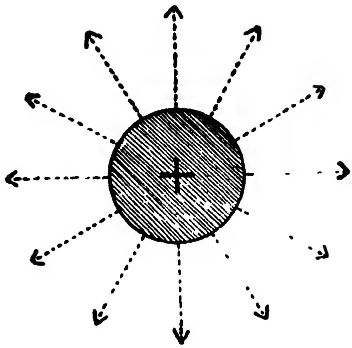
5. A sphere of radius 5 cms. is charged with 20 E. S. units of $+ve$ electricity; find its Potential. It is touched with another sphere of radius 3 cms. and having -5 E. S. units Calculate the common potential of the two spheres and the loss of energy during the process.

6. Two spheres of radii 5 and 7 cms. respectively, are separated by a distance of 20 cms. The potential of each is raised to 3 units. Calculate the force of repulsion, when air is the intervening medium and also when dielectric of S. I. C. $=8$ is the intervening medium.

CHAPTER VI

FIELD OF ELECTRIC FORCE

239. Field of Force :—Suppose a + *vely* charged body Fig. 30, is surrounded by air and a body *B* carrying a positive charge is placed at any point in its field; then on account of the force of repulsion between the two, the body *B*, if free to move, will move away from it along a line as shown in the figure. Similarly if the body *B* were situated at any other point, it would move along any one of the lines shown here. *Each of these lines is called a line of Force. It can be defined as a line, such that its direction at any point on it, through which it passes, gives the direction of the resultant electric force at that point. The whole space surrounding a charged body, in which the electrical force is exerted, is called a field of force due to the charge on that body.* Thus in the field due to a charged sphere, the lines of force are radial straight lines.



Field of force due to a charged sphere.

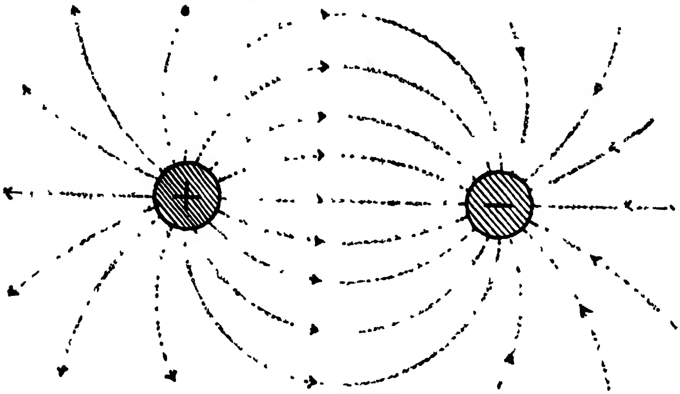
FIG. 30

The intensity of the field at any point is the magnitude of the force, which a unit positive charge experiences, when placed at that point.

A number of lines of force taken collectively is called a *tube of force*; and all the lines of force taken collectively, *which* take their origin from that portion of the surface of a conductor, which has a unit + *ve* charge, are known as a **unit tube of force**. The number

of the unit tubes starting from any charged body will thus be *numerically* equal to the amount of charge on it.

240. Properties of lines of force. The following



Lines of force due to two oppositely-charged spheres.

FIG. 31

properties are assigned to lines of force :—

(*i*) A line of force is conventionally supposed to take its origin from a +*ve* charge and end on a negative charge

(*ii*) The direction of a line of force is the direction in which a positively charged body would move.

(*iii*) The tubes of force tend to contract lengthwise like a stretched rubber tubing, and expand breadthwise.

Thus if a charged body be placed in a room, which contains no other conductor, then the lines of force start from the charged body and end on the walls and roof of the room: because a negative charge is induced on them. Now if a conductor be brought inside the room, negative electricity is induced on it; and a certain number of lines of force leave the roof and the walls and come to terminate over the conductor. The nearer the conductor is brought to the charged body, the greater is the number of lines of force, which terminate on it. Thus we see, that lines of force tend to contract

lengthwise and bulge out breadthwise. They represent a state of tension along their lengths and of pressure at right angles to them.

(*iv*) **No two lines of force can intersect.** Suppose two lines of force intersect each other at a point; then at that point, there would be two directions for the resultant force, which is impossible.

(*v*) A line of force cannot cut an *equipotential** surface more than once. For suppose a line of force cuts it more than once, then work must be done in moving a charge along a line of force; but no work is done in moving a charge from any one point of an equipotential surface to any other, whatever the path of motion may be. Hence no line of force can cut an equipotential surface more than once. From this it follows that no line of force can begin and end on the same conductor.

(*vi*) The lines of force are perpendicular to an equipotential surface. For if a line of force is not perpendicular to it, then there will be a certain component of the force acting along the surface of that conductor: but no work is done in moving a charge from any one point of an equipotential surface to another. Hence the lines of force must be perpendicular; because then they will have no component along the surface of the conductor, for a force has no component in a direction perpendicular to itself.

(*vii*) **No lines of force can exist within a hollow conductor, unless this surrounds other charged conductors.** Charges of opposite sign are always found at opposite ends of a line of force; but in the case of a hollow conductor charged only on the surface, no line of force can exist within the conductor. For, charges of the same sign will be found at both ends of a line, if such a one were imagined to exist. Thus no line of force can exist within a hollow conductor,

* An equipotential surface is that, on which all the points are at the same potential. There being no difference of potential, electricity remains at rest on its surface and cannot move from one point to any other.

and there must be the *same intensity within the conductor. Thus the potential inside a charged hollow conductor is uniform and equal to that of the conductor.*

SUMMARY

1. The space surrounding a charged body, where the forces of electric attraction or repulsion are perceptible, is known as a **Field of Force.**

2. **A line of force** is a line such that its direction, at any point through which it passes, gives the direction of electrostatic force.

3. **Intensity at any point** is the force acting on a unit $+ve$ charge.

4. **Unit tube of force** is a tube of force, which has a unit $+ve$ charge at one end.

5. The following are the peculiarities of the lines of force.—

(*a*) They are supposed to start from a $+ve$ charge.

(*b*) The direction of a line of force is the direction in which a $+ively$ charged body would move

(*c*) The lines of force tend to contract and repel each other

(*d*) No two lines can intersect.

(*e*) No line can start from and end on the same conductor.

(*f*) Lines of force are perpendicular to an equipotential surface.

(*g*) No line of force can exist inside a conductor.

EXAMPLES

1. What is a field of force?

2. Define intensity and find out the same at a point 30 cms. from a sphere having 20 units of charge

3. What is a unit tube of force? State the chief properties of the lines of force.

4. How would you show that the surface of a conductor is an equipotential surface?

5. 'Charge resides on the outer surface of a conductor.' How would you show that the inner and outer surfaces of a conductor are at the same potential?

CHAPTER VII

ATMOSPHERIC ELECTRICITY

241. It has been found by actual experiment, that the potential at any point in the atmosphere on a calm day is always higher than the potential of the earth. The electrification of the atmosphere varies not only with the hygrometric state of the atmosphere, but also with the season and hour of the day. The cause of this electrification is not known clearly as yet Some suggest that water, when it vaporizes, acquires a +*ve* charge : others say that electrification of the atmosphere arises from the friction of masses of air at different temperatures More recently it has been suggested that the electrification of the atmosphere is due to the ultra-violet rays of sunlight, which have the peculiar property of partially ionising the air

242 The Aurora. The Aurora Borealis, seen in the clear regions, is a form of electric discharge in the upper regions, of rarefied air. This is probably due to the difference of potential existing between the cold air of the Poles and the currents of warm air, which come from the Equator. Recent theory suggests that the phenomenon is due to streams of *electrons* discharged from the Sun, the paths of which are influenced by the earth's magnetic field

243 Thunder storms Thunder storms are probably the result of disruptive discharge, which takes place when two intensely charged clouds, having opposite kinds of charges, happen to come near each other The spark, accompanying such a discharge, is known as lightning: but nothing is as yet known, with certainty. This lightning assumes various forms, such as (*i*) **forked** (*ii*) **sheet** and (*iii*) **globular** lightning.

(*i*) *Fork lightning* usually takes a very irregular path with many ramifications. The path of the dis-

charge is always that of least resistance; and small specks of dust are enough to divert the passage of a spark. The flash is generally of an oscillatory character. The duration of the flash is always less than 1/10,000 of a second.

(*ii*) *Sheet lightning* is probably the reflection, from the surface of clouds, of distant fork lightning, which is hidden from view.

(*iii*) *Globular lightning* is very rarely seen and has been little understood as yet

EXAMINATION QUESTIONS X

1. Two small spheres are charged with 9 units of positive and 8 units of negative electricity respectively. Find the force between them, when they are placed 12 cms. apart.

$$f=\frac{qq'}{d^2} \quad \therefore f=\frac{-8\times 6}{12\times 12}=-\frac{1}{3} \text{ dynes}$$

Thus each sphere would attract the other with a force of $^1/_3$ dyne along the line joining them.

2. Two equally-charged spheres repel each other, when their centres are half a metre apart, with a force equal to the weight of 6 milligrams What is the charge on each in electrostatic units (*B. of E. 1892, London*).

$$f=\frac{qq'}{d^2}.$$

Weight of 6 in gms. is equal to $\frac{6}{1000}\times 981=5\cdot886$ dynes.

$$\therefore\ 5\cdot886=\frac{q^2}{(50)^2}, \text{ for } d=50 \text{ cms and } q=q'$$

$$\therefore\ q=(2500\times 5\cdot886)^{\frac{1}{2}}=121\cdot25 \text{ E. S units.}$$

3. A small brass ball is charged with 12 units of +ve electricity, and is then placed in contact with an equal and similar brass ball Find the force exerted between the balls, when they are separated by 6 cms.

After contact both shall have equal charges.

Thus the charge on each will be equal to 6 units.

$$\therefore\ f=\frac{6\times 6}{(6)^2}=1 \text{ dyne.}$$

That is each ball will repel the other with a force of one dyne.

4. If 100 units of work must be done in order to move an electric charge of 4 units from a place, where the potential is -10 to another place, where the potential is V. What is the value of V? (*B.* of *E.* 1898).

The work done will be $\frac{100}{4}=25$ units. when 1 unit of charge is moved.

$$V_a - V_b = W = 25$$
$$\therefore \quad V_a = V_b + 25 = -10 + 25.$$

5. A charge of 50 C G S. units raises the potential of a spherical conductor from 10 to 15 units. Find the radius of the conductor.

$$\text{Capacity } C = \frac{50}{15-10} = \frac{50}{5} = 10.$$

But since the capacity of a sphere is equal to its radius, therefore the radius $=10$ cms.

6. Two small spherical pith-balls, each one decigram in weight, are suspended from a point, by threads 50 cms. long, and are equally charged so as to repel each other to a distance of 20 cms. Find the charge on each ($g=980$).

7. A spherical conductor of 5 cms. radius and charged to a potential of 50 units is placed inside a sphere of radius 10 cms. Find the potential of the larger sphere.

The whole of the charge shall be transferred in this case.

Therefore $5\times50=10\times V$, where V=Potential of the larger sphere, *i. e.* $V=25$ units.

8. Two insulated metal balls of radii 3 and 5 cms. respectively, are connected by a very fine wire and charged. On testing, it is found that the smaller one has 9 units of charge. What is the total charge?

The Potential of the smaller ball would be $\frac{9}{3}=3$.

This is also the potential of the bigger ball, the charge on which would be $5\times3=15$

$\therefore$ the total charge$=9+15=24$ units.

9. Two spheres of radii 3 and 1 cms. respectively are placed far apart and a charge of 15 units is given to the larger sphere. What charge must be given to the smaller one, in order that the larger sphere may neither gain nor lose charge, when the two are connected by a wire?

(*B.* of *E.* 1903).

The potential of the bigger sphere would be $\frac{15}{3}=5$.

In order that there may be no transference, the potential of the smaller sphere should also be the same as that of

the bigger one *i.e.* 5; but its radius or the capacity is unity. Therefore $5 \times 1 = 5$ units of charge must be given to it.

10 To what potential must we charge an insulated sphere of 15 cms. radius, so that its surface density may be 2?

Surface density $= \frac{Q}{A}$, where A is the surface area of the sphere.

$$\therefore \quad 2 = \frac{Q}{4\pi (5)^2}. \qquad \therefore \quad Q = 200\pi \text{ units.}$$

$$\text{But potential} = \frac{Q}{v} = \frac{Q}{r} = \frac{Q}{5} = \frac{200\pi}{5}$$

$$= 40\pi = 125\ 7.$$

11. Equal quantities of electricity are communicated to two insulated metallic spheres, whose radii are as 5 1. What are their relative potentials? The spheres are then put in conducting communication, by means of a long thin wire, which is afterwards removed. What are the relative surface-densities of the two spheres now? (*Punjab, 1914—15*).

12. A Leyden jar has a diameter of 15 cms., height of tinfoils 18 cms. and thickness of glass 2 5 mms. If the value of specific inductive capacity of glass be 6·4, find the capacity of the jar. (*P. U. 1928*).

13 Eight equal globules of water, having equal and similar charges, unite to form a larger globule How will the electric capacity and potential change? (*P. U. 1922*)

14 Two conductors of capacities 10 and 15 respectively, are connected by a fine wire and a charge of 1000 units is divided between them Find the potential of either conductor and the charge on each. (*C. U. 1920*).

15. Two Leyden jars, whose capacities are 1 and 2, receive charges 3 and 4 respectively. Compare their combined electric energies, before and after their knobs have been in contact. (*B. of E 1898*).

MAGNETISM

CHAPTER I

FUNDAMENTAL PHENOMENA

244. Natural Magnets. The name magnet was first applied by the ancients to certain hard black stones found in Magnesia, in Asia Minor. These stones, now known by the name of Magnetite ($Fe_3 O_4$), were found to possess the property of attracting pieces of iron and steel, and besides this, there are reasons to believe that the Chinese, as far back as 2600 B. C., knew that pieces of magnetite possessed the property of pointing North and South, when suspended so as to move freely in a horizontal plane How and when they discovered the peculiar property of this stone, is not known exactly, but we must reckon this discovery, as a definite starting point, in our knowledge of magnetism. The earliest record of this knowledge in Europe, is found in the writings of a Norwegian (born in 1068), in which magnetite, having the above properties, is called as the **Lodestone** or leading stone

If a piece of lodestone be examined, it is found that its power of attracting pieces of iron or steel, is most marked at the two end-regions called the **poles** of the magnet, while the portion of the magnet lying between the two poles does not attract iron filings so strongly. It is also found that half way between the two poles, there is no attraction at all. This region is called the **equator** of the magnet An imaginary line joining the two poles of a magnet is called the **magnetic axis**

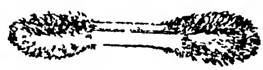


FIG 1

245. Two kinds of magnetic poles Take two magnets and suspend them in stirrups at a distance from each other. It is found that each of them points in the north and south directions. Mark the poles which point towards the North as N, and the others as S.

Disturb the magnets and see that the poles marked N again point in the northern direction. These poles are called the **North-seeking Poles** of the two magnets, while the others are called the **South-seeking poles.** Now bring the N-seeking pole of one magnet near to the N-seeking pole of the other, repulsion takes place. Similarly bring the S-seeking pole of one near to the S-seeking pole of the other, notice the repulsion again. Thus we see *similar poles repel one another.*

If however, the N-seeking pole of one be brought near the S-seeking pole of the other, attraction results; *i e dissimilar poles attract each other* These facts are similar to the law of electrostatic attraction and repulsion; and are known by the name *of the 1st law of magnetism*, which states that "*like poles repel, while unlike poles attract each other.*

246. Paramagnetic and Diamagnetic. Besides iron and steel there are other substances, such as nickel and cobalt, which are feebly attracted by a magnet; these substances are called *paramagnetic* substances While there are others, such as copper and bismuth, which are feebly repelled by a magnet: these are given the name of *diamagnetic* substances. It is interesting to note here, that even iron and steel cease to exhibit any magnetic property at a dull-red heat The temperature at which this occurs is called the *critical temperature*

247. Artificial Magnets. Natural magnets are not used in ordinary work; artificial steel magnets are generally used for that purpose These are of various forms, bar magnet and horse-shoe magnet are the commonest.

248. Methods of magnetisation. (*i*) *Single touch.* Lay the needle to be magnetised, flat on the table with its ends fixed by soft wax. Draw one pole (say the N-seeking pole) of a magnet along the whole length of the needle from A to B several times. Test the needle and see that it becomes magnetised.

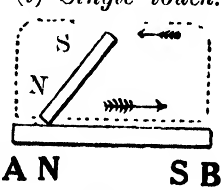


FIG. 2

The end B where the magnetising magnet leaves the needle becomes the S-seeking pole. In fact, it is found that the *last-touched point of the needle always becomes a pole of opposite kind to that used to rub it.* If the piece to be magnetised be thick, both sides should be similarly treated. This method however, is not suitable for strong magnetisation.

(*ii*) *Divided touch.* The bar or needle to be magnetised is laid flat on a table and its ends are fixed with soft wax as before. Dissimilar poles of two magnets are placed at its middle point and are then drawn apart towards the ends. The process is repeated several times. The bar becomes strongly magnetised, if instead of being held by soft wax, its ends are made to rest on the poles of two other magnets; such that the end, which is to be last touched by a N-seeking pole, rests on a N-seeking pole and that to be last touched by a S-seeking pole, rests on a S-seeking pole.

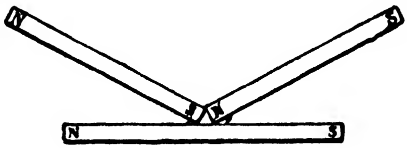


Fig. 3

(*iii*) *Double touch.* The bar or needle to be magnetised is laid on the table as before. Dis-similar poles of two magnets are then placed at its middle point, with a piece of wood in between them *i.e.* the dis-similar poles. Both together are moved backwards and forwards several times, along the bar to be magnetised and are, at the end, made to leave in the middle.

(*iv*) By none of the above methods can a steel bar be magnetised beyond a certain limit; hence to get very strong magnetisation, none of the above methods is used and the method of magnetising by an electric current is resorted to, for the

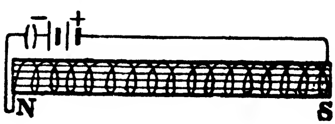


Fig. 4

purpose. The bar or needle to be magnetised is placed inside a thin-walled glass or cardboard tubing, round which cotton-covered copper wire is wound; and a strong current is made to flow through that wire for a short time. On stopping the current, the bar or needle is found to have acquired magnetism. The end, where the current runs in a *clock-wise* direction acquires the polarity of a *S-seeking* pole. The degree of magnetisation, acquired by any given piece of steel, depends upon: (*i*) the magnetising force and (*ii*) the quality of steel. Magnetisation is increased by increasing the magnetising force and its duration. But ultimately a limit is reached, beyond which magnetisation cannot be increased. The steel is then said to be **saturated**; *and a piece of steel or iron is magnetically saturated, when it fails to acquire a higher degree of magnetisation, however much the magnetising force may be increased.*

249. Consequent poles. Sometimes a magnet may be found which has similar poles at the ends. This is generally due to irregular magnetisation. A magnet showing this peculiarity, will always be found to have one or more poles of opposite kinds, somewhere along its length. These additional poles are called *consequent poles* and can be easily found out by dipping the magnet in iron filings. The consequent poles can be imitated, by placing two magnets with their similar poles together, as shown in fig. 5. Here *NN* is a consequent pole.

To get a consequent pole, take a needle and magnetise one-half of it by single touch; so that one end becomes a *S.S.* pole, and the middle point a *N.S.* pole. Magnetise the other half; such that the other end also becomes a *S.S.* pole and the middle point, a *N.S.* pole. Then this needle shall have a *N.S.* consequent pole at its centre.

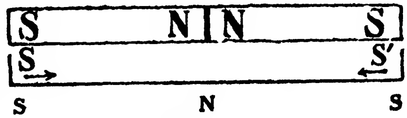
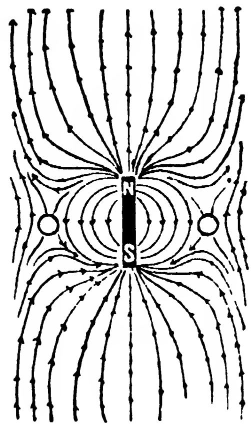


Fig. 5

Magnetic field. Magnetic field in magnetism is similar to electric field in electrostatics; and is the whole

space round about a magnet, in which forces due to the given magnet can be detected.

A magnetic field may be mapped out by sprinkling iron filings lightly on a sheet of thin glass, placed over a magnet. The filings arrange themselves in continuous curves, which indicate the direction of the magnetic force at every point. These continuous lines are called the lines of force; and a line of force is defined as a curve in a magnetic field, such that the direction of the tangent to it, at any point, denotes the direction of magnetic force at that point.



Field due to a bar magnet, *N*-pole pointing north.
Fig. 6

Hence an isolated *N*-seeking pole, when placed in a magnetic field, would always tend to move along a line of force.

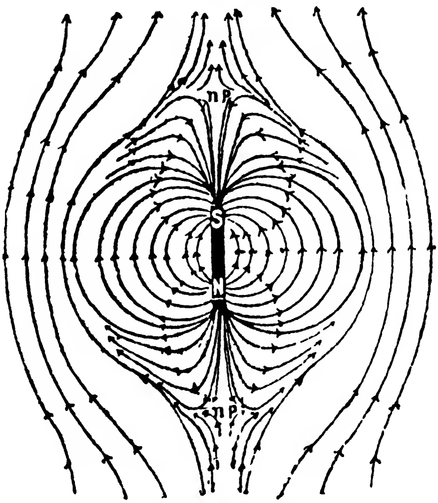
1. *The lines of force are supposed to start from the N-seeking pole and end on the S-seeking pole.*

2. *Lines of force cannot cross.* Because if two lines of force were to intersect, then a magnetic needle placed there should point simultaneously in two directions, which is an impossibility; hence no two lines of force can intersect each other.

3. *Lines of force represent a state of tension along their lengths.* The lines of force run from the *N*-seeking pole to the *S*-seeking pole and represent a force, tending to draw two poles near together; thus the lines of force represent a state of tension along their lengths.

4. *Lines of force represent a state of pressure, sideways.* Lines of force may be assumed to be similar to a stretched elastic string. When such a string is stretched, it grows longer but thinner in cross-section. It has a tendency therefore, not only to shorten length-wise but also to bulge out sideways.

Another method of mapping out a field of force is by a small compass-needle, enclosed in a small box with glass faces. Place a magnet over a sheet of paper and bring the small compass near it. The needle of this small compass would set itself in the direction of the magnetic force at that place. Make small dots on the paper, opposite each end of the needle. Move the compass from its position and place it so that the S-seeking pole of the compass-needle coincides with the previous dot. Mark again a dot opposite the N-seeking pole of the compass-needle. Move the compass again, so as to make the S*-seeking pole of the compass-needle coincide with the previous dot and mark out another dot opposite the N-seeking pole of the compass-needle. Repeat this till a series of dots are obtained. Join them by a line, this indicates a line of force. Drawing in this manner, many lines of force can be obtained, which together constitute a field of force. Various lines can be drawn by starting with the needle from different points in the field.



Field due to a bar magnet, N-pole pointing South.
FIG. 7

A glance over fig. 7 shows that there are two regions, marked *n. p.*, in which the small compass-needle will come to rest in all positions; these regions

* When the needle crosses the middle portion of the magnet, the N-seeking pole, instead of the S-seeking pole, would coincide with the dot.

are termed **neutral points**. In these places, the force due to the magnet and that due to the earth, exactly counter-balance.

It is very easy to determine the pole-strength of a given magnet, when the position of a neutral point in its field is known. The simplest case is that of a magnet placed in the magnetic meridian, with its S-seeking pole pointing towards the north, as shown in fig. 7. In this case, if d be the distance of the neutral point from the S. pole of a magnet of length l; then the intensity H due to the earth *plus* $\frac{m}{(d+l)^2}$, the intensity due to the north-seeking pole, together should be equal to $\frac{m}{(d-l)^2}$, the intensity due to the S-seeking pole, where m is the pole-strength of the given magnet. (Chapter II).

Thus $$H+\frac{m}{(d+l)^2}=\frac{m}{(d-l)^2};$$
when H is given and distances d and l are measured, the value of m becomes known.

Uniform field. *In a magnetic field, the area where the lines of force all run in the same direction and are parallel to each other, is said to be an area of uniform field.* The earth's field, in small areas over its surface, may be assumed to be uniform.

251. Magnetic Induction Bring a steel nail near to the pole of a bar magnet. See that the former remains attached to the magnet. Bring another nail of steel near to the lower end of the steel nail, already clinging to the magnet: notice that the second nail also remains attached. Similarly try with the third nail Keep these three attached to the magnet for a few minutes, and then gently detach the first nail from the magnet; it is found that the second and the third remain attached to the first. Now separate the nails. Test that they are magnets. It is further

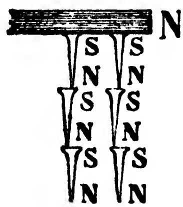


FIG. 8

found that dissimilar poles of the two pieces are in contact.

Place a soft iron piece on a wooden table, and sprinkle iron filings near this piece. Bring a permanent bar magnet near to the soft iron piece; see that the piece at once attracts the filings. Remove the permanent magnet to a distance and notice that the filings do not remain attached to the iron piece.

This experiment shows that a piece of soft iron behaves as a magnet, so long as a magnet is present in its vicinity; and it ceases to be a magnet when the magnet is removed. In the first experiment however, it is seen that pieces of steel retain their magnetism, even after the bar magnet is removed This power of steel, of retaining magnetism, is called *retentivity*.

The above effects are due to what is called **Magnetic Induction**; it may be defined as the process by which steel becomes permanently magnetised and soft iron only temporarily so, when placed in contact with or near to a magnet. A magnet attracts a magnetic substance by inductive action.

Induction in all cases must precede attraction: for when a magnetic pole is held near to a piece of iron, pole of opposite kind is induced in the nearer part of the iron and is attracted by the inducing pole.

This inductive action in magnetism is very similar to one, we have studied in electrostatics. The analogy goes even still further, for the iron piece magnetised by induction is found to have two poles; the end *nearest* to the pole of the inducing magnet being of opposite kind, while the *farthest* end of the bar is of the same kind as the inducing pole.

This analogy, between the electrostatic and magnetic phenomena of attraction, repulsion and induction, is *so close*, that students are in some danger of confusing the two. But magnetism is quite unlike the stationary charge, though it resembles a moving one. The following are the chief points of difference between magnetism and statical electricity :—

(*i*) It is possible to charge either + *vely* or − *vely*,

while it is impossible to give one kind of magnetism to anybody.

(*ii*) The phenomenon of electricity is more general, while magnetism must necessarily be confined to a few paramagnetic substances.

(*iii*) An isolated +*ve* or −*ve* charge is possible, while an isolated pole of either kind is an impossibility.

(*iv*) Electricity is lost by touching it with some conductor, while magnetism is not thus lost.

252. Susceptibility. The amount of induced magnetism depends both upon (*i*) the strength of the field and (*ii*) the nature of iron or steel. It is found however, that for a given strength of field, the induced polarity set up in soft iron is always stronger than that set up in hard steel. This is expressed by saying that the susceptibility of soft iron is greater than that of hard steel.

253. Coercivity—If steel and soft iron pieces be magnetised to the same strength, and if both be subjected to such a magnetising force, as tends to reverse their polarity ; then it is found that soft iron loses nearly all its polarity, while steel is affected only slightly. This property of steel of resisting demagnetising force is termed *coercivity*.

254. Theories of magnetism—Various theories known as one-fluid and two-fluids' theories were propounded in early times ; but they are all useless, because it is certain that magnetism is not a fluid at all. The modern theory propounded by Ewing is the one held in credit at the present day. According to this theory, the molecules of iron are supposed to be permanent magnets; and in an unmagnetised state, they are supposed to be arranged in groups in such a manner, that they produce no external field. The magnetisation consists in arranging them so that all the *N*-seeking poles point in one direction and the *S*-seeking poles in the opposite direction. Thus according to this theory, magnetisation consists simply in the arrangement of molecules. This view is supported by the following experiment :—

Take a test tube. Fill it with iron filings and close

its mouth with a cork. Magnetise it by the *divided touch* method and see that the tube of iron filings behaves as a magnet. Now on shaking the iron filings, it is seen that the tube at once loses its magnetism. This fact clearly shows that magnetism is closely related to the arrangement of the molecules of iron.

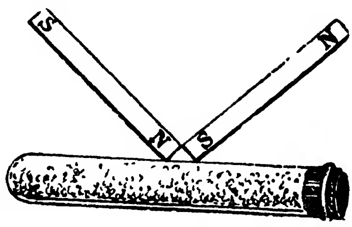


FIG. 9

Recently it has been postulated that magnetic phenomena are due to the rotation of electrons.

255. Demagnetisation. From the above consideration, it is clear that anything, which changes the arrangement of molecules must destroy its magnetism. Thus when a magnet is roughly used or is allowed to fall, it loses a part of its magnetism and becomes a feeble magnet. When a magnet is heated, its molecules acquire a rapid motion; then too, its magnetism is lost.

Self-demagnetisation. Besides these artificial methods of demagnetisation, there is a natural process by which a magnet tends to demagnetise itself; this is called *self-demagnetisation.* In a bar magnet, the attraction between its two dis-similar poles and also between one pole and the *constituent* molecular magnets of the magnet, tends to draw the two poles near together and destroy the arrangement of molecules. This destroying force would be great if the magnet be short, and less, if the magnet be long. If the magnet be bent round in the form of a horse-shoe, the attractive force acts mostly through the air-gap between the ends; and so the demagnetising force is lessened. In the case of permanent magnets, this self-demagnetisation is prevented by the use of *keepers*, which are pieces of soft iron put across the poles, when magnets are not in use. Two bar magnets with

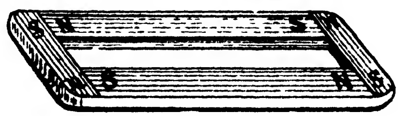


FIG. 10

dis-similar poles pointing in the same direction are put near each other separated by a wooden strip and two soft iron pieces are placed near their ends These soft iron pieces acquire temporary magnetism by induction and help to retain the magnetism of the permanent magnets, by tending to keep the magnetic poles at the ends of the bar

256. Laws of magnetic attraction and repulsion. 3 '*Like poles repel and unlike poles attract,*' has already been described The second law, which gives the magnitude of the force of attraction or repulsion is "*that the force of attraction or repulsion between two magnetic poles varies directly as the product of the two pole-strengths and inversely as the square of the distance between them*" Thus if m and m' be the pole-strengths of two N-seeking poles placed at a distance of d cms from each other, then the force of repulsion F would vary as $\frac{mm'}{d^2}$ so that $F \propto \frac{mm'}{d^2}$.

Or $F = \frac{1}{\mu} \cdot \frac{mm'}{d^2}$, where μ is a constant for any given medium, but is different for different media The value of μ for air is assumed to be unity

Thus the above expression, when air is the intervening medium can be written as

$$F = \frac{mm'}{d^2}$$

Now suppose $d=1$ cm., $F=1$ dyne and $m=m'$; then each shall be equal to unity. This gives us a definition of **unit pole** It is defined **as** *a pole, which when placed in air, at a distance of 1 cm. from an equal and similar pole, would repel it with a force of one dyne.*

257 Intensity of a Field of Force. –A magnetic pole, placed in a magnetic field, would experience a force and *the measure of the force experienced by an isolated unit north-seeking pole, placed at any point in a field of force, is called the intensity of the field at that point.* If the intensity is the same at all points, the field is

uniform. The force acting on a pole of strength m, when placed in a field of intensity f, would be equal to mf; and a magnetic field has unit intensity, when a unit magnetic pole situated in the field is acted on by a unit force.

258. Potential—The idea of potential in magnetic field corresponds exactly to the idea of potential in an electrostatic field. When a magnetic pole moves along a magnetic line of force, work must be done. If a unit N-seeking pole were to move from infinity to a point P in a magnetic field, then the measure of the total work done, in moving the unit N-seeking pole against the magnetic forces, gives the measure of potential at the point P. The difference of potential between two points P and Q is defined as *the amount of work required to move a unit N-seeking pole from P to Q*. The expression for potential at any point, distant d cms. from a pole-strength of m units, can be proved to be $\frac{m}{d}$; just as $\frac{q}{d}$ has been proved to be the expression for potential in electrostatics.

259. Magnetic screens.—Place a soft iron piece in a magnetic field; it is seen that the magnetic lines of force tend to travel through the iron, and the space around this small iron piece is run by fewer lines of force than when the magnet is removed. This is said to be due to the higher susceptibility of iron.

When a soft iron piece, placed in a magnetic field, causes the intensity of the field to diminish at any neighbouring point; it is said to screen that point magnetically. When a thick piece of soft iron is placed near the pole of a bar-magnet, as shown in fig. 11, many lines of force traverse the

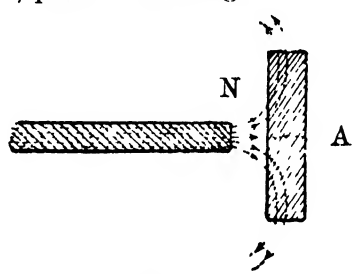


Fig. 11

iron from its centre towards its either end; while

only a few lines pass to the other side of the soft iron piece and the effect on a magnetic needle placed on the side *A* of the iron piece is very much diminished. To get the point *A* perfectly screened from the effect of the magnet, it should be surrounded by a hollow sphere of thick soft iron, as shown in Fig. 12. The space within the sphere may be said to be absolutely free from lines of force, because all of them would pass through the soft iron. This screening principle is applied, whenever a space is required to be perfectly free from magnetic forces.

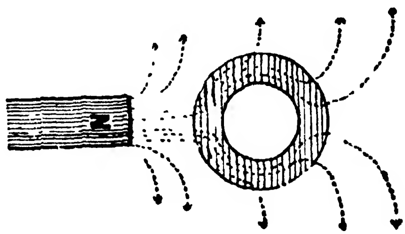


FIG. 12

SUMMARY

1. **Lodestone** or leading stone is the name applied to natural magnets, which are found in Asia Minor and other countries. Their composition is Fe_3O_4.

2 The ends of a magnet, where the power of attracting iron pieces is most marked, are called the **poles.** There are two poles (*i*) *N*-seeking pole and (*ii*) *S*-seeking pole.

3. The pole, which points towards the North, is called the **North-seeking** pole or briefly as *N*-pole. The pole, which points towards the south is called the **S-seeking pole** or *S*-pole.

4. Those substances, which are attracted by a magnet are called **paramagnetic** ; while those which are repelled are called **diamagnetic.**

5. Artificial magnets are formed by magnetising a steel bar by any one of the following methods. (*i*) Single touch, (*ii*) Double touch, (*iii*) Divided touch and (*iv*) by an electric current.

6 **Consequent poles** are the poles, which are found along the length of a magnet, in addition to the two poles at its ends.

7. **Magnetic field** is the space around a magnet, where the magnetic forces of the given magnet are perceptible.

8. **Neutral Point** is a point in a magnetic field, where the resultant magnetic intensity is zero and a small compass

needle set there will point in any direction.

9. **Lines of force** in a magnetic field are supposed to start from a N-seeking pole and end on a S-seeking pole. They have a tendency to shorten lengthwise and do not cross each other.

10 **Magnetic Induction** is the phenomenon, by which a piece of iron acquires magnetic properties, when placed near to a magnet

11 "Similar poles repel, while dis-similar poles attract each other." This is known as the **1st law of magnetism.** The **2nd law** is that the force of attraction or repulsion between two magnetic poles is directly proportional to the product of the two pole-strengths and inversely as the square of the distance between them

$F \propto \frac{mm'}{d^2}$ where m and m' are the respective pole-strengths and d the distance between them.

$F = \frac{mm'}{d^2}$, when air is the intervening medium

12. **Unit pole** is a pole, which when placed at a unit distance from an equal and similar pole, would repel it with a force of one dyne.

13 **Intensity** at any place due to a pole is the force acting on a unit N-seeking pole at that place

14. **Potential** at any point is the amount of work required to bring a unit N-seeking pole from infinity to that point

15 A magnetic needle surrounded by a soft iron cage is unaffected by external magnetic forces. This action is called **magnetic screening**

EXAMPLES

1. Describe the chief properties of a magnet and say how you would demonstrate them experimentally.

2. Define—pole, axis, pole-strength, intensity and potential

3. What do you understand by magnetic induction?

4 Describe the various methods of producing artificial magnets.

5 What are the laws of magnetic force?

6 What is a field of force and what are the properties of lines of force?

7 Define a unit pole. What must be the distance between two poles of 20 and 9 units respectively, so that the force between them shall be equal to the weight of half a gram ($g=981$)?

CHAPTER II

MAGNETIC MEASUREMENTS

260. Action of a magnet in a uniform magnetic field. Suppose a magnet is placed in a uniform magnetic field, so as to be free to move in a horizontal plane It is seen that it tends to set itself parallel to the lines of force, and if disturbed from its position of rest, it oscillates and again comes to rest in that very position *Why is it so?* The reason is that when the magnet is not parallel to the lines of force, each pole of the suspended magnet is acted upon by a force, which tends to bring it parallel to the lines of force.

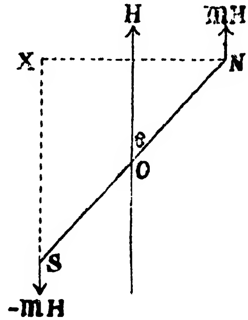


FIG. 13

Let NS fig 43 be a magnet with pole-strength equal to m, and let it be suspended in a uniform field of intensity H. Then the North-seeking pole N, would experience a force $m \times H$ in the direction shown in the figure; and similarly the South-seeking pole S would experience an equal force in the opposite direction. These two forces acting on the N and S-seeking poles respectively of the magnet are equal and parallel, and act in opposite directions, hence they constitute a couple, *the effect of which is always to turn it* This turning tendency of the couple is measured by the moment of the couple, which is equal to one of the two forces multiplied by the perpendicular distance between the two forces Thus the moment of the couple acting

upon the magnet $N.S.$, in the position shown in the figure, to bring it parallel to the lines of force, is equal to $mH \times NX$. But if SN the length of the magnet be denoted by l, we have $NX = l \sin\theta$, where θ is the angle made by the magnet with the direction of the magnetic lines of force. Hence the moment of the couple is equal to $mHl \sin\theta$. Now when the magnet is perpendicular to the lines of force, $\theta = 90$ and $\sin\theta = 1$; therefore the moment of the couple would be mHl. Further suppose H=unity, *i. e.* the field is of unit strength, then the moment of the couple would be ml. This is called the **magnetic moment** and is denoted by M. It is defined as *the moment of the couple acting on a magnet, when it is placed at right angles to a uniform field of unit strength and is equal to the product of pole-strength and the length of its axis.*

261. Intensity of Magnetisation. The intensity of magnetisation is defined as the pole-strength per unit area of the cross-section of a magnet. It can be obtained by dividing: (*i*) the pole-strength by the cross-section, or (*ii*) the magnetic moment by the volume of a magnet; for magnetic moment$= m \times l$ and this divided by $l \times a$ (the volume)$= \frac{m}{a}$, where a=area of cross-section.

262. Oscillation of a magnet in a uniform field. We have seen above that, when a freely suspended magnet is deflected from its position of rest, it experiences a couple tending to take it back to its position of rest. As a result of this couple, the magnet oscillates to and fro. This oscillation of the magnet is approximately isochronous for very small vibrations; hence the time of one complete vibration can be shown, both practically and theoretically, to be given by the equation, $T = 2\pi\sqrt{\frac{K}{MH}}$, where M is the magnetic moment of the magnet, H the intensity of the

earth's field and K is a quantity, called the *moment of inertia of the magnet.

From the formula, $T=2\pi\sqrt{\frac{K}{MH}}$, we see that if a magnet makes n oscillations in t seconds, then the time of one oscillation is $\frac{t}{n}$.

$$\text{Thus } \frac{t}{n}=2\pi\sqrt{\frac{K}{MH}}$$

$$\therefore \quad \frac{t^2}{n^2}=\frac{4\pi^2 K}{MH}$$

$$\text{or} \quad H=\frac{4\pi^2 K}{t^2 M}n^2.,\ \textit{i.e. } H \text{ varies as } n^2.$$

Thus the intensity of a field varies directly as *the square of the number of oscillations, made by a magnet in a given time.* Hence if the same magnet be made to oscillate in two different magnetic fields of intensities H_1 and H_2; it is quite clear that the intensities of the fields would be directly proportional to the squares of the numbers of oscillations, performed by the magnet in each field in the same time. Thus if N_1 and N_2 be the numbers of oscillations, made in fields of intensities H_1 and H_2, in the same time, we have $\frac{H_1}{H_2}=\frac{N_1^2}{N_2^2}$.

For example if in one field the magnet makes five oscillations and in the other ten oscillations per minute;

---

*Moment of inertia is defined as the sum of the products of the masses of various molecules into the squares of the distances of those molecules, from the axis about which moment of inertia is required; and in the language of Mathematics, it is denoted as Σmr^2.

For a cylindrical magnet of length l cms., radius r cms. and mass W grams, $K=W\left\{\frac{l^2}{12}+\frac{r^2}{4}\right\}$, and for a rectangular magnet of length l cms., breadth a cms. and mass W grams, $K=W\left\{\frac{l^2+a^2}{12}\right\}$.

then the intensity of the first field is to that of the second as $5^2 \cdot 10^2$, that is 25:100 or 1·4.

262. (*a*) This oscillation-method of comparing intensities is utilized in finding the relative quantities of free magnetism, at different points along a bar magnet. The magnet is set up vertically in such a position that the direction of a small compass-needle is not affected by it and the oscillations of this needle are counted at various points along its length Afterwards the number of oscillations, which the needle makes under the earth's influence only, is counted Then the number obtained by subtracting the square of this latter number of oscillations, which the needle makes under the earth's influence only, from the square of the number of oscillations, which it makes at any point along its length, due to both (*i. e.* the earth and the magnet), denotes proportionately the amount of free magnetism at that point.

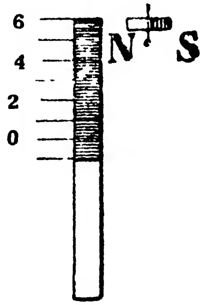


FIG 14

For instance, if a compass-needle makes 3 oscillations in one minute under the earth's influence only and 14 just near the pole of a vertical magnet, then the amount of free magnetism at the pole would be proportional to $14^2-3^2=187$. If at another point it makes 12 oscillations. then the amount of free magnetism at that point would be $12^2-3^2=135$; and so on. A graph may be plotted out in this way, as shown in fig. 15, to represent proportionately the amount of free magnetism along the whole length of the magnet. In this case, the heights of dotted lines represent the amounts of free magnetism,

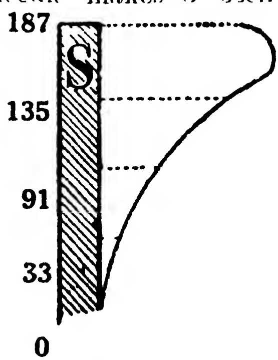


FIG. 15

distributed along various points of the length of the given magnet.

263. Magnetic Intensity due to a bar magnet at a point along its axis.

Let N. S. be a bar magnet, m its pole-strength $2l$ its length, O its centre and P a point on its axis at a distance d cms. from O.

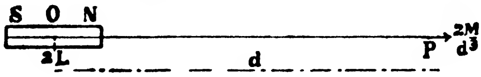


FIG. 16

Suppose an isolated unit N-seeking pole be at the point P; then a force $\frac{m \times 1}{NP^2}$ would act on it, tending to move it away from the magnet, in the direction OP; and a force $\frac{m \times 1}{SP^2}$ would act on it, tending to move it towards the magnet, in the direction PO.

The resultant force acting on an isolated unit N-seeking pole at the point P would therefore be $\frac{m}{NP^2} - \frac{m}{SP^2}$; and it would act in the direction OP, because $\frac{m}{NP^2}$ is greater than $\frac{m}{SP^2}$. Now $NP = d - l$ and $SP = d + l$

$$\therefore \frac{m}{NP^2} - \frac{m}{SP^2} = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}$$

$$= \frac{m[(d+l)^2 - (d-l)^2]}{(d^2-l^2)^2}$$

$$= \frac{4mdl}{(d^2-l^2)^2}$$

$$= \frac{2 \times 2l \times d \times m}{d^4 \left\{1 - \frac{l^2}{d^2}\right\}^2}.$$

If d is greater in comparison to l, then $\frac{l^2}{d^2}$ would be very small and may be neglected. The above expression

may be written as $\frac{2Md}{d^4}$

$$=\frac{2M}{d^3},$$

where M is the magnetic moment of the bar magnet.

264. Intensity of the magnetic field, due to a bar magnet, at a point on a line drawn through the centre and perpendicular to the axis of the magnet.

Let N. S. represent a magnet, m its pole-strength, $2l$ its length, O its centre, and P a unit N-seeking pole on a line drawn perpendicular to the axis from O, the centre of the magnet.

The force due to the N-seeking pole would be $\frac{m \times 1}{NP^2}$, in the direction NP and the force due to the S-seeking pole would be $\frac{m \times 1}{SP^2}$, in the direction PS.

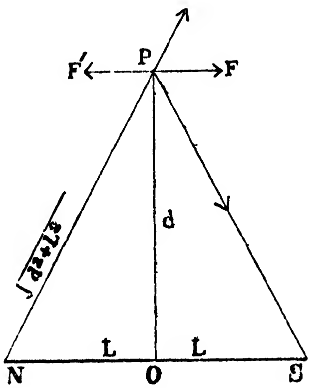


FIG. 17

But NP^2 and SP^2 are each equal to (l^2+d^2), because the $\triangle\, PON$ is a rt.-angled one. Therefore the force, acting on a unit N-seeking pole at the point P, would be the resultant of the two forces, each of which is equal to $\frac{m}{l^2+d^2}$. These two forces can be represented in magnitude and direction by the sides NP and PS of the triangle NPS; therefore their resultant F would be represented in magnitude and direction by NS, the third side of the triangle NPS, taken in the reverse direction, (by the principle of the triangle of forces). It would act in the direction PF parallel to NS at the point P. To get its magnitude, we have as before, $\frac{NS}{NP} = R \Big/ \frac{m}{NP^2}$.

$$\therefore\ R=\frac{m\times NS}{NP^3}=\frac{M}{NP^3}$$

$$=\frac{M}{(l^2+d^2)^{\frac{3}{2}}},\ \text{for } NP^2=(l^2+d^2),$$

$$=\frac{M}{d^3\left\{1+\frac{l^2}{d^2}\right\}^{\frac{3}{2}}}=\frac{M}{d^3};$$

because when l is small as compared to d, l^2/d^2 can be neglected.

Therefore magnetic intensity, due to a bar magnet, along an equatorial line is half of that along its axis at the same distance.

265. The Tangent Law. Suppose a magnetic needle is suspended at its centre of gravity in a field of intensity H. This sets itself in the position NS parallel to the lines of force. Now if another uniform field of intensity F be produced in a direction, at right angles to the first, either by placing a magnet perpendicular to the direction of H or by an electric current circulating in a coil; it is seen that the magnetic

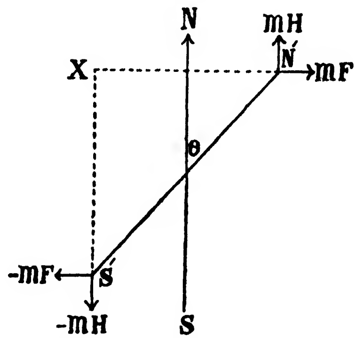


Fig. 18

needle is deflected from its first position of rest and comes to rest in the position $N'S'$, intermediate between the direction of the two fields and makes an angle θ with the direction of the field of intensity H. When the needle is at rest in the position $N'S'$, the forces acting on it must be in equilibrium. The forces acting on the needle in this position, due to the field of intensity H, are each equal to mH and act in the directions shown in the figure. Therefore they constitute a couple. The moment of this couple is equal to $mH\times N'X$; similarly the forces acting on the needle, due to the field of inten-

sity F, are each equal to mF and act in the directions indicated in the above diagram. These two forces being equal, opposite and parallel, again constitute a couple. The moment of this couple is equal to $mF \times S'X$. These two couples have a tendency to rotate the needle in opposite directions; and as the needle is in equilibrium position, their moments must be equal and opposite. Therefore we have $mH \times N'X = mF \times S'X$.

Now if the length of the needle be taken as l, then $N'X = l \sin\theta$ and $S'X = l \cos\theta$, where θ is the angle which the needle makes with the direction of H. Substituting these values in the above equation, we have

$$mH \times l \sin\theta = mF \times l \cos\theta$$

$$\therefore \; H \sin\theta = F \cos\theta$$

$$\text{or } F = \frac{H \sin\theta}{\cos\theta} = H \tan\theta$$

$$\text{or } \frac{F}{H} = \tan\theta.$$

Thus, if we know the angle of deflection of the magnetic needle and the intensity of one of the fields, the intensity of the other can be found from the above relation. This relation known as the tangent law is utilized in the comparison of magnetic moments by *magnetometer*.

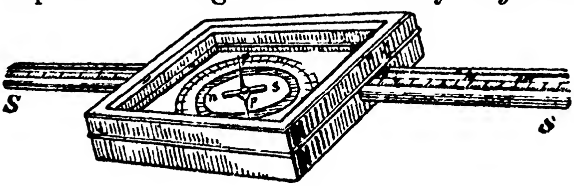


FIG. 19

266. The Magnetometer—It consists of a small magnetic needle pivoted so as to move freely in a horizontal plane, at the centre of a wooden box, containing a circular scale, divided to show degrees. At right angles to the needle is attached a light long pointer of aluminium, which denotes the deflections of the needle. To avoid errors, due to parallax in reading the positions of the needle, the bottom of the wooden box on which the scale is set, is made of plane mirror. The box is

fixed to a wooden stand, protruding both ways from its centre. This stand contains a groove and a metre scale. The circular scale is so fixed as to have its zero on the axis-line of the stand.

267. Comparison of magnetic moments of magnets by a magnetometer—(*i*) **Deflection method.** Set the magnetometer in the 'End on' position, so that the stand and the groove are perpendicular to the needle, *i.e.* the stand is *east* and *west* (position A* of Gauss), Fig. 20. Place the magnet A in the groove, with its centre d cms. from the needle and note down the deflection produced, by reading both ends of the pointer. Reverse the magnet keeping its centre still at the same point, *i.e.* d cms. from the needle and again take the two readings of the deflection produced. Similarly take two sets of two readings each by placing the magnet on the other side of the needle and then take the mean of these eight readings. Let it be θ_1. Similarly place the magnet B, d cms. from the needle. Take eight readings of the deflection and find the mean reading θ_2.

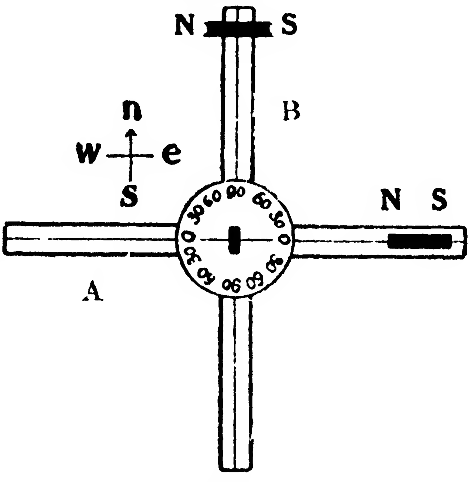


FIG. 20

Then $\dfrac{M_1}{M_2} = \dfrac{\tan\theta_1}{\tan\theta_2}$, where M_1 and M_2 are the magnetic moments of A and B respectively; because we have

*The position when the stand and the groove are parallel to the needle, as shown in fig 20 B, is called the position B of Gauss, or "Broadside on" position. In this case to get deflection, the magnet must be placed perpendicular to he groove.

$$\frac{2M_1}{d^3} = H \tan\theta_1 \quad . \quad (i)$$

and $$\frac{2M_2}{d^3} = H \tan\theta_2 \quad (ii)$$

$$\therefore \quad \frac{M_1}{M_2} = \frac{\tan\theta_1}{\tan\theta_2}.$$ Thus the magnetic moments of two magnets are directly proportional to the tangents of deflections, they produce in the magnetometer needle.

(*ii*) **The Null Method.** Set the magnetometer as before. Mark the middle points of the two magnets *A* and *B*, the magnetic moments of which are to be compared Place the magnet *A* in the groove, *d* cms. from the centre of the box, with its *N*-seeking pole towards the needle. Now place the magnet *B* in the groove on the other side of the needle, with its *N*-seeking pole towards the needle and move it along till the pointer is at zero. Let its distance be d_1 cms. from the centre of the box. Now reverse the polarity of both the magnets; and keep *A* at the same distance, *d* cms. from the centre of the box. Let the distance of *B* be d_2. Now interchange the two magnets, and find out d_3 and d_4 for the magnet *B*; while the magnet *A* is always kept at *d* cms. Let d' be the mean of d_1, d_2, d_3 and d_4.*

Then $\frac{M_1}{M_2} = \frac{d^3}{d'^3}$, where M_1 and M_2 are, as before, the magnetic moments of *A* and *B* respectively. Because no deflection is produced, intensity at the centre of the box due to one magnet must be equal and opposite to that produced by the other; therefore we have

$$\frac{2M_1}{d^3} = \frac{2M_2}{d'^3}$$

* Eight sets of readings in the deflection-method and four in the null method are necessary to eliminate errors due to the following causes --

(*i*) The centre of the scale and the centre of the needle may no coincide

(*ii*) The needle may not be pivoted at its centre.

(*iii*) The circular scale may not be correct

(*iv*) The magnetometer may not be set right exactly.

$$\text{or} \quad \frac{M_1}{M_2}=\frac{d^3}{d'^3}.$$

268. Oscillation-method of comparing moments of two magnets.

The magnetic moments of two magnets can be compared by making, first one magnet oscillate and then the other in the same field. It has been shown that the time of oscillation is given by the formula

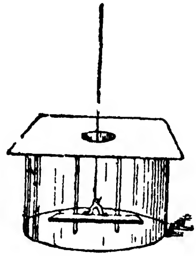


Fig. 21

$T=2\pi\sqrt{\dfrac{K}{MH}}$, where K is the moment of inertia of the oscillating magnet, then from the above equation we have

$$T_1^2=\frac{4\pi^2K_1}{M_1H} \quad \ldots \quad (i)$$

$$\text{and} \quad T_2^2=\frac{4\pi^2K_2}{M_2H} \quad \ldots\ldots \ldots \quad \ldots (ii)$$

where M_1 and M_2 are the magnetic moments of the two magnets. T_1 and T_2 their respective times of oscillations and K_1 and K_2, their moments of inertia.

Then dividing equation (ii) by (i), we have $\dfrac{M_1K_2}{M_2K_1}=\dfrac{T_2^2}{T_1^2}$; and if the magnets are of the same size and shape; $K_1=K_2$.

Then we have $\dfrac{M_1}{M_2}=\dfrac{T_2^2}{T_1^2}=\dfrac{N_1^2}{N_2^2}$; where N_1 and N_2 are respectively the numbers of oscillations made by the two magnets in the same time, for $N_1=\dfrac{1}{T_1}$ and $N_2=\dfrac{1}{T_2}$,

i.e. the moments vary inversely as the squares of the times of oscillations of the two magnets

269. Positions of the poles of a magnet.

Up to this time, we have been assuming the poles of a magnet to be situated at its extreme ends.

This assumption however, is unjustifiable, because the poles of a magnet are not always situated at its extreme ends, but are generally situated some distance

inwards from the ends. Therefore to find the position of a pole, the magnet is placed on a sheet of paper stretched on a drawing board and its boundary line drawn. A small compass-needle is placed near to the pole of the magnet; and the whole drawing board is rotated, till the *compass-needle points towards the magnetic N. and S**. Pencil marks are put on the paper to denote its direction (*i.e.* of the compass-needle). The process is repeated at the two other places. The magnet is removed and the three directions obtained are produced towards the magnet. The intersection of these three gives the position of the pole.

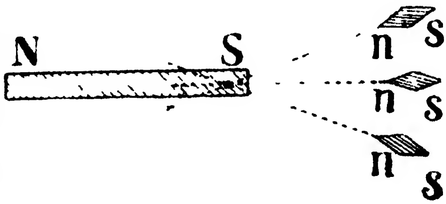


FIG. 22

270. To prove the Inverse Square Law. Set the deflection magnetometer in 'end on' position and get the mean deflection θ_1, due to a given magnet A. Set the magnetometer now in the 'broadside on' position (Fig. 20); and get the mean deflection θ_2, due to the *same* magnet A, placed at the *same* distance.

Now in the 'end on' position $\frac{2M}{d^3}=H\tan\theta_1$

and in the 'broadside on' position $\frac{M}{d^3}=H\tan\theta_2$

$\therefore$ dividing we have, $2=\frac{\tan\theta_1}{\tan\theta_2}$.

It will always be observed that the ratio of the tangents of the two deflections will be equal to 2. But the expressions $\frac{2M}{d^3}$ and $\frac{M}{d^3}$ have been deduced on the *supposition that the inverse square law holds good* and as

* This is necessary to avoid errors due to the earth's magnetic force. For if the needle were not to point in the N. and S directions, the earth's magnetism would make the needle point in some direction, other than that due to the magnet alone.

the theoretical result coincides with practical observations, the only conclusion is that the supposition must be correct.

SUMMARY

1. **Magnetic moment** is the moment of a couple acting on a magnet when placed in a field of unit intensity and in a direction perpendicular to the lines of force. It is measured by the product of m and l, where m is the pole-strength and l the distance between the two poles.

2. A magnet free to move in a horizontal plane oscillates, when disturbed from its position of rest. The time of oscillation T is given by the formula, $T = 2\pi\sqrt{\frac{K}{MH}}$, where K is the moment of inertia, H the intensity of the field and M its magnetic moment.

3. The **intensity** at a point on the axis, due to a bar-magnet, is given by $\frac{2M}{d^3}$, and at a point on a line drawn through the centre and perpendicular to the axis, by $\frac{M}{d^3}$.

4. When two fields of intensities H and F exist at the same place in directions perpendicular to each other; then $F = H \tan\theta$, where θ is the angle made by a freely suspended magnetic needle with the direction of H.

5. **Magnetometer** is an instrument for comparing magnetic moments of two magnets.

6. In the deflection-method of comparing magnetic moments, $M_1 : M_2 :: \tan\theta_1 : \tan\theta_2$; and in the null method, $M_1 : M_2 :: d_1^3 : d_2^3$.

EXAMPLES

1. Explain the terms magnetic moment of a magnet and intensity of field.

A magnet, placed at an angle of 30° with a uniform field of intensity ·32, experiences a couple, whose moment is 8. Calculate the magnetic moment of the magnet. The length of the magnet being 5 cms., calculate its pole-strength.

2. Find an expression for intensity at a point on the axis of a bar magnet.

3. Prove that $F = H \tan\theta$.

4. What is a magnetometer? How would you compare magnetic moments of two magnets by its aid?

5. How would you compare the intensities of two fields by the oscillations of a magnet?

6. Give a method of finding the magnetic length of a magnet.

7. Draw a graph, showing the distribution of free magnetism along the length of a given bar magnet. Describe in detail the precautions, which you would observe in performing the experiment.

8. Prove that the intensity due to an 'end on' magnet is twice that due to the same magnet 'broadside on', at the same distance; and prove the truth of inverse square law.

9. A magnet free to vibrate horizontally in the earth's field executes 20 vibrations per minute. Another small magnet is placed due North, with its *N*-seeking pole towards the North at a distance of 20 cms. from it, and the magnet takes two minutes to make 50 vibrations. Compare the strength of the field due to the magnet with the earth's horizontal field and find out its magnetic moment, if $H = {\cdot}18$ dynes.

We have $H \propto n^2$

$\therefore$ if intensity due to the earth be represented by H and that due to the magnet by f, then we have

$\frac{H}{H+f} = \frac{(20)^2}{(25)^2} = \frac{16}{25}$, for the intensity due to the magnet reinforces that due to the earth.

$$\text{or} \quad 16f = 9H \text{ or } f = \frac{9}{16} \times {\cdot}18 \text{ dynes.}$$

$$\text{But } f = \frac{2M}{d^3} \text{ approximately.}$$

$$\therefore \quad \frac{2M}{20^3} = \frac{9}{16} \times {\cdot}18$$

$$\text{or} \quad M = \frac{9}{16} \times \frac{18}{100} \times \frac{20 \times 20 \times 20}{2}$$

$$\therefore \quad M = 405.$$

10. A North pole of strength 30 units, experiences a force of 90 dynes, when placed at a given point in a certain field. Find the intensity of the field at the given point.

Pole-strength of 30 experiences a force of 90 dynes.

" " " 1 would experience " $\frac{90}{30}$ dynes.

Thus the intensity at the given point $= 3$ dynes.

11. Two magnets *A* and *B*, are made to oscillate in the same magnetic field; *A* performs 15 vibrations per minute

and B 10 vibrations per minute. The magnet A is then caused to oscillate in one magnetic field, and B in another. A now performs 10 oscillations per minute and B 20 vibrations. Compare the magnetic moments of two magnets; and also the intensities of the fields, in which A and B oscillate separately.

(i) $$\frac{\text{Moment of } A}{\text{Moment of } B}=\frac{15^2}{10^2}=\frac{9}{4},$$ when both of them are oscillating in the same field.

Let H be the intensity of the field, in which A and B oscillate together, H_1 and H_2 in which A and B oscillate separately.

Then we have $$\frac{H_1}{H}=\frac{10^2}{15^2}=\frac{100}{225}=\frac{4}{9}$$

$\therefore$ intensity of field H_1 is $\frac{4}{9}$ of H,

Also ,, ,, $$\frac{H_2}{H}=\frac{20^2}{10^2}=4$$

$\therefore$ the intensity of field H_2 is 4 times that of H.

Therefore the ratio of the intensities of the fields, in which A and B respectively oscillate, is given by

$$\frac{4}{9} : 4 \text{ or } 1 : 9$$

12. Compare the horizontal components of the Earth's magnetic fields at Lahore and Delhi; given that dip at Lahore is 45° and at Delhi 30°, while the total intensity at Lahore is ·24 gauss and at Delhi ·18 gauss.

As $H=I\cos\theta$

At Lahore we have, $$H=\cdot 24\cos 45^\circ=\cdot 24\times\frac{1}{\sqrt{2}}$$
$$=\cdot 168 \text{ gauss}$$

and at Delhi we have, $$H=\cdot 18\cos 30^\circ=\cdot 18\times\frac{\sqrt{3}}{2}$$
$$=\cdot 154 \text{ gauss}$$

13. A magnet, whose strength is 200, is placed in a uniform field of intensity 0·5 gauss. What are the forces which act upon its poles?

The force acting on N-seeking pole will be $200\times 0\cdot 5 =100$ dynes.

And the force acting on S-seeking pole will be $200\times 0\cdot 5 =-100$ dynes.

14. The magnetic moment of a bar magnet, which

weighs 65 grams, is found to be 1250. If the density of steel be 7·9, find the intensity of magnetization.

15. The length of a magnet is 16 cms. and the strength of its pole is 15 units. Find the intensity at a point on the axis of the magnet at a distance of 40 cms. from its centre.

16. The moment of a magnet is 1000 in C.G.S. units. How much work is done in turning it through 90° from the magnetic meridian, in a horizontal plane, at a place where the horizontal intensity is 0·16 gauss.

(*London, B. Sc. Pass-1902*).

17. The magnetic moment of a short magnet is 1000 C.G.S. units. Find the intensity of its field, at a point on its axis produced and 20 cms. from its centre. Find the intensity of magnetization, if the magnet is 10 cms. long and 2 sq. cms. area of cross-section.

18. A magnetic needle makes 12 oscillations a minute under the earth's force alone. Under the influence of the earth and a magnet A, it makes 14 oscillations; and 16 oscillations when A is replaced by another magnet B. Compare the magnetic intensities at the point, due to A and B. (*Punjab, 1915-16*).

19. A magnet AB, 24 cms. long, has poles of strength 27 units. What will be the force, due to the magnet upon a pole of strength 9, placed at a point C. 12 cms. from B on the line AB produced. (*Punjab, 1916-17*).

CHAPTER III
TERRESTRIAL MAGNETISM

271. The Earth as a Magnet.—The manner in which a freely suspended magnetic needle *swings to and fro*, and finally comes to rest, pointing approximately N and S, even in the absence of any neighbouring magnet, shows that at all points on the earth's surface a magnetic field exists, in which the lines of force are approximately parallel to the geographical meridian of the place. This magnetic field is supposed to be due to the earth's magnetism; but whether the earth itself is a permanent magnet or its magnetism is due to some other cause is as yet not known with certainty. The simplest explanation is to consider a very long magnet embedded in the earth's surface, with its north-seeking polarity in the neighbourhood of south geographical pole and its south-seeking polarity in the neighbourhood of north geographical pole; and further it is to be assumed that the distribution of free magnetism on this very long magnet is rather irregular. These assumptions are based on the fact that the actual field on the earth's surface resembles pretty closely with the magnetic field, that should result from a long magnet with somewhat irregular distribution of magnetism.

272. Magnetic elements.—The quantities, which completely determine the magnetic field of the earth, are called the magnetic elements.

These are: (*i*) **Declination,** (*ii*) **Inclination or Dip and** (*iii*) **the horizontal component of the Earth's Intensity at that place.**

The **Declination** *at any place is the angle which a magnetic needle, suspended so as to move freely in a horizontal plane, makes with the geographical meridian of the place.* Hence if we know the geographical meridian and the declination at the place, we can at once

get the magnetic meridian or the vertical plane, in which the magnetic axis of a freely suspended magnet would set itself. **The Inclination or Dip** *at any place, is the angle which a magnetic needle, free to move in a vertical plane, makes with the horizontal, when placed in the magnetic meridian.* This gives the direction of the Earth's Resultant magnetic intensity in the magnetic meridian.

273. Intensity of the Earth's force or *I*. It is the total *intensity* of the earth acting at the place. This we can find out, if we know the **horizontal component** of the *earth's intensity* and the *Inclination or Dip at that place.*

For if the total magnetic intensity at *any* place be represented both in magnitude and direction by AI; then this can be resolved into two components AH and AV, acting perpendicular to each other, as shown in fig. 23. The intensity AI of the field can be found by measuring AH, the horizontal component;

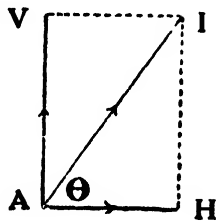


Fig. 23

$$\because \quad \frac{AH}{AI} = \cos \angle HAI,$$

$$\text{or that } AI = \frac{AH}{\cos \angle HAI},$$

where $\angle HAI$ = the Dip, as defined above; *i.e. the total intensity is equal to the horizontal intensity divided by the cosine of the Dip.*

274. Declination To determine the declination, it is necessary to find out the geographical and magnetic meridians and to measure the angle between them. The easiest and accurate way of determining geographical meridian is to fix a straight wire about a foot long, upright on level ground, where the Sun can shine upon it. About an hour before noon, mark the direction and length of the shadow of the wire. Draw an arc of the circle with radius equal to the length of the shadow and base-point of the vertical wire as centre. In the afternoon, when the end of the shadow again reaches the arc and hence is of the same length as an

hour before noon, mark its direction. Draw the bisector of the angle between these two directions of shadows of equal length: and this bisector gives the true geographical *N* and *S*.* To determine the magnetic meridian, a bar magnet is suspended in a stirrup by an unspun silk *fibre* and short *pieces of fine silver wire* are attached to its ends by a piece of sealing-wax. These fine wires should be perpendicular to the faces. When the suspended magnet comes to rest, points are marked to denote the positions of the two end wires. The magnet is then turned upside down, so that the top face becomes the bottom face and *vice versa*; and points are marked again to denote the positions of the two end wires as before. The respective points are joined and the angle between them is bisected. This bisector gives the position of the magnetic meridian. The angle between the magnetic meridian and the geographical meridian *NS*, as found before, gives the **Declination** of the place.

Declination varies from place to place on the earth's surface. In India the declination is zero at all places, having the same latitude as Pondicherry. To the north of this, the declination is easterly: while to the south of this, it is westerly. Lines joining those places on the surface of the earth, where the declination is the same, are called **isogonic lines;** and the lines joining those places, where the declination is zero (such as at Pondicherry), are called **agonic lines.**

275. Inclination or Dip. To find Dip at any place take a needle mounted so as to move freely in a vertical plane and place it in the magnetic meridian. The angle which such a needle makes with the horizontal is the angle of 'Dip.' An instrument for measuring the angle of Dip, is called a Dip-needle and is shown in figure 24.

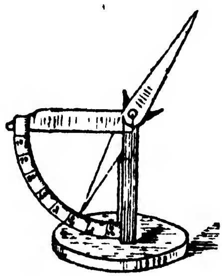


FIG. 24

* The direction of shadow at 12 noon gives the true geographical north, but it is rather difficult to get exact time.

For accurate measurement of the angle of Dip, a delicate instrument known as the Dip-Circle is used. It consists essentially, as shown in fig. 25, of a horizontal circular *wooden* table, with three levelling screws and this is provided with a circular scale divided into degrees. Over this horizontal table, moves a vertical rectangular frame with a vernier at its base; such that it moves along the horizontal circular scale, when the rectangular frame is rotated. This frame carries a vertical graduated circle, in front of which a needle supported on a rigid horizontal axis, moves freely in a vertical plane. The axis must coincide with the centre of gravity of the needle, in order that it may be absolutely independent of the force of gravity and be capable of being influenced by magnetic forces only. The axis should pass through the centre of the vertical scale, which is so graduated as to have its zero-line in the horizontal plane.

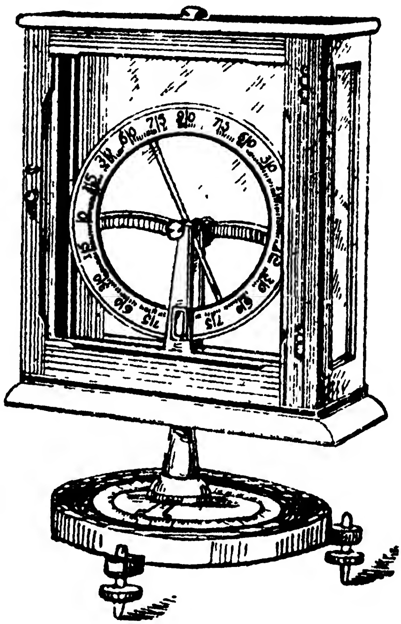


FIG. 25

To determine the Dip, the instrument is first perfectly levelled by means of levelling screws; and this is shown by the spirit-level at the top of the vertical frame, which is then turned round, until the needle is vertical. When this is the case, *the plane of the needle* is at right angles to the magnetic meridian; because the vertical position of the needle shows that the horizontal

component of the Earth's force is not effective in making the needle point North and South, but simply produces a pressure on its '*bearings*'. But " a force has no effect in a direction perpendicular to itself", therefore the direction of the plane of the suspended needle must be perpendicular to the magnetic meridian.

Hence to set the needle in the magnetic meridian, the whole vertical frame is now rotated through 90^0. The needle being thus set in the magnetic meridian, the reading of the angle, which the needle makes with the horizontal, *gives the angle* of **Dip** or **Inclination.**

In practice however, to ensure accuracy of the *Dip* and to eliminate all possible errors, due to want of exact adjustments between the needle and the scale, the mean of the following observations, gives the true angle of Dip:—

(*i*) Readings of both ends of the needle are taken to eliminate error, due to the axis of suspension of the needle, not passing through the centre of the circle.

(*ii*) The vertical scale is turned through 180°; and readings of both ends are taken again to eliminate errors due to: (*a*) non-coincidence of the magnetic axis of the needle with its geometrical axis, and (*b*) zero-line of the vertical circle, being not truly horizontal.

(*iii*) Observations (*i*) and (*ii*) above, are repeated after turning the needle over, with respect to the scale, to eliminate errors due to the centre of gravity, not lying on its geometrical axis.

(*iv*) Observations (*i*), (*ii*) and (*iii*) are repeated by remagnetizing the magnetic needle in the reverse direction, to eliminate error, due to non-coincidence of the centre of gravity with its point of suspension.

The mean of the sixteen readings thus obtained, gives true value of the *Dip*. The student should himself draw figures to explain the theory of the above four corrections.

The Dip, like the declination, differs in different localities. Near the equator, places are known where the Dip is zero. The line joining all these places, is called the magnetic equator or the **Aclinic line.** As the

needle is moved northwards or southwards, the Dip increases. Lines joining those places, where the angle of Dip is the same, are called **Isoclinic lines.**

276. The Horizontal component or H. It has been explained above that if H, the horizontal component and the Dip are known, the intensity due to earth's field becomes known. In fact, it is usual to determine H, because owing to the action of gravity, it is convenient to work in a horizontal plane. To get the value of H, it is necessary to perform two experiments:—

(*i*) A given magnet is set oscillating by a silk *fibre*, inside a glass-covered box, and the time of one complete oscillation is accurately measured; then we have:—

$$T=2\pi\sqrt{\frac{K}{MH}}$$

$$\text{or } T^2=\frac{4\pi^2K}{MH}$$

$$\text{or } MH=\frac{4\pi^2K}{T^2}\ldots\ldots\ldots\ldots\ldots(i)$$

(*ii*) The deflection produced in a magnetometer by the same magnet placed '*end on*', with its centre d cms. from the needle of the instrument, is measured; then we have $\frac{2M}{d^3}=H\tan\theta$

$$\text{or}\quad \frac{M}{H}=\frac{d^3}{2}\tan\theta\ \ldots\ldots\ldots\ldots(ii)$$

Dividing equation (*i*) by (*ii*), we get

$$H^2=\frac{4\pi^2K}{T^2}\times\frac{2}{d^3\tan\theta}$$

$$\text{or}\quad H=\frac{2\pi}{T}\sqrt{\frac{2K}{d^3\tan\theta}}.$$

If the units employed in K, t and d are in the C.G.S. system of units, then H is given in Gauss.

Also multiplying equation (*i*) and (*ii*),

we get $M^2 = \dfrac{2\pi^2 K d^3}{T^2} \times \tan\theta$

or $M = \dfrac{\pi}{T}\sqrt{2Kd^3 \tan\theta}$

277. Magnetic Variations. The magnetic *elements* vary not only from place to place, on the earth's surface; but also from time to time at the same place. These variations are: (*i*) *Secular*, (*ii*) *Diurnal* and (*iii*) *Aperiodic.*

(*i*) **Secular variations** are such, as would result, if the supposed long magnet embedded in the earth, were presumed to be rotating about the earth's axis from west to east, in a period of about five centuries.

(*ii*) **Diurnal variations** are the changes, which occur regularly at various hours of the day. These diurnal variations depend to a great extent upon the weather and season of the year.

(*iii*) **Aperiodic variations** are sudden changes in the magnetic elements. These are supposed to be due to what are called magnetic storms. The reason of these is an open question as yet.

Magnetic storms. Sometimes magnetic elements undergo violent changes; these are called magnetic storms. Such storms generally occur during the maximum display of *Aurora Borealis* and on this account it has been suggested that these storms are due to electrical disturbances in the atmosphere.

278. The directive action of the earth. Take a cork and over it place a magnetic needle. Let the cork float on the surface of water. The cork and the needle turn so that the needle points in the N and S directions; but the cork as a whole does not move either way. This shows that the action of the earth's field is directive only and not translatory. The reason is that the length of the needle is very small as compared to the diameter of the earth and hence the field in the small area occupied by the needle and the cork is uniform. The effect of this field on the two opposite poles of the mag-

net is equal in magnitude but opposite in direction and hence the magnet as a whole does not move either way.

Now bring a strong artificial magnet near to the floating magnetic needle. Notice that the needle and the cork move either towards or away from the magnet, depending upon relative positions of the two. The reason is that in this case, the field due to the artificial magnet is not uniform in the area occupied by the magnetic needle.

279. The Mariner's compass. The figure represents the modern form of the compass as designed by Lord Kelvin. It consists of 8 separate needles, varying in length from 2 to $3\frac{1}{2}$ inches, arranged as shown in the fig. The card consists of a thin aluminium rim, on which is fixed a paper-scale divided into 32 main divisions; the rim is connected by 32 silk-threads to a central disc, containing an inverted sapphire cup, which rests on a vertical iridium point. The total weight is about 11 gms. The advantage of this arrangement is that magnetic moment is small; while the moment of inertia is great, as compared to its weight. The period of vibration is thus considerable—a very important condition for steadiness in a heavy sea. To make the arrangement still more steady, the whole instrument is supported on *gimbals.*

FIG. 26

SUMMARY

1. The Earth's magnetism is supposed to be due to a very long magnet, embedded under its surface, with its *N*-seeking polarity near to the *S*-pole of the Earth; and *vice versa.*

2. The quantities which completely determine the magnetic field of the Earth are called the *magnetic elements.*

3. These are· (*i*) *Declination,* (*ii*) *Inclination* and (*iii*) *The Horizontal Intensity.*

4. **Declination** is the angle between the geographical meridian and the magnetic meridian at the place.

5. **Inclination** or **Dip** is the angle, which the direction of the total magnetic intensity makes with the horizontal, in the magnetic meridian.

6. Total **Intensity** is the force acting on an isolated unit *N*-seeking pole, when placed on the Earth's surface along the direction of the total intensity.

7. **Horizontal component or** H is the component of the Earth's intensity in the horizontal direction. If H is known, then the total intensity I is given by the formula, $I = \frac{H}{\text{Cos}\theta}$, where θ is the angle of Dip.

8. The value of H is found by (*i*) determining the time of oscillation of a magnet and (*ii*) the deflection produced in a magnetometer needle in the 'end on' position, by placing the magnet at d cms. from the needle.

9. Lines joining all places on the globe, where the declination is the same, are called **Isogonic lines;** and the line joining all places on the globe, where the declination is zero, is called the **Agonic line.**

10. Lines joining all places on the globe, where the Dip is the same, are called **Isoclinic lines**, and the line joining all places, where there is no Dip, is called the magnetic equator or the **Aclinic line.**

11. The magnetic elements at one and the same place change from time to time. These changes are classed as (*i*) *Secular,* (*ii*) *Diurnal* and (*iii*) *Aperiodic*

12. The Earth's force on a magnetic needle on the Earth's surface is **directive** only and not *translatory.*

EXAMPLES

1. Describe the nature of the magnetic field on the surface of the Earth. To what would you assign the Earth's magnetism to be due?

2. What are the magnetic elements of the Earth? Define them as clearly as you can.

3. Give methods of finding Declination and Inclination in the Laboratory.

4. Discuss the various sources of error in finding the angle of Dip by the Dip-circle and say how you would eliminate those errors?

5. What do you understand by earth's intensity? How would you calculate intensity, when its horizontal component H is known?

6. Describe a method of finding H and also the magnetic moment of a magnet.

7. What are magnetic storms? What is their effect and what can possibly be the cause of such magnetic storms?

8. It is said that earth's magnetic force is directive and not translatory. What is meant by the above statement and how would you furnish proof of it experimentally?

EXAMINATION QUESTIONS XI

1. Explain the terms magnetic pole, intensity, potential, permeability, magnetic moment, consequent pole and neutral point.

2. What do you understand by a line of magnetic force? A thick soft iron pipe is placed vertically between the poles of an electro-magnet, a card is placed over the poles and iron filings sprinkled over it and tapped. Very few iron filings collect over the hole Explain this. (*P. U.* 1929).

3. A short magnet is placed with its north pole pointing south. The neutral point is found to lie at a distance of 10 cms. from its middle point; calculate the moment of the magnet, $H=\cdot 32$ Gauss. (*P. U.* 1929).

4. Explain fully the terms. (*i*) Declination, (*ii*) Dip and (*iii*) Intensity of earth's magnetic field at a place. Show that a knowledge of these three completely determines the earth's field at a place.

Calculate the total force of the earth's field at the place, supposing the dip to be 30° and the horizontal component $=0\cdot48$ gauss. (*P.U.* 1930).

5. What are the magnetic elements of a place? Define magnetic dip, magnetic poles and magnetic equator.

A sailor observes the position of the *N*-point of his compass and then ascertains that the Declination is $22^\circ-20'W$. What angle must his course make with the magnetic needle so as to steer due west? (*P.U.* 1914).

6. Two exactly similar magnets are fixed horizontally

in a stirrup, with their axes at right angles. Explain how they set themselves when suspended. Illustrate by a diagram showing the forces acting. (*P.U.* 1915).

7. What is meant by the strength of a magnetic field at a point? Find an expression for the magnetic field at a point lying on the equator of a short magnet. (*P.U.* 1928).

8. A magnet lies with its axis at an angle θ to a uniform magnetic field, calculate the moment of the couple exerted. If in the above case, the magnet be freely suspended and the inclination θ be due to perpendicular field, show that

$$\text{Tan}\theta = \frac{\text{Deflecting field}}{\text{Controlling field}}. \quad (P.U.\ 1926).$$

CURRENT ELECTRICITY

CHAPTER I

VOLTAIC CELL

280. Introductory.—Take two conductors A and B, charge A with positive and B with negative electricity. Now connect A and B by a thin wire. Positive electricity would flow from A to B, the potential of the two conductors would be equalized and the flow stopped. This is quite similar to the flow of water from one cistern to another at a lower level. The water would go on flowing so long as the level of the liquid in one cistern is higher than that of the liquid in the other. In this case, it is clear that if we want to maintain a continuous flow from A to B, the level of the liquid in cistern A should always be kept higher than the level of the liquid in cistern B. This can be arranged by transferring the liquid in B to A by the aid of a water-pump, as shown in fig. 1. To work the pump, energy must be spent. Thus, it is clear that to keep a continuous flow of the liquid, two things are necessary: (*i*) The level of the liquid in A should always be higher than that of the liquid in B and (*ii*) Energy to work the pump, to maintain the difference of levels, is essential. This hydrostatic analogy holds good in the case of electricity. For, to maintain a continuous

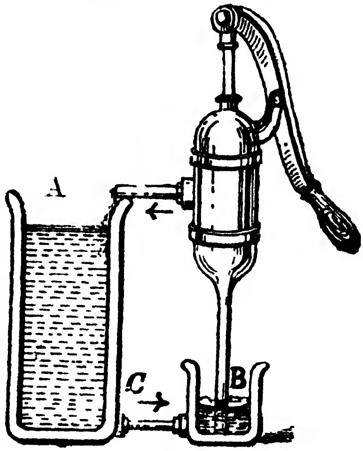


FIG. 1

flow of electricity from A to B, all what is required is· (*i*) *the potential of A should be higher than that of B and* (*ii*) *it should be possible to maintain this difference of potential by supplying energy.*

281. Voltaic cell.—Volta found that both the above conditions are fulfilled, when a strip of copper and a strip of zinc are placed in a vessel containing dilute sulphuric acid and connected by a wire. In this simple arrangement, it is found that the copper and zinc strips are kept at different potentials and the energy to maintain this difference of potential is supplied by the chemical energy. Thus a constant current of electricity flows from copper, which is at a higher potential *to* zinc*, which is at a lower potential.

282. Theory of voltaic cell.—To get an explanation of the working of a voltaic cell, we see that the essential difference between zinc and copper is that zinc has a far greater chemical affinity for oxygen than copper has. Next we see that the liquid, in which strips of these two metals are placed, is a dilute solution of sulphuric acid. In this solution, according to the theory of *electrolytic* dissociation, the molecules break up into their constituent atoms or groups of atoms, which differ from ordinary atoms of a Chemist in that they always carry a small amount of charge. To these is given the name of *ions.* Thus the molecules of sulphuric acid, in the solution, break up into hydrogen [H^+] and sulphion [SO_4^{-2}] ions. The latter are incapable of separate existence and so combine with the hydrogen ions of water, forming sulphuric acid once again and liberating oxygen [O^{-2}] ions. Hence to sum up the matter, we say that water molecules are broken up into ions. An oxygen ion carries a negative charge and is

* It is a conventional and most accepted way of describing the whole phenomena; for we might, equally well, say that negative electricity flows from zinc to copper Recent investigation has demonstrated the certainty of the fact, that an electric current consists in the transference of electrons from zinc to copper. For the sake of convenience and uniformity, it is advisable to stick to the old nomenclature and to describe the current as proceeding from points of higher to points of lower potential.

called *electro-negative element*, while a hydrogen ion carries a positive charge and is called *electro-positive element*. When copper and zinc plates are immersed in such a solution, the first chemical change which takes place is, that both copper and zinc attract oxygen ions towards themselves, on account of their chemical affinity for it. As oxygen ions carry a negative charge, both the copper and zinc plates become negatively charged; but the zinc plate is more highly charged with negative electricity than the copper one, for the affinity of the former for oxygen is far greater than that of the latter. So the copper plate would comparatively be at a higher potential than the zinc plate; and when the two are connected by a wire, positive electricity flows from copper to zinc and equilibrium is destroyed. The zinc plate becomes less strongly charged with negative electricity and again attracts oxygen ions; while the copper plate becomes more strongly charged with negative electricity and therefore repels the oxygen ions, instead of attracting them. In this way a continuous current of positive electricity flows from copper to zinc through the wire and from zinc to copper in the liquid. This last step is made intelligible, when we see that oxygen ions carrying negative charges move towards zinc in the liquid; they are equivalent to a negative current flowing towards the zinc plate or a positive current flowing from the zinc plate towards the copper plate. *The copper plate is called the positive pole, and the zinc plate the negative pole of the voltaic cell.*

In this cell, we have used copper and zinc plates; but any two substances, which attract oxygen to markedly-different extents, may be substituted for them.

283. Polarization—In theory, the simple cell described above should go on working so long as the chemicals in it are not exhausted; but in practice, it is found that after a short time, the current weakens and ultimately stops. Experiment has shown that it is due *to the deposit of a thin film of hydrogen on the copper plate, and the name given to this is Polarization.*

This layer of hydrogen affects the working of the cell in the following ways:—

1. Each small portion of the copper plate, to which a bubble of hydrogen adheres, is protected from the acid: and thereby the effective area of the copper plate is reduced.

2. Hydrogen is a bad conductor and therefore offers a great resistance to the passage of current.

3. The hydrogen attached to the copper plate attracts the oxygen ions in the liquid, even more strongly than the zinc plate does, the effect of which is to send a current in the opposite direction.

4. The hydrogen decomposes the zinc sulphate formed in the cell and produces a deposit of zinc on the copper plate.

284. Local Action. When plates of copper and zinc are placed in dilute sulphuric acid and connected by means of a wire, current flows from copper to zinc and bubbles of gas are seen to rise from the surface of both plates. On breaking the wire connection, bubbles are seen to rise from the zinc plate only; thus showing that the chemical action between zinc and dilute sulphuric acid goes on, even when no current is flowing and this represents waste of chemical energy. This action goes on only when ordinary zinc, with its impurities mostly consisting of carbon and iron, is used; and ceases altogether, if a plate of *pure* zinc be substituted for it. *Thus this action is due only to the presence of impurities in the zinc plate and is known as Local Action.* The impurities act like copper plate in a voltaic cell and so chemical energy is wasted in maintaining local circuits. *This local action is injurious to the cell in two ways*: First, the chemical energy is wasted without any gain of electrical energy; and secondly, this local action heats the contents of the cell and raises their temperature, by which the resistance of the cell is increased.

Local action can be prevented by *amalgamating* the zinc plate. The method consists in first cleaning the surface of the zinc plate with a little quantity of

dilute sulphuric acid and then rubbing it over with a drop of mercury. The mercury dissolves the zinc and covers the whole plate with a thin layer of this amalgam. The impurities, such as carbon and iron, being unaffected by mercury, are covered over by zinc amalgam; and thus local action is prevented.

285. Electromotive force. Generally written as *E.M.F.*, for the sake of brevity, it is popularly defined as whatever causes motion of electricity. But, this is not *force* in the *dynamical* sense of the word; because Newton has, once for all, defined force as that which produces or tends to produce motion in *matter*. *E.M.F.* is quite a different thing from Newton's force, for it acts not on matter but on *electricity*. It has already been explained, that flow of electric current is always due to the difference of potential existing between the two plates of a voltaic cell. This *difference* of potential is the *measure* of the *E.M.F.* of the cell, but it is not the *E.M.F.* itself. Just as in water pipes, difference of levels produces pressure and the pressure produces flow; so difference of potential produces electromotive force, which drives the current. The difference of potential is sometimes spoken of as *E.M.F.*, for the sake of convenience; but the distinction between the two must not be lost sight of. When the circuit of a cell is *open*; that is, when its poles are not connected by a wire, the difference of potential between the poles is equal to the *E. M. F.* But when the circuit is closed, the *E M.F.* is spent in driving the current; and therefore the difference of potential between the poles *cannot* be equal to the *E.M.F.*

The unit of *E M F.* is not the same, as the unit of potential in electrostatics. The actual practical unit, called the *volt*, would be defined later. However, to form a rough idea of the magnitude of the unit, we can say that the *E.M.F.* of a Daniell's cell is very nearly equal to a volt.

286. Resistance. The strength of a current does not depend *only* on the force tending to drive the

current round the circuit; but also on the *resistance*, which it has to overcome in its flow. Developing the hydrostatic analogy cited above, we see that the amount of water, which runs through the pipe, does not depend on the pressure alone, but also on the resistance that it meets with. For, if the bore of the pipe is very small or if it is choked with pebbles etc., the water will flow more slowly than otherwise.

From what has been said above, it follows that the resistance of a conductor depends both upon its dimensions and the nature of the material. The practical unit of resistance is called an *Ohm*, which will be defined later on.

287. Current. The strength of a current is defined as the *quantity of electricity*, which flows across any section of an electric circuit in one second. The practical unit of current is called the *Ampere*; and that of quantity of electricity, the *Coulomb*.

SUMMARY

1. To maintain a constant flow of electricity, it is necessary to ensure a difference of potential *and* a supply of energy to keep up that difference of potential.

2. A simple **voltaic cell** consists of a copper and a zinc plate, immersed in dilute sulphuric acid.

3. The current flows from copper to zinc in the wire and from zinc to copper in the liquid.

4. The difference of potential between copper and zinc exists, on account of different degrees of chemical affinity, which they have for oxygen.

5. The deposition of hydrogen on copper plate is called **polarization**. It affects the cell in the following four ways :—

(*a*) The effective area of copper plate in contact with dilute sulphuric acid is reduced.

(*b*) The resistance of the circuit is increased.

(*c*) It tends to send a current in the opposite direction.

(*d*) Zinc begins to be deposited on the copper plate.

6. The chemical action, which goes on between the zinc plate and dilute sulphuric acid, when no current is flowing, is called **local action**. This is always due to impurities in zinc. It can be prevented by amalgamating the zinc rod.

7. **Electromotive force** is that which drives the current. It is equal to the difference of potential. The practical unit of *E.M F.* is called a **volt** and is nearly equal to the *E.M.F.* of a Daniell's cell.

8. The quantity of electricity flowing through any section, in one second, is called the **Current.** The practical unit is called an **Ampere.**

9. **Resistance** is the obstruction offered to the flow of current. The practical unit is called an **Ohm.**

EXAMPLES

1. What are the necessary conditions for the flow of current? Illustrate your answer by hydrostatic analogy.

2. Describe a simple voltaic cell and give a very brief theory of its actions.

3. What are Local Action and Polarization? How would you prevent the former?

4. What do you understand by electromotive force, resistance and current? Name their practical units.

CHAPTER II

CELLS

288. The essentials of a good cell. A good cell should fulfil the following conditions:—

1. Its *E.M.F.* should be high and constant.
2. Its internal resistance should be small.
3. No chemical action should go on in the cell, when no current is flowing.
4. It should be *cheap* and durable.
5. It should not give off pungent fumes.
6. It should be free from polarization.

We have seen, how a deposit of hydrogen (called polarization) on the copper plate of a simple voltaic cell, prevents the flow of an electric current. Various forms of cells have been devised with the object of removing this deposit and of attaining as many conditions as possible, which are necessary for a good cell. To attain this end, three ways have up to this time been resorted to. They are:—

(*i*) **Mechanical.** In this method: (*a*) hydrogen bubbles are brushed off as they are formed, or (*b*) the positive pole is made rough, hydrogen bubbles collect on the numerous fine points and float off on account of buoyancy. An example of this form is Smee's cell.

(*ii*) **Chemical.** In this, some oxidizing agent is introduced in the cell; so that the hydrogen evolved, instead of being liberated on the positive pole of the cell, is oxidized. Examples of this class of cells are the Bichromate and the Bunsen's cell.

(*iii*) **Electrochemical.** In this method, hydrogen is exchanged for some other substance. An example of this class is the Daniell's cell.

(1) **Smee's cell.** This consists of a zinc and a platinized silver plate, dipping into dilute sulphuric

acid. The silver plate, on account of a rough coating of platinum, gives up hydrogen bubbles freely; but inspite of this, the current falls very quickly and the cell is not at all suited for any useful purpose.

(*ii*) **Bichromate cell.** This consists of a zinc plate attached to a brass rod, which slides up and down a brass tube passing through an ebonite cork. By this means, the zinc plate can be lifted out of the liquid, when no current is required. There are two carbon plates, one on each side of the zinc plate. These are attached to the cork and also have metallic connection with each other so as to form one plate. By this arrangement, the area of the carbon rod is increased and the resistance of the cell is greatly diminished. The liquid is a mixture of dilute sulphuric acid and potassium bichromate ($K_2Cr_2O_7$). The actions which take place are :—

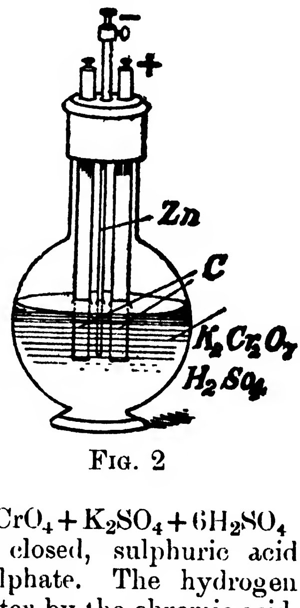


FIG. 2

(*a*) Chromic acid and potassium sulphate are formed according to the following equation—

$$K_2Cr_2O_7 + 7H_2SO_4 + H_2O = 2H_2CrO_4 + K_2SO_4 + 6H_2SO_4$$

(*b*) When the circuit is closed, sulphuric acid acts on zinc, forming zinc sulphate. The hydrogen produced is oxidized to form water by the chromic acid, which is reduced to chromic oxide, the latter dissolves in sulphuric acid and forms chromium sulphate, according to the following equation—

$$6H_2SO_4 + 3Zn + 2H_2CrO_4 = Cr_2(SO_4)_3 + 3ZnSO_4 + 8H_2O$$

(*c*) Further a secondary action takes place between potassium sulphate and chromium sulphate, forming chrome alum, according to the following equation—

$$K_2SO_4 + Cr_2(SO_4)_3 = K_2Cr_2(SO_4)_4$$

The last is a useless product.

Instead of potassium bichromate, chromic acid is now generally employed; because by so doing, chrome alum, a useless product, is not formed and also because chromic acid dissolves much more readily than potassium bichromate.

The *E.M.F.* of this cell is equal to 1·5 volts. This remains constant only for a short time and then rapidly falls; but it recovers considerably if the cell be put out of action for some hours. This cell is convenient to use, when current is required for short intervals only.

To keep the zinc plate amalgamated, a small amount of mercurous sulphate may be added to the solution.

Leclanche cell. It consists of a rod of carbon *C*, placed in a porous cell, which is then filled with manganese dioxide and pieces of carbon. The porous cell and the zinc rod are both placed in an outer vessel, which contains ammonium chloride solution.

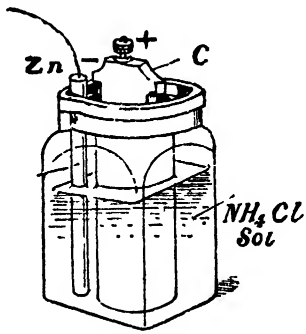


Fig. 3

The chemical actions which take place in the cell are:—

(*a*) The ammonium chloride acts on zinc, forming zinc chloride, hydrogen and ammonia gas.

$$2NH_4Cl + Zn = ZnCl_2 + 2NH_3 + H_2$$

(*b*) Ammonia gas comes out, while hydrogen penetrates into the inner porous cell and is *oxidized* by *manganese dioxide*, according to the following equation—

$$2MnO_2 + H_2 = Mn_2O_3 + H_2O$$

Thus manganese is converted to a lower oxide, which however, if left to itself acquires oxygen from the atmosphere and is again converted to manganese dioxide.

The *E. M. F.* of this cell varies from 1·48 to 1·61, which falls off rapidly on account of the fact, that hydrogen evolved is not readily oxidized by manganese

dioxide; but if left to itself, it regains its original strength. It is very suitable for getting intermittent currents and hence is largely employed for ringing bells, etc.

Grove's cell. It consists of an outer vessel of glass or china-clay, which contains a bent thick sheet of zinc, dilute sulphuric acid and a porous vessel. In the latter is placed, a thin sheet of platinum immersed in strong nitric acid, which acts as an oxidizing agent.

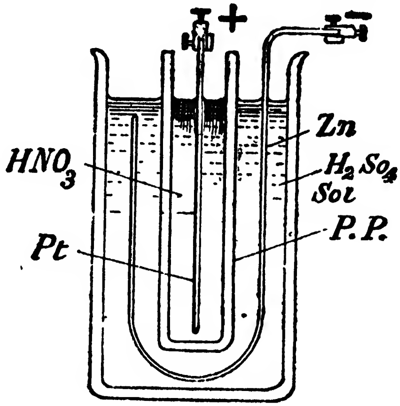


Fig. 4

The chemical actions, which take place are:—(*a*) zinc sulphate is formed and hydrogen evolved according to the following equation—

$$Zn+H_2SO_4=ZnSO_4+H_2.$$

This hydrogen penetrates into the porous cell and is oxidized to form water and nitrogen peroxide is given off—

$$H_2+2HNO_3=2H_2O+2NO_2$$

The *E.M.F.* of this cell ranges from 1·78 to 1·96 volts and remains fairly constant. This is very expensive on account of platinum and strong nitric acid used. It gives off nitrogen peroxide fumes, which are very injurious. It is used, when strong currents are required.

Bunsen's cell. It is very similar to the Grove's cell described above, with the modification that a carbon rod is used instead of a platinum sheet It makes the cell rather cheap. On this account, it has now chiefly replaced Grove's cell.

(*iii*) **Daniell's cell.** In this cell, hydrogen is not got rid off by oxidation, but by a substitution reaction. It

consists of an outer copper-vessel which contains saturated solution of copper sulphate and a porous cell. The latter contains a zinc rod immersed in dilute sulphuric acid. The outer copper vessel itself acts as the positive pole of the cell and to keep the copper sulphate solution saturated, crystals of copper sulphate are placed on the *ledge*.

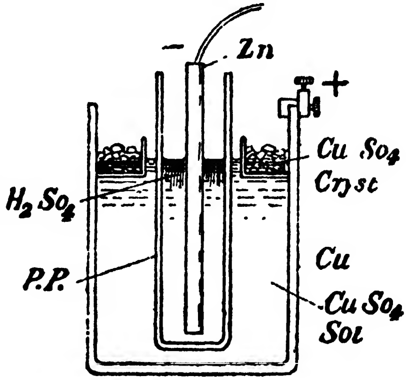


Fig. 5

The action of the cell is as follows :— Zinc sulphate is formed in the porous cell and hydrogen evolved comes out of it. It combines with the sulphate ion of the copper sulphate, to form sulphuric acid and copper is deposited on the outer vessel.

(*i*) $Zn + H_2SO_4 = ZnSO_4 + H_2$ (inner vessel)

(*ii*) $H_2 + CuSO_4 = H_2SO_4 + Cu$ (outer vessel)

Hence instead of hydrogen a thin film of pure copper is deposited on it.

The *E.M.F.* of the cell varies between 1·07 and 1·10 volts. It remains constant. It is used when a constant current is required for very long intervals.

Dry cells. The ordinary cells described above, are unsuitable for handy work, because they are not conveniently portable. For this purpose, the so-called dry cells are used. In reality, the cells are not dry and they can only work so long as the contents are moist. No doubt there is no liquid and hence there is no danger of its being spilt.

All the various forms of dry cells are in reality modified forms of Leclanche cell. The well-known is the Burnley or E.C.C. cell. It consists of a central carbon rod, surrounded by a black paste made of manganese dioxide, carbon powder, ammonium chloride,

zinc chloride and gum. Outside this is a thin layer of white paste, made of Plaster of Paris, ammonium chloride, zinc chloride and flour. All this is contained in a zinc case, which acts also as the negative plate. The top of this is covered over by a layer of melted pitch and through this upper cover passes a small tube, which allows the ammonia gas to escape.

The *E.M.F.* of this cell is ·75 volt. It is capable of giving current for about 48 hours.

The forms of cells suggested by electricians are too numerous to be dealt with here; but the most important of them have been described above.

SUMMARY

1. The essentials of a good cell are.—
(*a*) Its cheapness.
(*b*) Freedom from polarization.
(*c*) High E.M.F.
(*d*) Low internal resistance.
(*e*) No pungent odour.
(*f*) No local action.

2. The various methods of preventing polarization are·—(*a*) Mechanical, (*b*) Chemical and (*c*) Electro-chemical.

QUESTIONS

1. What are the essentials of a good cell?
2. Describe a Bichromate and a Leclanche cell.
3. Describe some form of a dry cell.

CHAPTER III

MAGNETIC EFFECTS OF CURRENTS

289. Take a magnetic needle pivoted on a point, so that it can move freely in a horizontal plane. Hold a straight wire AB above the needle with its axis parallel to that of the needle. Connect the straight wire to the terminals of a battery through a reversing key, so that a current can be reversed through the wire, whenever required.

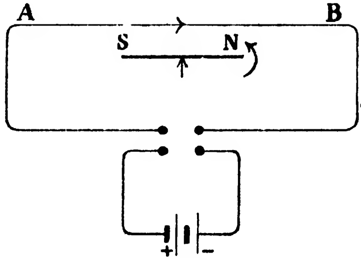


FIG. 6

Press the key to allow current to flow from A to B. Notice that the N-seeking pole is deflected in the direction, shown by the arrow-head, figure 6.

Now hold the wire under the magnetic needle and notice that the needle is deflected in the opposite direction. On reversing the direction of the current, the direction of deflection of the needle is also reversed. Keep the wire AB carrying the current perpendicular to the magnetic needle, no deflection is observed at all. Summarising, we have:—

(*i*) Wire above the needle	Deflection in one direction
(*ii*) ,, below ,, ,,	,, ,, opposite ,,
(*iii*) ,, above ,, ,, current reversed	,, ,, ,, ,,
(*iv*) Wire below the needle, current reversed.	,, ,, the same direction as in (*i*).

From the above results, it is evident that *a current*

passing over the needle in one direction and a current in the opposite direction, but under the needle, both tend to deflect the needle in the *same direction.* The effect of a weak current therefore, can be magnified by doubling the wire above and below the needle.

It is rather difficult to remember these results. The following rules have been given by various Physicists to express the relation existing between the direction of the current and the direction of deflection of the *N*-seeking pole.

(*i*) **Oersted's Rule.** Stand in such a way that the current flows away from you and hold a watch facing you so that you can read the time. Then if the current be supposed to flow through the spindle on which the hands of the watch are mounted, the *N*-seeking pole of the needle would turn in the same direction as the hands of the watch. Thus the *N*-seeking pole would turn from left to right, if it is above the wire carrying the current and from right to left, if it is below the wire.

(*ii*) **Maycock's Rule.** Grasp the wire carrying the current with your *right hand*, so that the thumb points in the direction of the current, then the *N*-seeking pole of a magnetic needle placed on the fingers, would turn towards the nails and its *S*-seeking pole, towards the knuckles.

(*iii*) **Ampere's Rule.** Suppose a man swimming with the current so as always to face the needle, the *N*-seeking pole will be deflected towards his left hand.

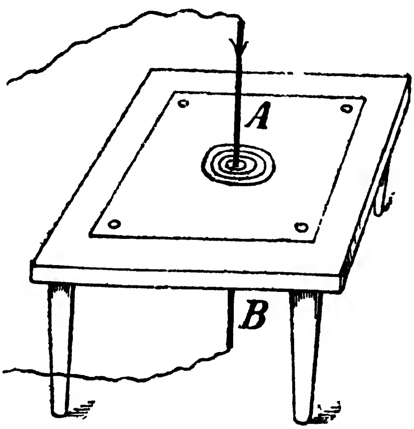


FIG. 7

The above mentioned rules give the same result; but Ampere's is by far the easiest in its application.

290. Magnetic field round a wire carrying a current. Make a hole in a piece of card-board and allow a wire carrying a current to pass through it. Keep the wire vertical and the piece of card-board horizontal. Sprinkle iron filings by a sieve and tap very gently. The iron filings arrange themselves in concentric circles round the wire as their common centre, as in figure 7.

To find whether the lines of force are in clock-wise or anti-clockwise direction, any one of the above three rules can be applied; but the relation is far better expressed by the following beautiful rule, which we owe to *Maxwell.*

(*iv*) **Maxwell's Rule.** The direction of the current and the positive direction of the lines of magnetic force, are related to each other in the same way, as the forward or backward motion of a right-handed screw is related to its direction of rotation.

Maxwell's rule is very useful, when we have to consider the magnetic effect produced by a current flowing in a loop of wire; because it applies either way *i.e.*: (1) if the forward or backward motion of the screw corresponds to the direction of the current, its direction of rotation gives the direction of the lines of force, or (2) if the screw be rotated in the direction of the current, its forward or backward motion gives the direction of the lines of force.

290. (*a*) Intensity due to a linear current. Biot and Savart showed that the magnetic intensity due to a current is directly proportional to the current and inversely proportional to the distance of the needle from the wire carrying the current. This can be shown to be true as follows :—

(*i*) Take a pivoted magnetic needle. Observe the number of oscillations, which it makes when placed at d_1 and d_2 cms. from the current-carrying wire. Let these be n_1 and n_2 per minute respectively. Stop the current and note down the number of oscillations, which it makes in the Earth's field alone. Let it be n.

Then it is seen that $\frac{n_1^2-n^2}{n_2^2-n^2}=\frac{I_1}{I_2}=\frac{d_2}{d_1}$;

i.e. the intensity of the field varies inversely as the distance.

(*ii*) Let n_1 be the number of oscillations per minute at a certain point, when current C_1 flows through the wire ; and n_2 the number at the *same point*, when current C_2 flows through the wire. Then it is found that $\frac{n_1^2-n^2}{n_2^2-n^2}=\frac{I_1}{I_2}=\frac{C_1}{C_2}$; *i e. the intensity of the field varies directly as the current strength.*

Laplace proved the above relations mathematically from certain assumptions ; but such a proof is beyond the Intermediate syllabus

291. Magnetic field due to a current in a circular wire. Pass the two ends of a piece of thick copper-wire through two holes in a piece of card-board and bend the wire into a circular form Keep the card-board in the middle, so that it cuts the circular loop into two hemispheres. Pass current through the loop and scatter iron filings over the card-board. Notice that the lines of force are circular near the wires; but at the centre, they are normal to the plane of the circular loop of wire The positive direction can be readily obtained by applying the Maxwell's Rule stated above.

Hence in a small space, near the centre of the loop, the magnetic field is very approximately uniform ; and is at right angles to the plane of the coil.

Magnetic Intensity at the centre of a circular loop of wire, carrying a current. It can be proved that the force on a unit North-seeking pole, when placed at the centre of a circular loop of wire of radius r and carrying a current of C units, varies as $\frac{2\pi C}{r}$; so that we can write $F\propto\frac{2\pi C}{r}$.

Or $F=\frac{2\pi C}{r}k$, where k is a constant, the value of which depends upon the units chosen.

Suppose we choose a unit of current, such that if it flows in a single circular loop of unit radius, it produces a magnetic intensity of 2π dynes at the centre; then if $C=1$, $r=1$, we have $F=2\pi$.

Substituting these in the above equation,

we have $2\pi = k\dfrac{2\pi \times 1}{1}$, $\therefore \; k=1$.

Or $F=\dfrac{2\pi C}{r}$, if the above unit of current be adhered to;

and $F=\dfrac{2\pi n C}{r}$, when there are n such circular loops.

The unit above considered is the *C.G.S.* electromagnetic unit of current. It may be defined as follows:— A current is said to be of *C.G.S. electromagnetic unit strength, which flowing in a single circular coil of unit radius, produces a magnetic intensity of 2π dynes at its centre.*

292. The Practical Units: The Ampere. As already stated, it is the *practical* unit of current; and is $\frac{1}{10}$ of the electromagnetic unit of current. *It may be defined as the current, which flowing in a single circular coil of unit radius, produces a magnetic intensity of $\dfrac{2\pi}{10}$ dyne at the centre of the circle.*

Coulomb. *It is $\frac{1}{10}$ of the electromagnetic unit of quantity of electricity; and is the quantity of electricity conveyed by one ampere in one second or the quantity required to deposit ·001118 gram of silver by electrolysis.*

Having defined the units of current, both electromagnetic and practical, it appears expedient to define the units of *E.M.F.* and resistance.

The electromagnetic unit of E.M.F. is defined as the *difference of potential existing between two points, such that one erg of work is done in moving an electromagnetic unit quantity of electricity from one point to the other.* This unit is too small to be used for practical purposes; and the practical unit employed, called

the **volt**, is 10^8 electromagnetic units. It is defined as the difference of potential between two points, such that one joule (10^7 ergs) of work is done in moving one *coulomb* (10^{-1} absolute unit) of electricity from one point to the other.

The *electromagnetic unit* of resistance is defined as the resistance of a conductor, which allows a current of one electromagnetic unit to flow, when the difference of potential existing at its ends is also equal to one electromagnetic unit of *E.M.F.* This unit is too small to be used for practical purposes; and the practical unit employed, is called the **Ohm**, which is 10^9 electromagnetic units. It is defined as the resistance of a conductor, which allows a current of one ampere to flow, when the difference of potential at its ends is equal to 1 volt. Its magnitude is very nearly equal to the resistance offered by a column of mercury, 106·3 cms long and one sq. mm. in cross-section at 0°C.

293. The Tangent Galvanometer. The fact, that magnetic intensity at the centre of a coil is uniform and varies with the strength of the current, is made use of in measuring the current by the instrument, known as the *tangent galvanometer*. The essential parts of such an instrument are: (*a*) *a magnetic needle*, (*b*) *a coil of wire and* (*c*) *a scale to read the deflections.* A simple form of such an instrument is shown in fig. 8. It consists of 3 separate coils of 2, 50 and 500 turns each, of well-insulated copper wire, wound round the circular frame; and the ends of these coils are attached to the screw terminals fixed in the base-board. The latter is provided with three levelling screws to level the instrument. The ends of the coils are so connected, that any one coil may be used separately, or any two or more together as one coil. In the middle of the vertical frame is a horizontal circular box: and at its centre is pivoted a very small magnetic needle, carrying a light long pointer, at right angles to its length. This pointer moves over a scale graduated in degrees. The needle is pivoted at a point, which is

the common centre of both the vertical frame and the horizontal circular box. To use the instrument for measuring currents, it is first levelled by the levelling screws in such a manner, that the needle and the pointer move freely and do not touch the scale. After this adjustment has been made, the vertical frame is turned and placed in the *magnetic meridian.* The needle is thus made to lie in the plane of the coil and the pointer reading arranged to

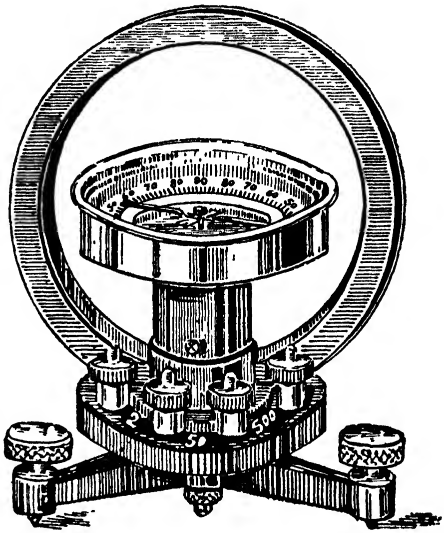


FIG. 8

be zero. When a current begins to flow round any of the coils, the needle is deflected; because then in addition to the Earth's intensity, force due to the current flowing in the coil comes into existence and acts in a direction perpendicular to it. The deflection produced is proportional to the strength of the current.

293. (*a*) **Theory of the relation between the current and the deflection.** Let $N'S'$ fig. 9 represent the needle in the deflected position: NS being the position, when no current is flowing. The needle takes the position $N'S'$ under the action of two forces, one due to the Earth and the other due to the current. If the Earth's horizontal component be denoted by H and m be the pole-strength of the small magnetic needle; then a force equal to mH acts at each end of the needle, tending to bring it back to the magnetic meridian. These two forces are equal in magnitude, parallel

and act in opposite directions; and hence they constitute a *couple*. The effect of a couple is measured by its moment, which is equal to the product of one of the forces and the perpendicular distance between them. If the length $N'S'$ of the needle be equal to l and the deflection equal to θ; then from fig. 9, it is clear that the perpendicular distance between these

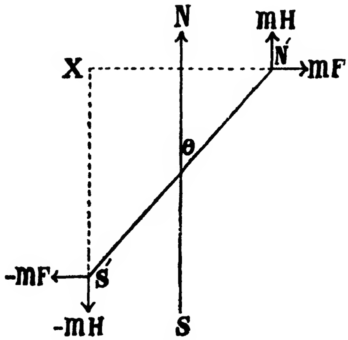


FIG. 9

forces$=XN'=l\sin\theta$. Therefore the moment of the couple due to the Earth$=mH.l\sin\theta$.

Now consider the effect of the intensity due to the current. Let this be denoted by F, then the force acting on each pole of the needle is equal to mF in magnitude, which tends to turn the needle in the opposite direction. The forces acting on the two poles of the needle due to the current are parallel, equal and opposite; therefore they also constitute a couple and the moment of this couple is similarly equal to

$$mF.XS'=mF.l\cos\theta$$

As in the position $N'S'$ the needle is at rest, the forces acting on it must be in equilibrium. But we have seen that the forces acting, constitute two couples. Therefore their moments must be equal. Hence we must have—

$$mH.l\sin\theta=Fm.l\cos\theta$$

But the force F at the centre of a coil of radius r and n turns is given by $\frac{2\pi nC}{r}$, where C is the current flowing through it, when measured in electromagnetic units. Therefore, substituting this value in the equation, we have— $mH.l\sin\theta=\frac{2\pi nC}{r}.ml\cos\theta$

or $C = \dfrac{Hr}{2\pi n} \tan\theta$, in electromagnetic units.

or $C = 10\ Hr. \tan\theta/2\pi n$, in Amperes.

Thus C is proportional to the tangent of the angle of deflection.

The quantity $\dfrac{10Hr}{2\pi n}$, by which the tangent of the deflection must be multiplied to give the current in *Amperes*, is called the **Reduction factor** of the galvanometer and is denoted by K.

Thus we have $C = K \tan\theta$.

From this we see, that if H be known, current can be measured; because quantities n and r admit of easy measurement. The quantity $\dfrac{2\pi n}{r}$, called the galvanometer-coil constant, is denoted by G.

Thus the expression for current can be written as

$$C = \frac{H}{G} \tan\theta$$

In using a galvanometer, after setting it according to the directions given above, the undernoted precautions must always be borne in mind.

Read both ends of the pointer. Reverse the current and again read both ends of the pointer. The mean of the four readings is the correct reading.

This is essential to safeguard against the following sources of error —

(*i*) The pointer may not be perpendicular to the needle.

(*ii*) The pivot-point of the needle may not be in the centre of the horizontal scale

(*iii*) The frame may not be exactly in the magnetic meridian.

294. Sensitiveness of a Tangent Galvanometer.— A galvanometer is said to be sensitive, when for a given current, it gives a large deflection. The expression for current C is $\dfrac{rH}{2\pi n} \tan\theta$. Thus to get a large deflection for the same current, one of the three things must be fulfilled :—

Either (*i*) r the radius of the coil must be decreased;

or (*ii*) n the number of turns of the coil must be increased;

or (*iii*) H the value of the Earth's horizontal component must be decreased. This can be done by the control magnets, so arranged as to produce a field in the direction opposite to that of the Earth's.

The fourth way to increase the deflection is by mechanical means, such as (*i*) by making the pointer very large, or (*ii*) by attaching a mirror to the needle and reading the deflections by the reflected beam. The deflection in the latter case is doubled on account of the fact, that when a mirror is turned through any angle, the reflected ray is turned through double that angle.

Caution—While using a Tangent Galvanometer, it is essential that the deflection should neither be too large nor too small For accurate work, it should be 45° or somewhere near that.

Let us suppose that a current C produces a deflection θ in a tangent galvanometer and that a current $C+dC$ produces a deflection $\theta+d\theta$, where dC and $d\theta$ represent large increments of current and deflection respectively. We want that for dC, $d\theta$ should be as great as possible.

We have $C=K\tan\theta$ . . . (*i*)

Differentiating C with respect to θ, we get

$$\frac{dC}{d\theta}=K\sec^2\theta \qquad (ii)$$

$$\text{or } dC=K\sec^2\theta\, d\theta \qquad (iii)$$

Dividing (*iii*) by (*i*), we get

$$\frac{dC}{C}=\frac{\sec^2\theta}{\tan\theta}\cdot d\theta \qquad (iv)$$

$$\text{or } dC=C.\frac{1}{\cos^2\theta}\cdot\frac{\cos\theta}{\sin\theta}\cdot d\theta$$

$$dC=\frac{C}{\cos\theta\sin\theta}\,d\theta$$

$$=\frac{2C}{\sin 2\theta}\cdot d\theta$$

In order that $d\theta$ should be as large as possible, the expression $\frac{2C}{\sin 2\theta}$ should be as small as possible. It is so,

when $\sin 2\theta$ is maximum, *i e.* when $2\theta=90^\circ$ or $\theta=45^\circ$.

295. Astatic galvanometer.—Another method of increasing the sensibility of a galvanometer is to use an astatic pair, first devised by Nobili. In this instrument, two needles of very nearly the same strength are rigidly connected with their axes parallel and their poles turned in opposite directions. This pair is hung by means of a fine quartz *fibre*, which has a small mirror attached to it. If the two needles be of exactly the same strength, then the pair should come to

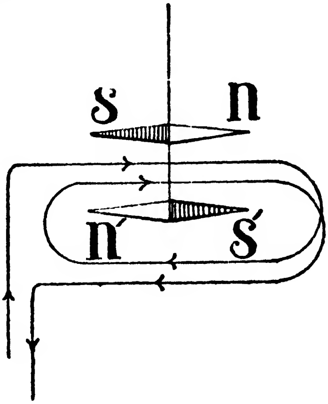


FIG. 10

rest in any position; but as it is very seldom the case, the pair points north and south, due to the greater force acting on the stronger of the two magnets. For if m and m' be the pole-strengths of the two needles, then the Earth's force would be $(m-m')\ H$. This pair would then behave like a single magnet, the pole-strength of which is very small. The coil is wound round the lower needle, as shown in fig. 10. By applying Ampere's Rule, it is evident that the force, due to the current in the upper portion of the coils, tends to turn both the needles in one direction; while the force, due to the current in the lower portion of the coils, tends to turn the lower needle also in the same direction, but the upper needle in the opposite direction. Thus we see that, three forces tend to turn the pair in one direction, while the fourth tends to turn it in the opposite direction. But as the distance of the upper needle from the lower turns of the coil is very great, the fourth force, which acts in opposition to the other three, is very feeble and can be neglected. Thus we see that the deflection is enhanced on account of: (*i*) the decrease in the Earth's controlling force and (*ii*) the multiplicity of the effect of the current.

SUMMARY

1. When a wire carrying a current is held over a magnetic needle, the latter is deflected when the former is parallel to it It is not deflected, when it is held at right angles to its length.

2. The direction, in which the *N*-seeking pole is deflected, is given by —

(*a*) **Oersted's Rule.** "Hold a watch so that you can read the time. Then if the current be supposed to flow through the spindle away from you, the direction of motion of the hands of the clock, gives the direction of deflection of the *N*-seeking pole."

(*b*) **Maycock's Rule** Hold the wire carrying the current in your right hand, in such a manner that the thumb points in the direction in which the current is flowing . then the *N*-pole of a needle placed on the fingers, would be deflected towards the nails and the *S*-pole, towards the knuckles.

(*c*) **Ampere's Rule** Suppose a man swimming with the current, with his face always towards the needle, then the *N*-pole would be deflected towards his left hand.

(*d*) **Maxwell's Rule** The direction of current and the positive direction of the lines of force, are related to each other in the same manner, as the forward and backward motions of a right-handed screw, are related to the directions of motion of its head.

3. The *force* at the centre of a coil of radius r and n turns is $\frac{2\pi nC}{r}$, when C is measured in *C G.S. electromagnetic units* of current

4. The *electromagnetic unit of current* is that, which when flowing in a circle of radius one cm., produces a magnetic intensity of 2π gauss at its centre.

5 The practical unit, called the **Ampere,** is $\frac{1}{10}$ of the electromagnetic unit and is equal to the current, which flowing in a single circular coil of unit radius, produces a magnetic intensity of $\frac{\pi}{5}$ gauss at its centre.

6 The electromagnetic unit of *E M F.* is the difference of potential between two points, such that an **erg** of work is done, when an electromagnetic unit quantity of

electricity is moved from one point to the other.

The practical unit of *E.M.F.*, called the **volt**, is the difference of potential between two points, such that a joule (10^7 ergs) of work is done, when a practical unit of current, *i.e.* an *Ampere* is moved from one point to the other. It is thus equal to 10^8 times the electromagnetic unit.

7. The electromagnetic unit of resistance is the resistance of a conductor, which allows an electromagnetic unit of current to flow, when the difference of potential at its ends is equal to an electromagnetic unit. This unit being too small, the practical unit employed, called the **Ohm**, is 10^9 times the electromagnetic unit. It is the resistance of a conductor, which allows a current of one Ampere to flow, when the difference of potential at its ends is equal to one Volt.

8. Tangent Galvanometer is an instrument for measuring currents, the formula is

$$C \text{ in amperes} = \frac{10Hr}{2\pi n} \cdot \tan\theta$$

9. Astatic Galvanometer is a sensitive form of galvanometer.

EXAMPLES

1. What are the various rules for finding the direction of deflection of a magnetic needle?

2. What are the electromagnetic units of : (*i*) *E M F.*, (*ii*) Current and (*iii*) Resistance? Name the practical units and express them in electromagnetic units

3. Describe a Tangent Galvanometer, and give its theory briefly.

4. How would you make a Tangent Galvanometer sensitive?

5. Describe the construction and working of an Astatic Galvanometer. Is it possible to measure current by its means?

CHAPTER IV

ACTION OF A MAGNET ON A CURRENT

296. Suppose W represents the cross-section of a wire carrying a current downwards in the paper and N represents an isolated N-seeking pole; the latter will tend to move round the wire in the clockwise direction. But if N be fixed and W free to move; then it will rotate in such a manner, as to occupy the same relative position with respect to N, as would have been the case if W were fixed and N free to move, because the force is mutual between the two and by Newton's III Law of Motion, "Action and Reaction are equal and opposite." Hence W will move in a direction opposite to the direction of motion of N and will occupy the position W', shown in fig. 11. This motion will continue, so long as the current continues to flow. It is also clear, that if the direction of the current be reversed, the direction of motion of the wire will also be reversed.

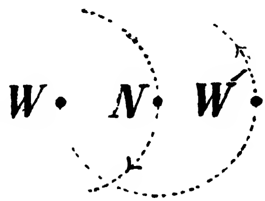


FIG 11

Experiment —Take a powerful electromagnet, send a current by one flexible wire and let it go up by the other. When the current begins to flow, the two wires separate. The direction in which the wires move can be found by applying **Fleming's Rule:**—

Extend the first and second fingers and the thumb of the **left hand**, *so that all the three are at right angles to one another. Point the first finger along the direction of magnetic field, the second in the direction of the current; then the thumb gives the direction in which the wire carrying the current will move, or the direction of*

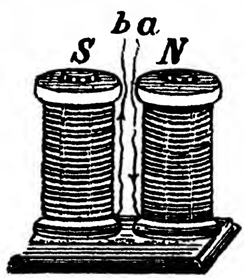


FIG. 12

the **Electromagnetic force**

The rotation of a conductor, carrying a current in a magnetic field, is shown by the aid of the apparatus of fig 13

(1) N is the north-seeking pole of a magnet, in the centre of a mercury cup. Wires from a battery dip into it, when the current is turned on, the upper wire rotates round the magnetic pole. On reversing the current, the wire begins to rotate in the opposite direction.

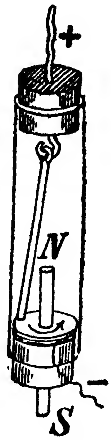


FIG. 13

(2) It is further demonstrated by **Barlow's wheel** fig. 14, which consists of a toothed-wheel, supported on a horizontal metal axis and capable of rotation about the same. The teeth of the wheel, in their lowest positions, dip into a mercury cup. A strong horse-shoe shaped magnet is placed, so as to enclose the teeth of the wheel in between its two poles. If now a current be made to flow from the centre of the wheel to its periphery, the wheel begins to rotate and the direction of rotation is given by **Fleming's left hand rule.**

297. Solenoid. A wire wound round a circular cylinder in such a manner, that it is equivalent to a number of circular loops, arranged so that the current runs simultaneously in them, is called a solenoid.

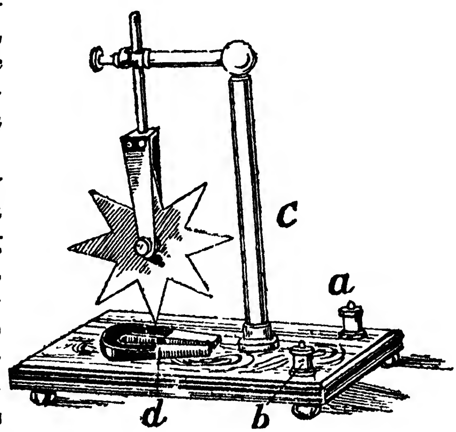


FIG. 14

It has already been explained that a current, running in a circular loop, produces a magnetic field inside it. The field produced inside a solenoid, which consists of a number of such loops, is of the nature similar to that of a bar magnet, the axis of which

coincides with the axis of the solenoid. If a piece of iron be placed inside this solenoid, as shown in fig. 15, the magnetic field due to the current running in the solenoid, becomes strong on account

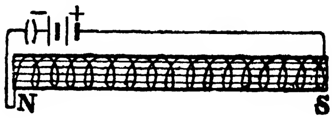


FIG. 15

of the high permeability of iron as compared with air. The iron piece, placed inside the solenoid, becomes a magnet; and if made of steel, it retains its magnetism, even after the current is stopped. This method of magnetizing by an electric current, is very satisfactory for making powerful magnets.

A solenoid as described above, capable of free motion in a horizontal plane, moves and sets itself in the magnetic meridian, when traversed by a current.

The polarity acquired by a solenoid, when traversed by a current, is given by the following rule.— Look at the face of the solenoid in such a manner that the line of sight is perpendicular to it and coincides with its axis; then if the current runs in anti-clockwise direction, it will exhibit *N*-seeking polarity and *S*-seeking polarity, if the direction is clockwise.

Take a rectangular coil of thick copper wire, suspend it by means of a fine wire attached to one of its terminals and allow the other end of the wire of the coil to dip into a small mercury cup. Pass current through the coil, by attaching the battery terminals to the mercury cup and the suspension-wire. Notice that this coil acts like a solenoid and turns round in such a manner, that the Earth's field runs through it at right angles

Place a strong magnet near the coil and see that it now turns in such a manner, that the lines of force due to the magnet run through it. Also notice that attraction or repulsion takes place between the magnet and the coil, as if the latter were a magnet. This happens, because the force due to the magnet is stronger than that due to the Earth.

It is evident that when a current flows through a coil, free to move in a magnetic field, it turns in such a manner, that the greatest number of lines of force due to the field pass through it in the same direction, as the lines of force due to the current in the coil itself. In other words, that end of the coil, which becomes north pole, will turn towards the south pole of the magnet, in whose field the coil is suspended.

298. Moving-coil Galvanometer. This form of galvanometer, of which D'Arsonval is a typical example, is shown in fig. 16. It consists of a vertically fixed strong horse-shoe shaped magnet, between the poles of which is suspended a rectangular coil of wire, capable of deflection round the suspension-wire at its axis. The rectangular coil is suspended from above by a fine metal wire, which acts as a *lead* for the current to the coil and is connected to one of the connecting screws. Another fine wire has its one end connected to the lower terminal of the coil, and the other to the end of a spring, which is connected to the second screw and acts as the second lead.

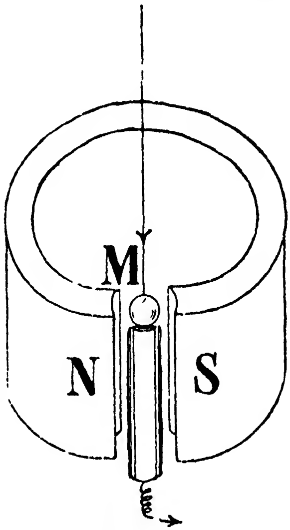


FIG. 16

The spring besides serving as a *lead* keeps the suspension-wire stretched and prevents it from breaking, when the coil rotates; it also keeps the rectangular coil in position between the two poles. In order to make the instrument very sensitive. the field in which the coil is to move, is made very strong by fixing a small cylinder of soft iron in the coil, with its axis parallel to the suspension-wire. To begin with, the coil is adjusted in position, so that with no torsion on the suspension-wire, the coil

stands between the poles, with its plane parallel to the direction of the field. When a current flows through the coil, it tends to set itself with its plane perpendicular to the field. But as soon as it begins to turn, a torsion is set up on the suspension-wire, which tends to bring the coil to its original position. Under the action of these two couples, *i.e.*: (*i*) due to the current and (*ii*) due to the torsion, the coil takes a position, where the two couples are equal in magnitude. Thus we see that the deflection is proportional to the current. This instrument is very useful for the following reasons.—

(*i*) The deflections are practically independent of external magnetic fields

(*ii*) The instrument may face in any direction, because the zero-position of the coil does not depend upon the direction of the Earth's field.

(*iii*) It is dead-beat in its action, *i.e.* the needle does not swing but takes a definite position

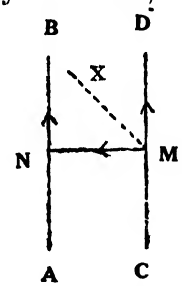


FIG. 17

299. Mutual Action of Parallel and Oblique currents.

1. *Parallel currents.* Let AB and CD represent two parallel wires, carrying currents in the directions shown by arrow-heads. If AB were fixed, then the wire CD would move towards AB along MN, by Fleming's Rule; for the direction of the magnetic field, due to the current in AB, may be represented by MX perpendicular both to CD and N. *Hence two parallel wires attract each other, if currents traverse them in the same direction and repel each other, if the currents are in opposite directions.*

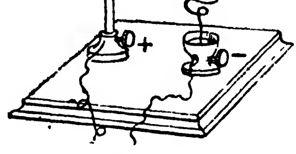


FIG. 18

2. *If two wires carrying the current are inclined to each other, then they would*

attract *each other, if currents traversing them proceed from or to the apparent point of intersection, but* **repel** *each other, if one current proceeds from and the other towards that point.*

3. The mutual attraction between currents is demonstrated by **Roget's Vibrating Spiral**, Fig 18 It consists of a coil of copper wire, with one end fixed up in the screw and the other dipping into a cup of mercury. As soon as a current flows through the coil, attraction between the successive turns of the coil takes place and it jumps up. The lower end of the coil gets out of the mercury and the current stops. There being no current, the coil comes back due to the action of gravity, and makes contact with mercury. Again it jumps up; and such series of movements continue, so long as the current is allowed to flow.

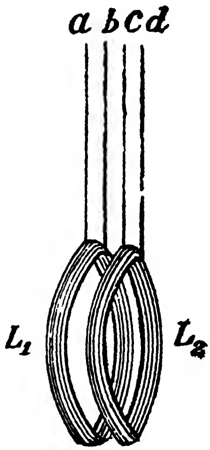


Fig 19

This is further beautifully demonstrated with the help of the simple apparatus, shown in fig. 19, where L_1 and L_2 are two coils of wire suspended from terminals *ab* and *cd*. First connect *bc* and pass current through the terminals *a* and *d* Notice the attraction between L_1 and L_2. Next join *b* to *d* and again pass current by joining terminals *a* and *c* to the battery Notice the repulsion.

SUMMARY

1 **Fleming's Rule** Extend the first and second fingers and the thumb of the left hand, in such a manner that each of them is perpendicular to the other two; now if the first finger denotes the direction of magnetic lines of force, the second, the direction of the current, then the thumb denotes the direction of motion of the wire carrying the current

2 Any circular wire of several turns is called a solenoid. When a current runs through a solenoid, it tends to turn the moving coil in such a manner, that its plane is perpendicular to the lines of force.

3 **D'Arsonval's** galvanometer is a form of galvanometer, which is very sensitive and deadbeat. It is very useful for accurate work.

EXAMPLES

1. Describe experiments to show that a wire carrying a current would move, when placed in a magnetic field. State Fleming's Rule to find the direction of motion of a current-carrying wire.

2. What is a solenoid? Sketch the field existing inside it, when a current is flowing through it.

3. What would happen, if a circular wire free to rotate, were run by a current?

4. Describe D'Arsonval's Galvanometer. What are its various advantages over an ordinary Tangent Galvanometer? Can it be used to measure currents?

CHAPTER V

ELECTROLYSIS

300. With regard to their behaviour towards an electric current, liquids can be divided into three classes:- (*i*) Insulators, which do not allow electricity to flow through them, such as oils ; (*ii*) Those, which allow the current to flow through them without any change, such as mercury; and (*iii*) Those, which undergo a chemical change, when electric current passes through them, such as solutions of salts, etc. These are called **Electrolytes.**

The decomposition of an electrolyte by the passage of electricity is called **Electrolysis.** Current is conveyed to and from an electrolyte, by immersing in it rods or plates of some metal or carbon. These rods or plates are called **electrodes**—that by which the current is conveyed is called the **anode** and that by which it leaves, is called the **kathode**. The product of electrolysis, *i.e.* the substances liberated are called the *ions*—that liberated at the anode is called the **anion** and that liberated at the kathode is called the **kation.**

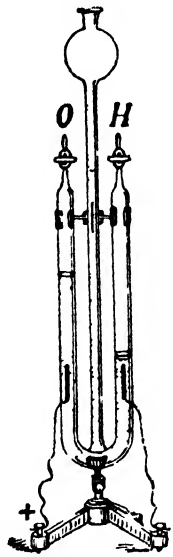


FIG. 20

An apparatus in which electrolysis of an electrolyte may be suitably conducted is called a **voltameter.** Fig. 20 shows a form of voltameter, suitable for the electrolysis of water. The taps are opened; and water mixed with a small quantity of sulphuric acid (ratio 7 : 1), is poured in by the funnel, till the vertical tubes are filled. The taps are then closed and current is passed through it by joining the terminals to the poles of a battery. The gas liberated at each electrode rises in the tube and collects in its upper part. It is seen that the volume of one gas is nearly double

than that of the other. On testing the gases with a splinter of wood and a lighted taper, it is found that they are oxygen and hydrogen, and that the latter has double the volume of the former. On carefully examining, it is found that *oxygen is liberated at the Anode* while *hydrogen is liberated at the Kathode.* Thus we see that the passage of electricity, through acidulated water, decomposes it. If instead of acidulated water, hydrochloric acid be used as the electrolyte, then chlorine collects at the anode and hydrogen at the kathode.

Next use a solution of copper sulphate as electrolyte and pass the current. Oxygen appears at the anode as before, while copper collects at the kathode.

From the above examples, we notice *that oxygen and chlorine always appear at the anode*, while *hydrogen and copper appear at the kathode.*

301. Theory of Electrolysis. The reason why electrolytes are decomposed by the passage of electricity is, that according to the theory of dissociation, molecules of a substance in a solution are broken up into parts called ions. These are of two kinds: one known as *electro-positive*, which carry a positive charge; and the other known as *electro-negative*, carrying a negative charge. *The amount of charge, carried by an ion is always directly proportional to its valency.* When the current begins to flow, the anode becomes $+$*vely* charged, while the kathode becomes $-$*vely* charged. The anode attracts the electro-negative ions and the kathode attracts the electro-positive ions; and the separation is effected by the migration of ions to the respective electrodes. To sum up, we have the following assumptions:—

(*i*) *Molecules in electrolytes are broken up into electro-positive and electro-negative ions, carrying charges proportional to their valencies.*

(*ii*) *Passage of electricity consists in the migration of electro-positive ions to the Kathode and that of electro-negative ions to the Anode.*

(*iii*) *Oxygen and all other gases, except hydrogen, are electro-negative and thus appear on the Anode. Hydrogen*

and metals are electro-positive and appear on the Kathode.

302. Secondary reactions. If hydrochloric acid or copper chloride be decomposed by electrolysis, chlorine appears on the anode, while hydrogen or copper appears on the kathode. The electro-chemical reactions can be expressed as follows:—

$$(i)\ 2HCl = H_2 + Cl_2$$
$$(ii)\ CuCl_2 = Cu + Cl_2$$

But now if instead of hydrochloric acid or copper chloride, sulphuric acid or copper sulphate be similarly treated; we get hydrogen or copper at the kathode, but never do we get the substance SO_4 at the anode. Instead of that we get oxygen. The reason of this is, that sulphate ion *i.e.* SO_4, is incapable of existing and so is broken up into SO_3 ion and oxygen ion. The former combines with water to form sulphuric acid; while oxygen is given up at the anode Such a reaction is called a secondary reaction. The reactions can be expressed as follows:—

$$(iii)\ H_2SO_4 = H_2 + SO_4 \quad (a)$$
$$2SO_4 = 2SO_3 + O_2 \quad (b)$$
$$2H_2O + 2SO_3 = 2H_2SO_4 \quad (c)$$
$$(iv)\ CuSO_4 = Cu + SO_4 \quad (a)$$
$$2SO_4 = 2SO_3 + O_2 \quad (b)$$
$$2H_2O + 2SO_3 = 2H_2SO_4 \quad (c)$$

The reactions expressed in equations *i*, *ii*, *iii* (*a*) and *iv* (*a*) are *primary reactions*, because they are brought about *by the passage of electricity through them*; while the reactions expressed in equations (*iii*) (*b*) and (*c*) & (*iv*) (*b*) and (*c*) are called *secondary reactions*, because they are not brought about by the flow of electric current, but by *chemical affinities*. To sum up, *a primary reaction is that, which is necessarily brought about by the flow of electricity; while a secondary reaction is that, which is brought about by the chemical affinity*

Thus in the electrolysis of sodium sulphate, the following Primary electro-chemical reaction takes place:—

$$(i)\ Na_2SO_4 = Na_2\ (\text{Kathode}) + SO_4\ (\text{Anode}).$$

After this the following secondary or chemical reactions take place:—

(ii) $Na_2 + 2H_2O = 2\ NaOH + H_2$

(iii) $2\ SO_4 + 2H_2O = 2\ H_2SO_4 + O_2$

Thus at the kathode, hydrogen and caustic soda are produced, while at the anode, oxygen and sulphuric acid are produced

303. Faraday's Laws of Electrolysis— Before discussing in detail, Faraday's laws of Electrolysis, it is expedient to define clearly the following chemical terms, which are frequently used —

(i) *Atomic weight*— It is the weight of an atom of the substance, relative to the weight of an atom of hydrogen, which is taken as unity.

(ii) *Valency*— It is the number of hydrogen atoms, which one atom of the substance can chemically displace.

(iii) *Chemical equivalent* of a substance is the ratio between its atomic weight and valency.

(iv) *Electro-chemical equivalent* of a substance is the weight in grams, which the passage of one coulomb of electricity would liberate

Faraday's First Law of Electrolysis— *The mass of an ion, liberated by a current, is proportional to the quantity of electricity, which passes through an electrolyte.*

Arrange two voltameters, one containing water, the other copper sulphate, in series with a battery of six Daniell cells, fig 21. Pass current through them for 15 minutes and measure the volume of *hydrogen* on the *Kathode* and that of *oxygen* on the *Anode* in the water-voltameter. Also find the amount of copper deposited on the kathode of copper sulphate voltameter Now in the above experiment, allow the same current to flow for 30 minutes, the quantity of each substance set free, is found to be twice as much as was liberated, when the current was allowed to pass for 15 minutes, provided however, the current strength remains the same This law can be put to test in several ways and would always be found to be strictly true.

Faraday's Second Law of Electrolysis. *If several different electrolytes are included in the same circuit, the quantities of different ions liberated are proportional to their chemical equivalents.* In the above experiment it is seen, that the oxygen liberated is half in volume of the hydrogen; but oxygen is 16 times as heavy as hydrogen. Therefore the quantity by weight of oxygen liberated is 8 times as much as that of hydrogen; and the weight of copper deposited on the kathode in the copper voltameter, is found to be 31·8 times as much as that of hydrogen.

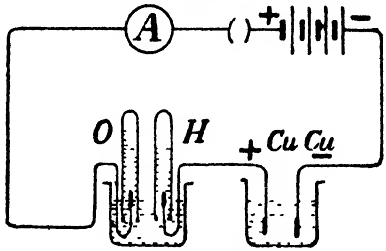


FIG. 21

The chemical equivalent of hydrogen is $\frac{1}{1}=1$

The " " of oxygen is $\frac{16}{2}=8$

and the " " of copper is $\frac{63{\cdot}6}{2}=31{\cdot}8$

We see that the quantities of different ions, liberated by the same quantity of electricity, are exactly proportional to their chemical equivalents.

This second law of Faraday enables us to find the electro-chemical equivalents of different ions, if we know the electro-chemical equivalent of one substance. Careful experiments show, that the electro-chemical equivalent of hydrogen is ·00001035 gm. and its chemical equivalent is equal to unity. The chemical equivalent of silver is 108, because its atomic weight is 108 and valency unity; therefore its electro-chemical equivalent, *i. e.* the quantity liberated by 1 coulomb, is $\cdot 00001035 \times 108 = \cdot 001118$ gm.

In the same way, electro-chemical equivalents of other substances may be obtained. Thus for example, the electro-chemical equivalent of copper would be

$$\cdot 0000135 \times \frac{63 \cdot 6}{2} = \cdot 000332 \text{ gm.}$$

Moreover it affords an accurate method of measuring currents; for ampere, the practical unit of current, has been defined as that current, which flowing in a silver voltameter, deposits ·001118 gm. of silver per second.

Faraday's Unit. The electro-chemical equivalent of hydrogen is ·00001035, therefore $\frac{1}{\cdot 00001035} = 96{,}600$ coulombs of electricity would liberate one gram of hydrogen. This quantity of electricity is called Faraday's unit.

304. Measurement of Galvanometer constant. From Faraday's 1st law, it is evident that if w be the electro-chemical equivalent of a substance, then the quantity m gms. deposited in t seconds by a current of C amperes would be wCt, so that $m = wCt$;

$$\text{or } C = \frac{m}{wt}.$$

If a galvanometer and a voltameter are in the same circuit, then the expression for current C can be written

as, $$C = \frac{m}{wt} = K \tan\theta.$$

Knowing the weight of the substance deposited during t seconds and also the value of w, the electro-chemical equivalent in the above equation, it is easy to find K, the galvanometer constant, which is equal to $\frac{C}{\tan\theta}$.

305. Laws of Electrolysis in Cells. The laws of electrolysis hold good in each cell. To produce every coulomb of electricity, ·000332 gm. of copper is deposited and 000338 gm. of zinc is dissolved, excluding that wasted in local action. Every such coulomb liberates ·00001035 gm. of hydrogen and ·0000832 gm. of oxygen.

306. Secondary batteries or Accumulators.

Experiment. Arrange the apparatus, as shown in fig. 22. A voltameter, containing dilute sulphuric acid, has lead plates immersed in it, and by means of a three-way key, it can be connected either to a battery or to an electric bell.

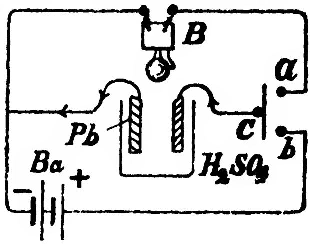


FIG 22

Connect *bc* and pass current through the voltameter for about 3 minutes. Disconnect *bc* and join *ac*. Notice, the bell begins to ring, showing that the current is now flowing from the voltameter.

This current, due to the strong tendency of liberated ions to re-unite, constitutes the *principle of secondary batteries or accumulators*. The explanation is this:—During electrolysis, **water** is decomposed and the chemical potential energy of the system is increased. This increase is furnished at the expense of the electrical energy of the current, employed to effect the decomposition. If however, when electrolysis has been performed, the external current be removed and the electrodes of the voltameter be connected by a wire; chemical action involving re-union between the products of electrolysis takes place and a current is sent **in the opposite direction**, *i e* the current now flows from c to *aB*.

Thus we see that an ordinary voltameter **may** act **as** an accumulator or a storage cell, but it is quite **useless from** the commercial point of view. The back **current is transient** only, and the capacity of such an **accumulator** is very small.

The modern commercial accumulators, devised by Plante and improved by Foure, consist of lead plates covered over by a layer of red-lead ($Pb_3\ O_4$), immersed in dilute sulphuric acid. On charging, the red-lead on the kathode is reduced to spongy metal and that on the anode becomes peroxidized. The actions which take place may be represented by the following equations—

At the kathode.—

$Pb_3O_4 + 3H_2SO_4 = 3PbSO_4 + 3H_2O + O$

$PbSO_4 + H_2 = Pb + H_2SO_4$

At the anode.—

$Pb_3O_4 + 3H_2SO_4 = 3PbSO_4 + 3H_2O + O$

$PbSO_4 + O + H_2O = PbO_2 + H_2SO_4$

If after charging in this manner, the two plates, which have been reduced to Pb and PbO_2, be connected by a wire, a current flows in the opposite direction; *i.e.* from PbO_2 to Pb in the wire and from Pb to PbO_2 through the electrolyte. Both the plates are reduced to $PbSO_4$ according to the following equations:—

(*i*) $2Pb + 2H_2SO_4 = 2PbSO_4 + 2H_2$

(*ii*) $2PbO_2 + 2H_2SO_4 + 2H_2 = 2PbSO_4 + 4H_2O$

When the cell has been wholly discharged, it can be recharged by passing a current from an external source. The water is electrolysed and the nascent oxygen on the anode converts the $PbSO_4$ into PbO_2, according to the equation—

$PbSO_4 + O + H_2O = PbO_2 + H_2SO_4$...... ...(*i*)

while the nascent hydrogen on the kathode reduces the $PbSO_4$ to the spongy metal thus—

$PbSO_4 + H_2 = Pb + H_2SO_4$ ... ... (*ii*)

The secondary batteries are easily portable and handy. In addition, they possess the following advantages —

(*i*) Their internal resistance is very low, being about ·02 Ohm per sq. foot of plate.

(*ii*) Their *E.M.F.* remains fairly constant.

(*iii*) Their *E.M.F.* is fairly high, being about 2·1 volts.

(*iv*) The density of the solution is the most convenient index of the condition of the cell. When fully charged, it should not be above 1·21 and at discharge, it should not be below 1·17.

These lead accumulators, inspite of their possessing the above advantages, are very heavy and suffer from the additional defect of '*buckling*' (*i.e.* disintegration of plates), when left uncharged. These defects have been removed or minimized in **Edison's Accumulator,** which consists of nickel and iron peroxide plates respectively, dipping in 20 per cent. caustic potash solution

The plates do not *'buckle'* and the battery is not so heavy as lead accumulators. The *E.M.F.* however, is only 1·3 volts instead of 2·1.

SUMMARY

1. The process of decomposing an electrolyte, by the passage of electricity through it, is called **electrolysis.**

2. Solutions of salts, which undergo decomposition, when a current flows through them, are called **electrolytes.**

3. Plates, carrying currents in an electrolyte, are called electrodes. That, which takes the current *in*, is called the **Anode** ; and that, which brings it *out*, is called the **Kathode.**

4. The products of electrolysis are called the ions.

(*a*) **Primary action** is that change, which is brought about, by the passage of electricity.

(*b*) **Secondary action** is that, which follows the primary, and is brought about, only by the chemical affinity.

5. **Faraday's laws:**—

(*i*) The mass of an ion liberated by a current is proportional to the quantity of electricity, which passes through an electrolyte.

(*ii*) The quantities of different ions liberated, are proportional to their chemical equivalents.

6. Secondary battery or an **accumulator** is simply a storage cell. When a current is passed through it, certain chemical changes take place. When the external circuit is removed, then on joining the two electrodes, a current flows in theo pposite direction and the chemical changes are reversed.

EXAMPLES

1. Define anode, kathode, ion, electrolyte, chemical equivalent, Primary action and Secondary action.

2. Enunciate and prove Faraday's Laws of Electrolysis.

3. Describe Plante's secondary battery and explain the principle on which it is based.

4. How would you arrange to measure a current by a copper voltameter?

5. How may electrolysis be used to test the accuracy of an instrument, designed to measure a current in Amperes.

6. What will happen when (*a*) one Daniell cell and (*b*) two Daniell cells, are connected with a water-voltameter?

CHAPTER VI

OHM'S LAW

307. In its simplest form, Ohm's Law states: 'At uniform temperature, the current produced in any portion of a homogeneous conductor, the terminals of which are kept at different potentials, is directly proportional to the difference of potential between those two terminals.'* Thus if C be the current in a conductor, when the difference of potential existing between its ends is E; then by Ohm's Law stated above, we have C varies as E.

Or $\frac{E}{C}=K$, where K is a constant for the given conductor. This constant K is known as its *Resistance*; so we can write the above equation as

$$R=\frac{E}{C}; \text{ or } C=\frac{E}{R}; \text{ or } E=RC.$$

The second equation is the full expression of Ohm's Law, which expresses that the strength of the current varies directly as the electromotive force and inversely as the resistance of the circuit.

Using the practical units, which have already been described, we can restate Ohm's Law as follows :—The number of amperes of current, flowing through a circuit, is equal to the number of volts of *E.M.F. divided by* the number of Ohms of resistance in the entire circuit.

This is experimentally proved in the following manner:—Arrange a circuit, as shown in fig. 23, consisting of a battery, a standard wire-resistance R, an ammeter A

* It should be noted however, that the resistance of *selenium* decreases, when exposed to light and that of bismuth increases, when exposed to a strong magnetic field.

to read the current flowing in the circuit and a voltmeter V to denote the $E.M.F.$ at the terminals of the resistance R. Carefully note the resistance of R in Ohms, the current in Amperes and the $E.M.F.$ in volts.

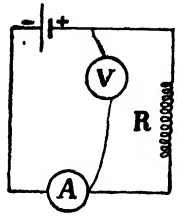


FIG. 23

Now increase the number of cells in the battery and repeat the former readings again. By successively increasing the number of cells, a series of such observations may be taken and results entered in the following manner:—

	Amperes	*Volts*	*Volts÷Amperes*
1	0·2	4	$4\div0{\cdot}2=20$
2	0·3	6	$6\div0{\cdot}3=20$
3	0·4	8	$8\div0{\cdot}4=20$

The results obtained in the last column are the same for the same resistance. This shows that for a given wire, the ratio of the $E.M.F.$ at the terminals of a wire to the current flowing through it, is *constant.* This result, discovered by Ohm in 1826, is known as **Ohm's Law.**

$$\text{Thus}\ \frac{E}{C}=R,\ \text{or}\ C=\frac{E}{R}.$$

The above experiment may for the sake of convenience, be slightly modified, by having a standard battery, changing the resistance R successively and showing that the products of Ohms and Amperes are always equal to Volts.

Careful observations show that the resistance of a conductor varies directly as the length and inversely as the area of cross-section, and depends upon the material; so that $R\propto\frac{l}{a}$, or $R=S\frac{l}{a}$, where S is a constant depending on the nature of the material of

the wire and is known as the *Specific resistance* of the material. In the above equation, if l and a be each equal to unity, we have $R=S$. *Thus the specific resistance of any substance is the resistance of a cube of that substance of unit edge, when the current enters its one face and leaves the opposite face.*

Specific conductivity of a substance is defined as the reciprocal of the specific resistance of its material.

Thus the specific conductivity $k=\frac{1}{R}$.

It should be borne in mind that the $E. M. F.$ goes on falling along the length of a wire; for by Ohm's Law, we have $C=\frac{E}{R}$; but current remains constant in every part of the circuit and the resistance decreases as the length of the wire decreases, therefore the $E. M. F.$ falls proportionately.

308. The Resistance-Box. To determine $E.M.F.$ and resistance practically by the application of Ohm's Law, need arises of coils of known resistances. For this purpose, wires of standard resistances, arranged in the manner shown in figure 24, are sold by instrument-makers under the name of Resistance-box. This consists of coils of german silver of such lengths as to have resistances of a definite number of Ohms. These coils are highly insulated and after being doubled

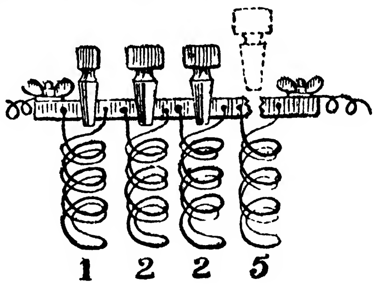


FIG. 24

on themselves, they are wound in the manner shown in the figure to avoid *self-induction*. Each end of a coil is soldered to two adjacent solid brass pieces, which are fixed on an ebonite block, which forms the cover of a closed box. Sufficient space is left between successive brass pieces and stout brass plugs, with ebonite heads,

fit tightly, when inserted in between the two pieces. So long as the plugs remain *in*, the current flows through the solid brass pieces and plugs, without encountering any resistance in its passage, but when any plug is removed, current can only pass from one brass piece to another by traversing the coil, connecting those two brass pieces. Thus a resistance equal to the resistance of the above coil is introduced in the circuit. The series of resistance coils, usually fitted in a resistance-box, are 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, 200, 500 and 1000 Ohms. By this arrangement, any resistance, correct up to whole numbers, can be introduced in the circuit. Thus if we want to introduce 173 Ohms, we should take out the following plugs, 100+50+20+2+1.

309. Ohm's Law applied to the comparison of E M.F's. The following are some of the various methods employed for comparing *E M.F.'s* of two cells :—

(*i*) **Variable Deflection method.** The apparatus is arranged, as shown in fig. 25, C_1 and C_2 are the two cells, the *E. M. F.'s* of which are to be compared. *R. K.* is a reversing key, *R. B.* a resistance-box and *G.* a Tangent galvanometer. The deflections of the galvanometer-needle due to C_1 and C_2 are noted, then the ratio of the tangents of the angles of deflection is the ratio of *E.M.F.'s* of the two given cells. The resistance of the circuit should be the same in both the cases.

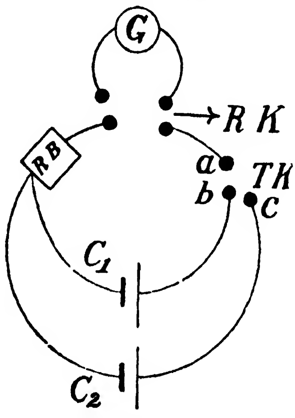


FIG. 25

Theory—According to Ohm's Law, we have

$$E_1 = i_1 R = R\ K \tan\theta_1{}^* \quad . \quad . \quad (i)$$

and $$E_2 = i_2 R = R.K \tan\theta_2 \qquad (ii)$$

Dividing equation (*i*) by (*ii*),

---

* $i = K \tan\theta_1$, when it produces a deflection θ_1 in a tangent galvanometer.

We have $\dfrac{E_1}{E_2}=\dfrac{\tan\theta_1}{\tan\theta_2}$,

where E_1 and E_2 are the *E.M.F.'s* of the two cells and i_1 and i_2 the currents respectively.

(*ii*) **Equal Deflection method.** The arrangement of the apparatus is the same as shown above. Current due to C_1 is made to flow and resistance R_1 introduced from the resistance-box, till a suitable deflection θ is obtained. Next C_1 is put out of circuit. Current due to C_2 is allowed to flow and a resistance R_2 introduced from the resistance-box, till the deflection is the same as before, *i.e.* equal to θ.

Then $\dfrac{E_1}{E_2}=\dfrac{\text{Total resistance in the circuit in the first case}}{\text{Total resistance in the circuit in the second case}}$.

By Ohm's Law, we have

$$E_1=i_1\,(R_1+G+r_1). \qquad (i)$$

and $$E_2=i_2(R_2+G+r_2). \qquad (ii)$$

where G is equal to the resistance of the galvanometer, r_1 and r_2 are the internal resistances of the cells C_1 and C_2 respectively. Therefore dividing (*i*) by (*ii*), we get

$$\frac{E_1}{E_2}=\frac{R_1+G+r_1}{R_2+G+r_2}, \text{ because } i_1=i_2;$$

for the deflection of the galvanometer is the same in both the cases.

(*iii*) **Sum and difference method.** In this method the two cells, the *E.M.F.'s* of which are to be compared, are first arranged in series. The deflection, which they produce, is noted. Let it be θ_1. The cells are then arranged so as to send currents in opposite directions, *i.e* the $+ve$ pole of one is connected with the $+ve$ pole of the other and the negatives of the two cells are taken as the battery terminals. The deflection produced, which would be very small, is noted. Let it be θ_2; then

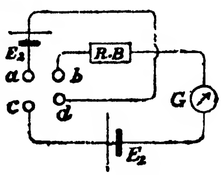


FIG. 26

$$\frac{E_1}{E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_1-\tan\theta_2},$$

for by Ohm's law we have, when the cells are in series,

$$E_1+E_2=Ri_1=RK\tan\theta_1 \quad . \qquad (i)$$

but when arranged to send currents in opposite directions,

$$E_1-E_2=Ri_2=RK\tan\theta_2 \quad \ldots \quad (ii)$$

Dividing (i) by (ii), we have

$$\frac{E_1+E_2}{E_1-E_2}=\frac{\tan\theta_1}{\tan\theta_2}.$$

Subtracting unity from both sides, we have

$$\frac{E_1+E_2-E_1+E_2}{E_1-E_2}=\frac{\tan\theta_1-\tan\theta_2}{\tan\theta_2}$$

$$\text{or } \frac{2E_2}{E_1-E_2}=\frac{\tan\theta_1-\tan\theta_2}{\tan\theta_2} \quad \ldots \quad (iii)$$

Similarly adding 1 to both sides, we have

$$\frac{E_1+E_2+E_1-E_2}{E_1-E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_2}$$

$$\therefore \quad \frac{2E_1}{E_1-E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_2} \quad \ldots \quad (iv)$$

Dividing (iv) by (iii), we have

$$\frac{E_1}{E_2}=\frac{\tan\theta_1+\tan\theta_2}{\tan\theta_1-\tan\theta_2}.$$

If *ac* and *bd* fig 29, be connected, the cells become arranged in series; and if *ab* and *cd* be connected, the cells become arranged in opposition.

(iv) **Deflection method.** The arrangement of the apparatus is, as shown in fig. 25. Put C_1 in circuit and take out plugs from the resistance-box, till the deflection in the galvanometer is d_1 and the resistance introduced from the resistance-box R_1. Then by Ohm's Law the current flowing is

$$\frac{E_1}{R_1+G+r_1}=i_1=K\tan\theta_1-i \quad \ldots \quad (i)$$

where $E_1=E.M.F$ of the cell C_1
$G=$Resistance of galvanometer
$r_1=$ „ cell C_1
$i_1=$current
$K=$Reduction factor

Now alter the resistance in the box, till the deflection is θ_2 and the resistance introduced R_2,

Then we have, as above

$$\frac{E_1}{R_2+G+r_1}=i_2=K\tan\theta_2 \quad \ldots \quad (ii)$$

Inverting the equations and subtracting, we have

$$\frac{R_1+G+r_1}{E_1}-\frac{R_2+G+r_1}{E_1}=\frac{R_1-R_2}{E_1}$$

$$=\frac{1}{K\tan\theta_1}-\frac{1}{K\tan\theta_2} \quad \ldots\ldots \quad (a)$$

Remove cell C_1 out of the circuit and put C_2 in the circuit. Put in a resistance R_1' in the circuit from the resistance-box, till the deflection with cell C_2 is d_1; then we have

$$\frac{E_2}{R_1'+G+r_2}=i_3=K\tan\theta_1 \quad \ldots\ldots \quad (iii)$$

Now alter the resistance in the box, till the deflection $=d_2$ and the resistance introduced R_2'.

Then we have similarly

$$\frac{E_2}{R_2'+G+r_2}=i_4=K\tan\theta_2 \quad \ldots\ldots \quad (iv)$$

Then inverting equations (*iii*) and (*iv*) and subtracting as before, we have

$$\frac{R_1'-R_2'}{E_2}=\frac{1}{K\tan\theta_1}-\frac{1}{K\tan\theta_2} \quad \ldots\ldots \quad (b)$$

Then equating (*a*) and (*b*), we have

$$\frac{R_1-R_2}{E_1}=\frac{R_1'-R_2'}{E_2}$$

$$\text{or} \quad \frac{E_1}{E_2}=\frac{R_1-R_2}{R_1'-R_2'}.$$

This method, though somewhat more tedious, gives better results.

(*v*) **The Potentiometer method.** A Potentiometer consists of a thin wire of manganin * of uniform area of cross-section, generally 1 to 10 metres long (according to requirements), stretched on a wooden board, provided with binding screws and a movable jockey point, to make contact with any point of the wire.

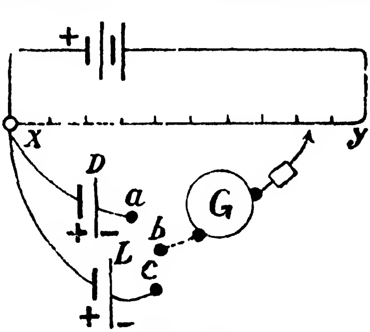


FIG. 27

When current from a battery, consisting of a few accumulators, is passed

* Manganin is chosen on account of its high resistance.

through such a wire, the potential falls uniformly from one end X to the other end Y; and the potential difference between any two points on the wire is proportional to their distance apart.

To compare the *E.M.F.'s* of the two cells, their positive terminals are connected at X, *where the positive terminal of the battery is connected.* Their negative poles are connected to the screws of a three-way key. as shown in fig. 27. The third terminal, which can be connected to either of the cells, is joined through a galvanometer and a high resistance to the movable jockey. First one cell, say D, is put in circuit by connecting *ab* and the jockey is shifted to a point A along the potentiometer wire to get no deflection in the galvanometer (which will happen, when the *E.M.F.* of the cell D is equal to the Potential difference between X and A on the potentiometer wire, due to the battery). Let this length be l_1.

Now disconnect D, introduce the cell L and proceed accordingly, to find another position B of no deflection in the galvanometer. Let this length from X be l_2.

$$\text{Then } \frac{E.M.F. \text{ of } D}{E.M.F. \text{ of } L} = \frac{P.D. \text{ between } XA}{P.D. \text{ between } XB} = \frac{l_1}{l_2};$$

because the potential difference along a uniform conductor is proportional to its length, as explained above.

The important points to be noted are: (*i*) The *E.M F.* of the battery should always be much greater than the *E.M F.* of either cell. (*ii*) The positive poles of the cells and the battery must be connected to the same binding screw. (*iii*) A high resistance should be put in series with the galvanometer to prevent any damage to it, due to accidental strong current passing through it.

310. Resistance of conductors. If a number of conductors of resistances r_1, r_2 and r_3 be arranged *in series* as shown in the upper diagram of fig. 28; then the combined resistance of the three, is simply $r_1+r_2+r_3$. For let C be the current flowing through the circuit, then the differences of potential at the terminals of

various resistances, by Ohm's Law, are given by Cr_1, Cr_2 and Cr_3. Therefore the total difference of potential

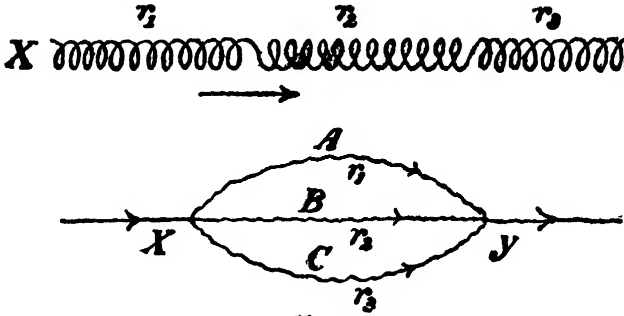


FIG. 28

between the extreme ends of these will be $C(r_1+r_2+r_3)$; but if R is the combined resistance, then the difference of potential must be CR.

$$\therefore \quad CR=C(r_1+r_2+r_3)$$
$$\text{or} \quad R=r_1+r_2+r_3.$$

When however, the conductors are arranged *in parallel* as shown in the lower diagram of fig. 31, *i.e.* the current entering the end X has three paths open to it and the resistance of the three together, when arranged *in parallel*, will be less than the resistance of any one of them. To find the resistance of such a system, let us suppose C_1, C_2 and C_3 be the currents flowing through r_1, r_2 and r_3 respectively; then by Ohm's Law,

$$C_1=\frac{E_1-E_2}{r_1}, \quad C_2=\frac{E_1-E_2}{r_2} \text{ and } C_3=\frac{E_1-E_2}{r_3} \quad \ldots (i)$$

where E_1 and E_2 are the potentials of points X and Y respectively. If C be the total current, it must be equal to $C_1+C_2+C_3$; but by Ohm's Law, $C=\dfrac{E_1-E_2}{R}$ $\quad .\,.(ii)$

where R denotes the effective resistance due to r_1, r_2 and r_3, when arranged *in parallel*. From equations (i) and (ii), we have

$$\frac{E_1-E_2}{R}=\frac{E_1-E_2}{r_1}+\frac{E_1-E_2}{r_2}+\frac{E_1-E_2}{r_3}$$

$$\text{or } \frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}. \qquad (iii)$$

Now $\frac{1}{R}$, $\frac{1}{r_1}$, $\frac{1}{r_2}$ and $\frac{1}{r_3}$ are the reciprocals of resistances and hence the *conductances* of the conductors. Therefore we can say that the *conductance of a system of conductors arranged in parallel, is the sum of their separate conductances*; also it is evident from equation (*i*) *that the current flowing through any branch is inversely proportional to its resistance.*

311. Arrangement of Cells.—The current given out by one cell is generally too feeble for ordinary domestic purposes; hence usually batteries of two or three cells are used. The following are the various ways of grouping cells to form batteries.

311. (*a*) Ohm's Law as applied to groups of cells.

(*i*) **In series.** Cells are said to be connected in series, when one pole of one cell is connected to the opposite pole of the next, and so on. The poles of a battery are the free poles of the end cells. In this arrangement, the *E.M.F.* of the battery is equal to the sum of the *E.M.F.'s* of the cells so arranged; and so is the resistance.

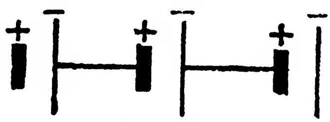


Fig. 29

If m cells, each of *E.M.F.*, E and internal resistance r are arranged in series; then their *E.M.F.* $= mE$ and their internal resistance $= mr$. If the external resistance be R, then the current would be equal to

$$\frac{mE}{R+mr} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (i).$$

(*ii*) **In parallel.** Cells are said to be arranged in parallel, when all the positive poles of the constituent cells are connected together and so are all the negative poles, fig. 30. The cells are in

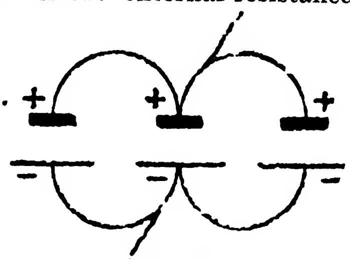


Fig. 30

fact converted into one big cell. The *E.M.F.* remains the same, as of a single cell; but the resistance is decreased to $\frac{1}{n}$th of that of one cell, if n cells are connected in parallel. The current, when the external resistance is equal to R, is given by

$$\frac{E}{R+\frac{r}{n}}=\frac{nE}{nR+r} \quad \ldots \ldots \ldots (ii)$$

where E is the *E.M.F.* and r the internal resistance of each cell.

(*iii*) **Mixed circuit.** Cells are said to be arranged in mixed circuit, when they are arranged both in series as well as in parallel. In figure 31, there are six cells. They are arranged in two rows of three cells each, which are connected in series; and these again are connected in parallel.

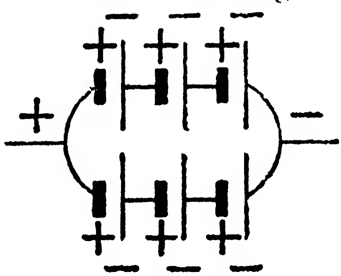


FIG. 31

When $m.n$ cells are arranged in a compound circuit, in n rows each of m cells in series, the $E.\ M.\ F.=mE$

and the resistance$=\frac{m}{n}r$.

$\therefore$ the current C, when R is equal to the external resistance is given by $\dfrac{mE}{R+\frac{m}{n}r}$

$$i.e.\ C=\frac{mnE}{nR+mr}$$

$$\text{or } C=\frac{mnE}{(\sqrt{nR}-\sqrt{mr})^2+2\sqrt{mnRr}}.$$

This expression will have the greatest value, when

$$(\sqrt{nR}-\sqrt{mr})^2=0$$

$$i.e. \text{ when } nR = mr$$

$$\text{or } R = \frac{m}{n} r;$$

i.e. The current will be maximum when the external resistance is equal to the total internal resistance.

312. Shunts. An important application of the principle of arranging conductors *in parallel* is found in the use of *Shunts.* Suppose we want to measure a strong current by means of a sensitive galvanometer; then if this current be passed directly through the instrument, it would damage it. To avoid this, a resistance is arranged in parallel with the galvanometer, as shown in Fig. 32, so that only a part of the current flows through the galvanometer. If the galvanometer resistance be G and shunt resistance arranged in parallel with the galvanometer be S, the effective resistance R is given by the equation

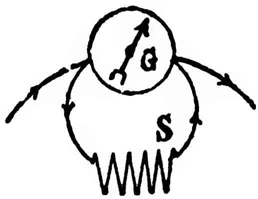


Fig. 32

$$\frac{1}{R} = \frac{1}{G} + \frac{1}{S} \quad \text{or} \quad R = \frac{GS}{G+S};$$

and if C be the current through the primary circuit, then the current through the galvanometer would be

$$\frac{CR}{G} = \frac{CGS}{G(G+S)} = \frac{CS}{G+S}.$$

Thus if the shunt resistance be one Ohm and the galvanometer resistance 9 Ohms; then the current through the galvanometer will be only $\frac{1}{10}$ of the main current.

313. Ammeters. An ammeter is an instrument, which directly measures the current in Amperes. There are various types of instruments in general use, but the underlying principle is the same in all forms. An ammeter is *a very low resistance* moving-

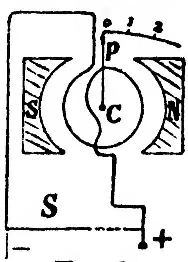


Fig. 33

coil galvanometer, which is placed *in series* in the circuit of which the resistance is to be measured. It is very essential that the resistance of the instrument should be small, so that its introduction into the circuit may not appreciably alter the current. When very strong currents are required to be measured, the instrument is shunted with a low resistance coil, as shown in fig. 33.

314. Voltmeters. A voltmeter is an instrument which measures the *E.M.F.* in volts. It is in fact a *high resistance galvanometer*, connected in *parallel* to the circuit of which the *E.M.F.* is required to be measured. If the resistance of the instrument is not high enough, a high resistance is connected in series with it, as shown in fig. 34. Figure 35 shows how an ammeter and a voltmeter are connected to measure current and *E.M.F.* respectively.

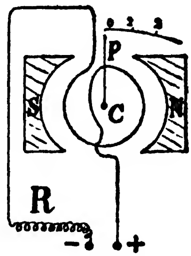


FIG. 34

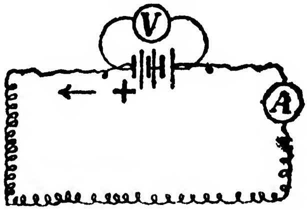


FIG. 35

315. Wheatstone's Bridge. If a circuit as shown in fig. 36, be arranged; then a difference of potential must exist between A and C, and the same fall of potential must take place along ABC and ADC. Points such as B and D however, can be found in each branch at which the potential would be the same; *i.e.* fall of potential from A to B would be the same as that from A to D. But by Ohm's Law, fall of potential is proportional to the resistance. Therefore, if the fall of potential from A to B is the same as that from A to D, then the resistance AB must bear the same ratio to resistance ABC as AD bears to ADC. Hence if P, Q, R and S denote the resistances of the portions AB, BC,

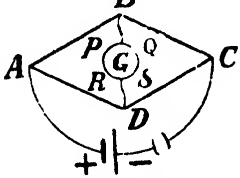


FIG. 36

AD and DC respectively;

then we must have $\frac{P}{P+Q}=\frac{R}{R+S}$ ..... (i)

By cross-multiplication, $P(R+S)=R(P+Q)$

i.e. $PS=RQ$ ....(ii)

or $\frac{P}{Q}=\frac{R}{S}$ ....(iii)

315. (a) The following is another method of proving the above relation:—Let V_a, V_b, V_d and V_c be the potentials of A, B, D and C respectively; then by Ohm's Law, we must have

$V_a-V_b=C_1P$ and $V_b-V_c=C_1Q$ . . . .(i)

$V_a-V_d=C_2R$ and $V_d-V_c=C_2S$ (ii)

where C_1 and C_2 are the currents, flowing through ABC and ADC respectively. But $V_b=V_d$, when no current is flowing through the galvanometer; then dividing equation (i) by (ii), we have

$$\frac{C_1P}{C_2R}=\frac{C_1Q}{C_2S}$$

or $\frac{P}{Q}=\frac{R}{S}$.. . ... (iii)

From equation (iii), it is evident that if three of the four resistances P, Q, R and S be known, the fourth becomes known by the above relation. *Or even if, the ratio of any two of them be known* and the third resistance be also *known*, then the fourth can be found out.

Such an arrangement as shown in figure 36, by the help of which we can find an unknown resistance, is called a *Wheatstone's Bridge or net.*

The points B and D will be at the same potential, when there is no deflection in the galvanometer.

316. The Metre Bridge. It is a practical form of Wheatstone's bridge, which is generally employed in laboratories for accurate work.

It consists as shown in figure 37, of a fine german silver wire, about a metre long, stretched between two binding screws A and C; these points are joined by a thick copper strip, having two gaps in it. In one of the gaps is put the unknown resistance P, and in the other

a resistance-box Q. The battery terminals are joined to A and C. One of the galvanometer terminals are joined to X and the other, by means of a sliding jockey, to the point D on the wire AC, so that there is no deflection in the galvanometer. Then

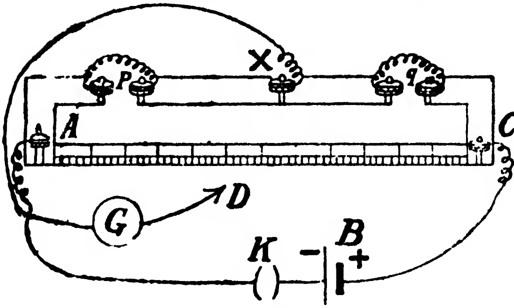


FIG 37

$$\frac{P}{Q}=\frac{AD}{DC} \text{ or } P=\frac{Q\times AD}{DC}.$$

317. The Post-office Box. It is so called after its use in Post Offices, for finding resistances of telegraph lines. It is a very convenient and handy combination of a resistance box (already described) and a Wheatstone's net.

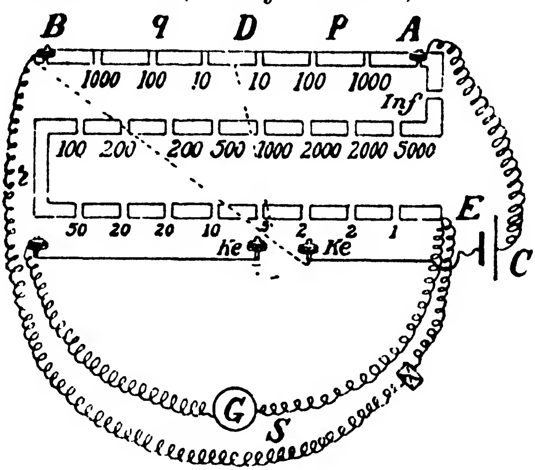


FIG. 38

It consists as shown in fig. 38, of a set of resistances and spring keys. The battery C is connected

to points A and B through a spring key and the galvanometer G is similarly connected to the points D and E. The unknown resistance X is connected to points B and E; and constitutes the fourth arm of the Wheatstone's bridge, the other three arms being constituted by BD, DA and AE.

At first P and Q are made equal to 10 ohms each, by taking out plugs of 10 ohms resistance from the arms AD and DB. The plugs taken out of the third arm r denote the unknown resistance X, when no current flows through the galvanometer. To get accurate results, P may be made 100 and 1000 in turn, while Q remains 10 ohms. Then X will be $\frac{1}{10}$th. and $\frac{1}{100}$ths. part respectively, of the resistance in the arm r.

The student should note very carefully the connections, shown in the figure.

SUMMARY

1. Ohm's Law states that the current in a circuit, is directly proportional to the $E. M. F.$ and inversely proportional to the resistance.

2. The **specific resistance** of a substance is the resistance offered by a unit cube of the substance, when current enters its one face and leaves the opposite face.

3. When cells are arranged in a mixed circuit, the strongest current is produced, if the external resistance is equal to the internal resistance.

4. **Shunt** is a small resistance arranged in parallel with a galvanometer, used to measure strong currents.

5. **An ammeter** is a low resistance galvanometer used to measure currents.

6. **A voltmeter** is a high resistance galvanometer used to measure $E.M.F.$'s.

EXAMPLES

1. What is Ohm's Law? How would you prove it experimentally?

2. Upon what factors does the resistance of a wire depend? What do you understand by specific resistance?

3. What are the various modes of arranging the cells? Which is more economical and what arrangement would you adopt to get a strong current?

4. Describe the various methods of comparing *E.M.F.'s* of two cells.

5. A cell of *E.M.F.* 2 volts is connected to the terminals of a tangent galvanometer, the coil of which has a resistance of 200 ohms; a deflection of 45^0 is produced. What is the reduction-factor of the galvanometer? (Loc. Camb. '07).

6. If 5 cells of 2 volts each, and $\frac{1}{4}$ ohm internal resistance, be connected *in series* with a wire of $^1/_2$ ohm resistance. What current in absolute units will be flowing?

7. What are the chief differences between electrolytic and metallic conduction? (Loc. Camb. '05).

8. A metal wire 2·5 metres long, and 0·12 cm. diameter, has a resistance of 0·5 ohm. What is the conductivity of the material?

9. Describe the metre form of Wheatstone's Bridge and explain its theory very briefly.

10. What is the essential difference between an ammeter and a voltmeter?

How would you determine the resistance of an electric lamp while glowing, if you were provided with an Ammeter and a Voltmeter? Draw a diagram of the experimental arrangement. (*P. U.* 1931).

CHAPTER VII

THERMAL EFFECTS, THERMO-ELECTRICITY AND ELECTRIC LIGHTING CIRCUITS.

318. An electric current is capable of doing various kinds of work, such as chemical, magnetic, mechanical and thermal. By Ohm's Law, the current produced by a given cell diminishes, as the resistance of the circuit is increased. Why is it so? The reason is, "Whenever electricity in motion is stopped by resistance, the energy of the flow is frittered away by the resistance into heat; just as whenever matter in motion is stopped by friction, the energy of motion is frittered away into heat." Heat in fact, always appears whenever a conductor offers resistance to the flow of electricity. For instance, if the terminals of a battery of six accumulators be joined by a thick copper wire, no appreciable heat is generated in the wire, while the battery becomes hot; but if the terminals be joined by a long thin wire of platinum, the wire may begin to glow, while the battery will remain cool.

We have defined Volt as the difference of potential existing between two points, such that 10^8 ergs of work are done in one second, if a current equal to one *C.G.S.* electromagnetic unit is maintained between those points. Or one joule, *i.e.* 10^7 ergs are done, if a current equal to one Ampere, *i.e.* $\frac{1}{10}$th of the *C.G.S.* electromagnetic unit, is maintained for one second.

If the difference of potential between two points be E volts and a current of C amperes be maintained between them, the work done in one second would be $E \times C$ Joules. As the difference of potential$=E$, therefore E Joules of work would be done, if a current of one ampere were flowing; but as the current is equal to C amperes, therefore work done in one second$=E.C$ joules,

and the total work done in t seconds would be ECt joules or $ECt \times 10^7$ ergs.

By Ohm's Law, $E = C.R$

$\therefore \quad ECt = C^2R.t$

Now if the current be not employed to do any other work, the whole of its energy will appear as heat, but the heat generated is always measured in *calories*. Joule found out that $4{\cdot}24 \times 10^7$ ergs of mechanical work are required to produce one calorie; therefore the amount of heat generated by the current will be equal to $\frac{E.C.t \times 10^7}{4{\cdot}24 \times 10^7}$ or $\frac{C^2Rt}{4{\cdot}24}$ calories. This is *Joule's Law*, which states that the amount of heat generated in a circuit is proportional to (*i*) the square of the current, (*ii*) the resistance and (*iii*) the time during which the current flows.

319. Joule's Law can be demonstrated by the apparatus shown in figure 39. It consists of a highly polished thin copper calorimeter, having an ebonite cover, which has two holes, one for the thermometer and the other for the stirrer. Inside the calorimeter is a wire of known high resistance R, the ends of which are connected to two binding screws fitted to the ebonite cover.

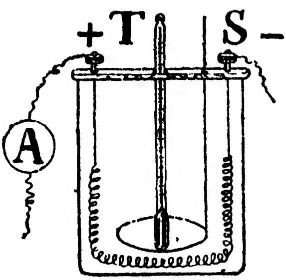


FIG. 39

The calorimeter is half filled with some oil of low specific resistance. After noting the weight and the initial temperature t, a current of definite strength C (as indicated by an ammeter A, connected in series) is passed for a time θ. The contents in the meanwhile, are kept well stirred and the resulting temperature T is carefully noted.

By equating the energy of the heat produced with the electric energy expended, we get $JW(T-t) = C^2R\theta$, where W is the water-equivalent of the calorimeter

and its contents.

From this J, the mechanical equivalent of a therm, is given by the equation, $J=\dfrac{C^2R\theta}{W(T-t)}$.

By sending currents of various strengths and for different times, the value of J is always found to be the same. Thus we conclude that the quantity of heat produced in a conductor varies directly as the square of the current, directly as the resistance of the conductor and directly as the time.

320. Electric lamp. One of the most important applications of heating effects of currents is the electric lamp. We know, the hotter a body is, the more efficient it is as a source of light. For an efficient electrically-heated lamp, we require a substance, which should neither fuse nor disintegrate at very high temperatures. On this account the first lamp, introduced by Edison in 1878, consisted of a fine platinum wire; but its commercial success was prevented both on account of its heavy cost and its fusion at high temperatures. Later on, it was found, that carbon filament answered the purpose even more satisfactorily than platinum; but since carbon burns in air, the filament must be enclosed in an exhausted glass vessel.

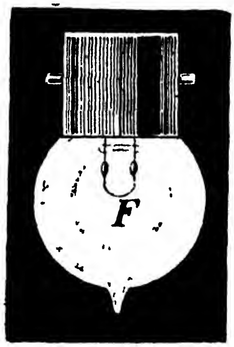


FIG. 40

An example of such kind of lamp, is the well-known incandescent lamp shown in fig. 40. It consists of a filament of carbon enclosed in an exhausted glass bulb. The current passes into the filament by means of platinum wires, fused into the ends of the glass vessel. The filaments are prepared from a soluble cellulose, which is obtained by dissolving cotton-wool in zinc chloride. Thick paste of the cellulose is drawn into a fine thread, which on drying, assumes the form of a catgut. This thread is then 'carbonized.'

The carbon filament disintegrates at a temperature of about 1600°C. and moreover its resistance diminishes with rise of temperature, so that it is very sensitive to fluctuations of voltage.

Of late years, certain metals such as tantalium and tungsten have been discovered, which are extremely difficult to fuse and do not combine with oxygen at high temperatures. They are more refractory than carbon, hence they are extensively used for lamp filaments.

The resistance of these metals is not so high as that of carbon. For this reason metal filaments are always longer than carbon filaments and are to be coiled in a special manner. Dr. Bolton's tantalium lamp and the well-known Osram lamp are examples of metallic filament lamps. The chief advantage of these lamps, lies in the fact that for the same consumption of power, these give three times the light of a carbon filament and possess the same life. On account of these, the metallic filament lamps are now generally used in preference to carbon filament lamps.

321. The Arc Lamp. An arc lamp consists of two carbon rods with their pointed ends near each other and is best suited for powerful lighting. Its light is more like the Sun than that of any other illuminant. If the rods of an arc lamp be examined, it is found that the positive pole is hollowed out. To work the lamp, the two pieces of carbon are made to touch lightly and a current passed. A little spark is produced, which volatilizes some of the carbon from the positive pole. The pieces of carbon are then gradually drawn apart and the current continues to flow through the vapour. This white-hot vapour emits light.

As carbon burns in air, the rods are gradually eaten up. In order to maintain the arc, it is necessary to bring the rods slowly near together as they are used up. Thus to get a self-starting arc, the following conditions must be fulfilled:—

(*i*) The rods should touch each other as soon

as the current is turned off.

(*ii*) The rods should be drawn off to a reasonable distance as soon as the arc has been formed by turning on the current.

(*iii*) The rods should be brought near together as they are eaten away.

The above conditions are fulfilled by the aid of electromagnetic arrangements.

322. The general plan of electric lighting installation is that lamps, fans, motors etc., are all arranged **in parallel**. The advantage of this arrangement is, that *any one lamp may be put off without affecting the others*, besides as the *lead* wires are always of thick copper, all lamps etc. are supplied with the *same voltage*, for the *fall of potential* along those wires is negligibly small.

The general plan of electric installation in a building is shown in fig. 41. The wires from the electric power-house enter the building generally through a hollow metallic pipe fixed at the top of the building. The Company's *Meter M* is connected *in series* as shown, to measure the electric energy consumed and a safety-fuse (*c.f.*) is also installed in series, to guard against accidental short-circuiting.

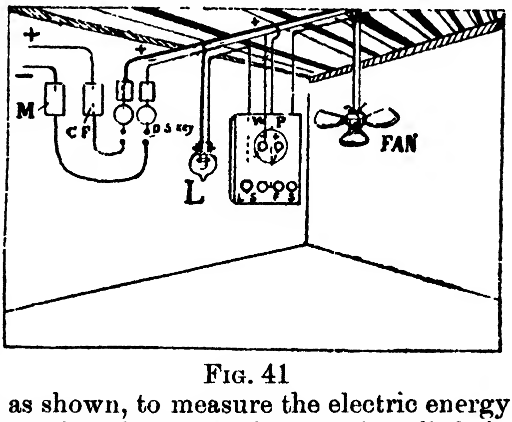


Fig. 41

Beyond these is a double switch-key (*D.S. Key*) to cut off or put on the supply of energy to the building as desired. Then come a pair of safety-fuses having fuse wires thinner than those used by the Company, to safe-guard the building against casual short-circuiting. After this, lamps, fans and wall-plugs are installed **in parallel**; and they are controlled by switches.

It should be carefully observed that to set up one lamp-switch, two wires must be brought to the switch-board. But to set up two switches, only three wires need be brought and to set up three switches, *i.e.* to get three points, only four wires need be brought to the switch-board, as shown in the figure. On the switch-board, we see four wires coming to provide for a wall plug and two switches, one for a lamp and the other for a fan.

323. Double Switch Key. In order to fit up a lamp for a staircase, so that a person using the stairs may be able to switch on or off the light from either end as desired, arrangement, as shown in fig. 42, is made. At each end is a double switch-key (*i.e.* one having three terminals.)

FIG. 42

The lamp terminals are connected to the middle terminals of the switch keys, as shown. This middle terminal at each end can be connected to the positive or negative wire as desired, by simply turning the handle of the switch. There will be light when the two terminals of the lamp are connected to the two wires, as shown in the upper fig. There will be no light, when the terminals are connected to the same wire. It will be observed that the lamp can be lighted or extinguished from either end as desired.

The Board of Trade Unit. The ordinary *practical units of current*, *E.M.F.*, *work*, *etc. are too small* to be used for *commercial* purposes. The *unit of power* employed for commercial purposes, also known as the Board of Trade unit (B.O.T.), is the *kilowatt* or 1000 joules per second; and the unit of quantity* of electrical *energy* is the **kilowatt-hour** or the amount of work

*Care must be taken to distinguish between unit of electrical energy and unit of quantity of electricity.

done in an *hour* at the rate of 1000 watts or joules per second. Thus it is equal to $3{\cdot}6 \times 10^7$ joules.

324. Thermo-electricity. About the year 1821, **Seebeck** discovered that if a closed circuit be made of two different metals and one of the junctions be heated: then a current begins to flow. Thus if a piece of bismuth and a piece of antimony be welded together and their free ends joined with a short-coil sensitive galvanometer, fig. 43 and if the junction J be heated; then a current, whose direction at the heated point is from bismuth to antimony, flows.

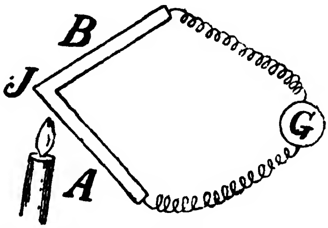


FIG. 43

Currents so produced are called **thermo-electric currents** and the *E.M.F.'s*, to which they are due, are called **thermo-electromotive forces.** The strength of the current is proportional to the difference of temperature between the hot junction and the cold ends. If however, the junction is cooled below the temperature of the ends; then the current begins to flow in the opposite direction. The current continues to flow, so long as the difference of temperature is maintained. The energy of the current is due to the heat-energy, which is absorbed at the junction.

325. Thermo-electric Inversion. A simple thermo-electric couple, as shown above, exhibits many interesting phenomena ; if the junction of a copper-iron couple be placed in a hot bath, while the other ends remain at the temperature of the melting ice. It will be noted that as the temperature of the junction rises, the deflection of the galvanometer increases, showing an increase of thermo-electric current. The increase in current-strength is rapid at first, but becomes less and less rapid as the temperature rises. After a time the increase comes to a stop and the current attains a maximum value. If the heating be still further continued, the current begins to decrease, till it reaches zero-value. If

however, heating be still further continued, the current is reversed in the circuit. Thus to sum up, the current increases at first, attains its maximum value and then decreases. Ultimately the current is reversed after reaching its zero-value.

The following interesting points can be experimentally verified:—

(*i*) For a given thermo-couple, the current attains its maximum value at a fixed temperature, which is always the same, whatever the temperature of the other ends may be.

(*ii*) The temperature, at which the **reversal** of the current takes place, is as much above the temperature of maximum current as that is above the temperature of the cold ends. That is, the temperature of the maximum current is the mean of the temperatures of the cold ends and of reversal.

Thus for example, in a copper-iron couple, if the cold junctions be at 35°C., the temperature of maximum current will be about 260°C. and the temperature of reversal, about 485°C. If the temperature of cold junction be 0°C.; then the temperature of maximum current will still be the same, *i.e.* 260°, but the temperature of reversal will be 520°C. The temperature of maximum current for any couple is known as the **neutral point** for that couple. It should be clearly noted that it is always constant for a given couple and is the mean of the junction temperatures, when reversal takes place. The temperature of reversal depends upon the temperature of the cold junction.

326. Thermo-electric Pile. A single thermo-couple gives a very feeble current; thus a bismuth-antimony couple, which perhaps gives the greatest *E. M. F.* for a given difference of temperature between its junctions, gives an *E.M.F.* of about 117 micro*-volts. In order to increase the electromotive force

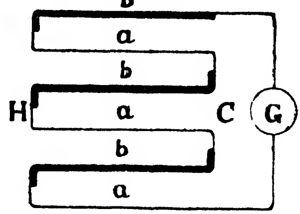


Fig. 44

* One microvolt is one-millionth of a volt.

of thermo-electric pairs, it is customary to join a number of pairs of metals *in series*; so bent that the alternate junctions can be heated, as shown in fig. 44, while the other junctions are kept cool. The several *E.M.F.'s* all act in the same direction and current is increased in proportion to the number of pairs of junctions. **Melloni** constructed a thermopile of very large number of bismuth-antimony couples. In his hands, this proved a very sensitive thermometer, when used in conjunction with a sensitive short-coil mirror galvanometer. This instrument is extremely useful for detecting very small differences of temperature. The arrangement of the thermopile for this purpose is shown in fig. 44.

SUMMARY

1. The work done by a current is ECt and the heat produced, measured in calories, is $\frac{ECt}{4\ 24}$. This is known as Joule's Law.

2. Incandescent electric lamp consists of a filament of carbon in an exhausted bulb. Instead of carbon filament, rare metal filaments can be used.

3. An arc lamp consists of two pointed carbon rods with their pointed ends near each other. To start the action, it is necessary to bring them very close together; this is done by electromagnets.

4. The ordinary practical units are not employed, because they are too small for commercial purposes. The **Board of Trade Units** are employed for the purpose. The B.O.T. unit of power is 1000 Joules per second and is called the kilowatt. The unit of electric energy is called the **kilowatt-hour.** It is the amount of work done in an hour at the rate of one kilowatt per second.

5. **Thermo-electric current** is that, which is produced when a junction of two metals is heated, while the other ends are kept cool.

6. **Neutral point** is the temperature of maximum current for a given thermo-couple.

7. **Temperature of Reversal** is that at which the thermo-electric current changes direction.

8. **Thermopile** consists of a large number of thermo-couples arranged in series.

EXAMPLES

1. A tangent galvanometer, having a coil of 10 turns of 40 cms. radius, gives a deflection of 45° with a current of one ampere. Calculate the Earth's horizontal component at the place.

$$H=\frac{2\pi nc}{r}\times\frac{1}{tan\theta}$$
$$=\frac{2\pi.10}{40}\times\frac{1}{10}$$
$$\therefore \frac{\pi}{20}=\cdot 157 \text{ gauss.}$$

2. If one ampere flowing through a T. galvanometer causes a magnetic force of ·20 dyne at the centre of the coil; find the reduction-factor for the galvanometer, if H be equal to ·32 gauss.

$$F=\frac{2\pi nc}{r}$$
$$\cdot 20=\frac{2\pi n}{10r}=\frac{\pi}{5}\times\frac{n}{r}.$$
$$\therefore \quad \frac{n}{r}=\frac{1}{\pi};$$
$$\text{but} \quad K=\frac{r}{n}\cdot\frac{H}{2\pi}=\frac{\pi\times\cdot 32}{2\pi}=\cdot 16$$

3. 40 Bunsen cells ($E.M.F.=1\cdot 8$ volts each) are connected in series and the circuit is completed by a wire of 10 ohms. Supposing the internal resistance of each cell to be 0·25 ohm, calculate the strength of the current.

The total $E.M.F.=40\times 1\cdot 8=72\cdot 0$ volts.

The total resistance in circuit$=10+40\times\cdot 25$
$=10+10=20$ ohms

$$\therefore \quad \text{the current}=\frac{V}{R}=\frac{72}{20}=3\cdot 6 \text{ Amperes.}$$

4. Taking specific resistance of copper as 1642 absolute units; calculate the resistance of a kilometre of copper wire, whose diameter is 1 millimetre.

The resistance of a wire is given by

$$R=\frac{l}{a}\cdot s$$
$$\therefore \quad \text{Resistance}=\frac{1000\times 100\times 1642}{\frac{\pi}{400}};$$

and in ohms$=\frac{100000\times1642\times400}{\pi\times10^9}=20{\cdot}89$ ohms.

5. Calculate by Joule's Law, the number of calories of heat developed in a wire, whose resistance is 4 ohms, when a steady current of ·14 ampere is passed through it for 10 minutes.

By Joule's Law, heat developed in calories is $\frac{C^2R.t}{4{\cdot}24}$, when all the quantities are measured in practical units.

$$\therefore \text{ we have}=\frac{{\cdot}14^2\times4\times10\times60}{4{\cdot}24}=11{\cdot}2 \text{ calories.}$$

6. A 50 C.P. lamp on a 150 volts circuit, consumes one ampere. If electrical energy costs 9*d.* per B.O.T. unit, how much will such a lamp cost per hour?

The lamp uses 150×1 watts per sec.$=0{\cdot}15$ kilowatt per hour.

$$\therefore \text{ the cost per hour}={\cdot}15\times9d.=1{\cdot}35d.$$

7. A current of 9 amperes works an electric arc light and the difference of potential between the two carbons is 140 volts. What must be the horse-power of the supply dynamo?

The quantity of work done by the current in one second $=9\times140$ Joules or watts

$=1260$ watts per second.

This is equal to 1·260 B O.T. units per hour. Now 746 watts are equal to 1 H. P.

Therefore the quantity of power absorbed in H.P.

$$=\frac{1260}{746}=1{\cdot}69 \text{ H.P.}$$

8. A voltmeter, connected in parallel with a given coil in a circuit, gives a potential difference of ·4 volt and an ammeter in series with the coil shows ·5 ampere. Find the resistance of the coil.

9. A lamp connected on 220 volts supply uses ·5 ampere. Find its resistance and also the watts consumed per second.

10. An arc lamp requires 5 amperes and the difference of potential between its terminals is 45 volts. What resistance must be connected in series with it, so that it may be worked from 220 volts supply?

11. The Lahore Electric Supply Co. is to transmit equal amounts of power to Lahore and Amritsar, distances 4 and

36 miles respectively, from the "Power" plant. The Company wishes that heat losses in the two line-wires should be the same. If the voltage of the Lahore current be 200, what should be the voltage of the Amritsar current?

12. An ammeter is graduated to read 1·6 amperes and has a resistance of 0·1 ohm. What resistance should be connected in parallel with it as a shunt, so that it may be used to measure a current upto 16 amperes?

13. An ammeter and a silver voltameter are connected in series. The deflection noticed in the ammeter is $\frac{1}{10}$ of the whole range, provided $\frac{1}{2}$ gram of silver is deposited in 10 minutes. What is the total range of the ammeter?

14. What do you understand by a half-watt lamp. A 40 C.P. half-watt lamp is connected to a 220 volts circuit. How much current will it take and how much will it cost per hour at 8 annas per B.O.T. unit?

15. A lamp, consuming 30 watts per second, is connected in series with a battery and a copper voltameter. The deposition of copper after $\frac{1}{2}$ hour is 0·089 gm. Find the potential difference between the terminals of the lamp. Electro-chemical equivalent of copper=·00033 gm. per coulomb. (*P.U.* 1929).

16. An electric lamp of 100 candle power is found to take a current of 0·225 ampere, when the voltage is 220. Calculate: (*i*) the consumption of power in watts and (*ii*) the cost of using the lamp for 6 hours, if one B.O.T. unit costs 6 annas. (*P.U* 1928).

17. A 220 volts electric tea-kettle takes 990 watts. Calculate: (*i*) the resistance of the heating element, (*ii*) the time required to melt and boil away 1 kilogram of ice, assuming a heat loss of 40 per cent by various sources. (*P.U.* 1927).

18. A 100 *c p.* lamp is lighted and when immersed in a beaker of water, is found to impart 5700 therms per minute. If the bulb is blackened, the number of therms rises to 6000 per minute. Neglecting radiation; calculate :—

(*i*) percentage of energy converted to light the lamp.
(*ii*) current taken by the lamp, (lamp voltage=210).
(*iii*) watts per candle power. Therm=$4{\cdot}2\times10^7$ ergs. (*P.U.* 1926).

19. A flat iron weighing three kilograms uses 4·5 units of current, when operating on a 110 volts circuit. How much time will it take to heat the flat iron from 20° C. to 200° C.?

sp. heat of iron is 0·113 and a therm=4·2 Joules.

20. Find an expression for the work done by a current. What is a Joule?

21. Describe an Incandescent lamp. Why is an arc lamp unsuitable for domestic purposes?

22. What is the B O.T. unit of electric energy? Why was it introduced?

23. Describe the construction and use of a thermopile.

24. Give a diagrammatic sketch of electric installation in a house.

25. How will you connect a two-way staircase lamp?

CHAPTER VIII
ELECTRO-MAGNETIC INDUCTION

327. Faraday discovered that, when a magnet was moved in a closed circuit, a current was generated. His method of observation was as shown in fig. 45. The two ends of a coil of insulated wire are joined to a delicate Ballistic galvanometer and if the N-seeking pole of a magnet be rapidly inserted into the hollow of the coil, it is observed that the magnetic needle of the galvanometer is deflected, showing that a momentary current has been produced. Keep the magnet in the hollow and observe that no current whatsoever flows. Now take the magnet out of the hollow and see that a deflection to the opposite side is produced. This shows that a momentary current is again produced in the coil, but in the opposite direction. This experiment clearly shows, that a momentary current is produced in the coil, when the magnet is moved either in or out of the hollow of that coil; and that no current is produced, if the magnet keeps lying in the hollow. These momentary currents produced in a circuit by the motion of a magnet in its hollow are called **induced currents.**

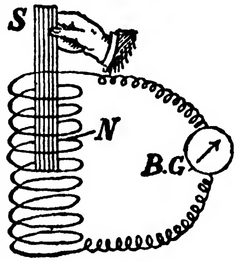


FIG. 45

On repeating the above experiment with the S-seeking pole of a magnet, it is observed that the current induced when S-seeking pole is inserted, is opposite in direction to that induced, when the N-seeking pole is inserted; but is in the same direction as that produced, when a N-seeking pole is taken out. The magnitude of the deflection however, remains the same, if the rapidity with which the S-seeking pole is inserted is the same,

as was in the case of the *N*-seeking pole. *Thus it is clear that the direction of the induced current depends upon the direction of the magnetic pole introduced.*

We have already learnt in magnetism, that lines of force originate from the *N*-seeking pole of a magnet and end on the *S*-seeking pole; or we can say that +*ve* lines are always linked with the *N*-seeking pole and −*ve* lines with the *S*-seeking pole. Therefore theoretically, on the above basis, we can say that the effect of introducing a *N*-seeking pole is to insert +*ve* lines of force in the hollow and that of taking it away is to remove those +*ve* lines of force. Similarly the effect of inserting and taking away a *S*-seeking pole is to insert −*ve* lines of force and then to remove them. Thus the changes in the lines of force on inserting and removing a magnet in the hollow of a coil, can be represented in the following manner, if *n* lines of force are supposed to be linked with the magnet:—

Introducing	N pole	from 0	to n	Change of	n lines	(*i*)
Removing	N. ,,	,, n	to 0	,,	—n	(*ii*)
Inserting	S. ,,	,, 0	to—n	,,	—n	(*iii*)
Removing	S. ,,	,,—n	to 0	,,	n	(*iv*)

Thus we see that experiments (*i*) and (*iv*) are magnetically the same and so give currents in one direction; while experiments (*ii*) and (*iii*) are similar to each other but opposite to (*i*) and (*iv*), and hence give currents in opposite direction to (*i*) and (*iv*).

328. Currents produced by Mutual Induction. Faraday showed that induced currents are produced in a circuit, whenever a current is started or stopped in a coil placed in its hollow. The plan of the apparatus is shown in fig. 46.

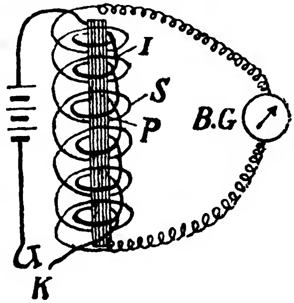


FIG. 46

As before, the terminals of a coil are joined to a Ballistic galvanometer; and another coil of stout wire, which can be inserted in the hollow of the first coil, is connected to the terminals of a battery

The outer coil is called the *Secondary* and the inner coil the *Primary*. The two together are known as *Inductorium*. On inserting the primary, through which a current is flowing, into the secondary, a momentary current is produced in the latter; and on taking the primary out of it, a current in the reverse direction is produced. But so long as a steady current is traversing the primary, no induced current is produced; unless there is relative motion between the two. **This phenomenon is called Mutual Induction.** *Whenever a current is started or stopped in the primary coil of an inductorium, a momentary current is produced in the secondary.*

We have already seen, that a coil of many turns when traversed by a current has a field similar to that of a magnet; hence the effect of making a circuit is similar to that of the insertion of a magnet in the primary. The effects produced in this latter case are more pronounced, if a soft-iron core is inserted in the primary coil.

329. On the basis of the above experimental facts, Faraday formulated the following laws, which are known by the name of **Faraday's Laws of Electromagnetic Induction.**

(1) *A current is induced in a circuit, whenever the number of lines of magnetic force running through the circuit is changed.*

(2) *The induced current is transient; and lasts only while the change in the flow of magnetic lines of force, through the circuit, is taking place.*

(3) *The direction of the induced current depends on the direction of the lines of force; and is such that its reaction tends to stop the motion, to which it is due.*

(4) *The magnitude of E. M. F. induced is proportional to the rate at which the lines of force change.*

On examining the direction of the induced current, we see that when a *N*-pole is inserted, the induced current has such a direction, that the face of the coil approached becomes a north face, *which therefore opposes the motion of the magnet.* When the *N*-pole is withdrawn, the induced current has such a direction

that this face is a south face, *which therefore tends to draw the magnet back again.* Or to sum up: *the induced current is in such a direction, that its reaction tends to stop the motion or displacement, to which the induced current is due. This statement is known as* **Lenz's Law.**

330. Self-induction. When a current is started in a coil of wire, a magnetic field due to this current is suddenly set up. The effect of this is to induce an opposing *E.M.F. in that very coil itself,* in a direction opposite to that of the current, which sets up the magnetic field. Due to this, the primary current takes longer time to attain its full strength. Similarly, when the current is stopped, the magnetic field is suddenly destroyed and an induced *E.M.F.*, in the same direction as the primary current, is produced. The effect of this is that the current is not stopped all at once. Such currents are said to be due to **Self-Induction.**

Experiment. Self-induced currents can be beautifully demonstrated with the help of the apparatus shown in fig. 47. Current from a battery of 12 accumulators, divides itself into two portions, one of them flows round the coil of a strong electro-magnet and the other lights a small electric lamp dimly. On breaking the battery-circuit by releasing the key, current due to self-induction flows through the lamp, which for a moment glows brightly.

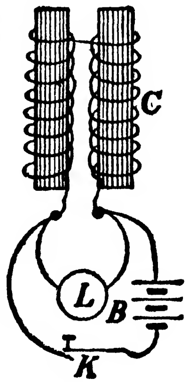
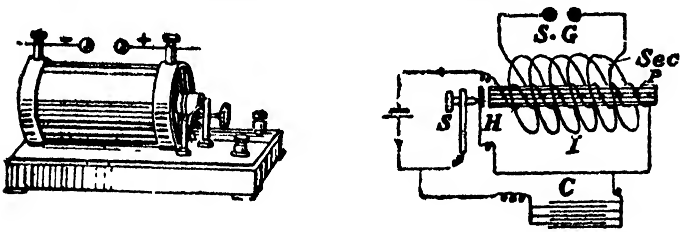


FIG. 47

331. Energy of Induced Current. Induced current is capable of doing work; hence the question arises: Wherefrom is this energy derived in the secondary? According to the universal law of conservation of energy, this cannot be created by itself and evidently there is no source of energy in the secondary itself. The energy of the induced current must be due to some external agency. The straight-forward answer to this is, that the energy is derived from the *mechanical work done in overcoming the mutual forces of attraction and repulsion between the secondary circuit*

and the primary or the magnet. Thus when a magnetic North pole is approaching, the direction of the induced current is such as to produce a North pole at the upper end of the coil; and hence a force of repulsion is exerted. When however, the magnet is receding, the induced current exerts a force of attraction. Extra work done against these opposing forces represents the energy of the induced current.

332. Ruhmkorff's Induction Coil. This is an



(*a*) FIG. 48 (*b*)

inductorium, provided with an automatic arrangement for rapidly starting and stopping the current. The primary consists of a few turns of thick well-insulated wire, wound round a soft-iron rod. The secondary consists of a very large number of turns of thin, highly insulated wire, wound over the primary. The whole is covered over by a highly insulated ebonite cover. The two ends of the secondary terminate in screws over the ebonite cover; and to them, sparking rods can be attached at will.

The connections of the primary with the battery are shown in fig. 48 (*b*). One end of the primary is directly connected to the battery; while the other is connected through the hammer *H* and the screw *S*. In addition to this, in good instruments there is always attached a condenser *C*, as shown in the figure. One end of this is connected to the hammer and the other to the screw. The hammer *H* consists of a spring, fixed at the lower end and to its upper end is attached a piece of soft iron. *S* is a screw with a platinum tip, which can be made to touch the hammer lightly.

The working of an induction coil is as follows:—When the current is first started, the iron core inside the primary coil becomes magnetized and attracts the soft iron piece of the hammer H towards itself. This results in breaking the contact with S. The circuit is broken and the current stopped. The iron core ceases to be a magnet and hence does not attract the soft iron piece of hammer, which by force of the spring, to which it is attached goes back to its original position and makes contact with the screw S. On so doing, the current is again started and the same operation repeated. Thus the current is made and broken rapidly. The effect, of this rapid *make* and *break* in the primary, is to set up corresponding induced currents in the secondary. At make, the induced current is in one direction; while at break, it is in the other. Thus an alternating current is produced in the secondary.

Now the real working is not so simple as described above. Because when a current is started, an induced current is produced in the opposite direction, on account of self-induction; and for this reason, the current in the primary does not assume its full value all at once, but is spread over a longer period. Hence the *E. M. F.* induced in the secondary at make is not high, because the induced *E. M. F.* depends upon the rate at which the change takes place.

The break however, is done mechanically by the attraction of the hammer towards the iron core and is instantaneous. Therefore, the induced *E. M. F.* in the secondary will be very high. When the gap between H and S is produced, due to the attraction of H towards the iron core; then the self-induced current in the primary, on account of the sudden stoppage of the primary current, tries to jump from the hammer over to the screw and thus produces a spark, which acts injuriously, in wearing the platinum tip of the screw.

Thus we see, that in an induction coil without a condenser, a spark is first produced between the hammer and the screw and secondly the make is slow and the break comparatively rapid. Now a condenser is

attached to the induction coil, to remedy the first defect; and so to modify the second, that the 'make' is accomplished more slowly and the 'break' more rapidly. The effect of this is that very high *E.M.F.* is induced at *break*, while comparatively very small *E.M.F.* is induced at *make*. This *results in an* **intermittent current** *always in one direction,* rather than an alternating current. And this is what is generally wanted; *i.e.* a current at high *E.M.F.* should be flowing always in the same direction. This is achieved in the following manner:—

(*i*) The self-induced current, produced in the primary at break, does not produce the spark; but on the contrary follows the path of least resistance and charges the condenser. Thus the spark is avoided and the break is done very rapidly.

(*ii*) At make, the charged condenser is first discharged and then the current can be set up in the primary. Thus the make is still more delayed; and on this account, the *E. M. F.* induced would be very low.

To sum up. The action of the condenser consists in preventing the spark and also in producing an intermittent current, in the same direction in the secondary; rather than an alternating current.

333. The Transformer. The Ruhmkorff's Induction coil is a sort of (Step-up) **transformer** This is used to transform a current of low *E. M. F.* and high current-value into one of high *E. M. F.* and low current-value. The ratio of *E. M. F.* induced in the secondary to that in the primary, is equal to the ratio of the number of turns of the secondary to that of the primary; *i.e.* $\frac{E}{e}=\frac{N}{n}$, where E is the *E. M. F.* induced in the secondary, e that at the ends of the primary, N the number of turns of the secondary and n that of the primary.

SUMMARY

1. Whenever a change in the number of lines of force enclosed by a circuit takes place, an induced current is produced. The direction of this **induced current** depends

upon the direction of the lines of force.

2. The induced current is momentary; and the *E.M.F.* induced is directly proportional to the rate, at which the change takes place.

3. A combination of two coils, one over the other, is known as **inductorium.**

4. In an inductorium, if the current in one of the coils be altered, an induced current is produced in the other. This phenomenon is called **Mutual Induction.**

5. Whenever a current is started, stopped or varied in a coil, then an induced current is produced in that very coil. On this account, the current at the time of starting does not attain its full value all at once. This is called **Self-induction.**

6. **Faraday's Laws.** Whenever a magnetic flux, enclosed by a circuit, is changed, an induced current is produced; the *E.M.F.* induced, depends upon the rate of change of the magnetic flux.

7. **Lenz's Law.** The direction of the induced current is such, that its reaction always tends to stop the motion or the change to which the induced current is due.

8. **Ruhmkorff's** induction coil is an inductorium, in which the secondary consists of a very large number of turns of thin wire; while the primary consists of a small number of turns of thick wire. In addition, there is an arrangement for rapid automatic *make* and *break.*

9. The function of a condenser, usually attached to good induction coils, is· (*i*) To prevent the spark at the screw and (*ii*) To make the 'break' very rapid and the 'make' very slow.

EXAMPLES

1. What are Faraday's and Lenz's Laws of Electro-magnetic Induction? How would you prove them experimentally?

2. Define an inductorium, Self-Induction and mutual induction.

3. Describe Ruhmkorff's induction coil and state its utility. Also describe the action of the condenser.

CHAPTER IX

PRACTICAL APPLICATIONS

334. Electric Bell. This is a common application of electromagnets. A small horse-shoe electromagnet is mounted on a wooden base. One end of the wire is directly connected to a Leclanche cell, through a bell-push P and the other is attached to one end of the spring S. The spring, which is always of steel, is so adjusted that when no current is passing, it makes light contact with the screw H and a wire from this screw, is connected to the second pole of the battery.

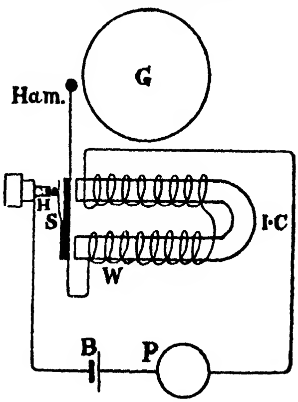


FIG. 49

The spring at its other end carries a hammer, which lies close to the gong G, as shown in figure 49.

When contact is made by pressing the button of the *bell-push*, a current begins to flow round the electromagnet and the horse-shoe shaped piece of iron becomes magnetized. It attracts the spring towards itself and makes the hammer strike against the gong. But when the spring is attracted towards the electromagnet, it does not remain in contact with the screw. Thus the circuit is broken and consequently the current is stopped. The core, *i.e.* the horse-shoe shaped piece of iron, becomes demagnetized and the spring goes back to its original position; in so doing it makes contact with the screw. The circuit is again

completed and the core magnetized. The spring is once more attracted, but goes back as soon as contact with the screw is broken. In this way the spring trembles between the screw and the electromagnet, strikes against the gong and produces sound. The tip of the screw and that point of the spring, which comes in contact with the screw, are both made of platino-iridium to guard against wear and tear.

334. (*a*) **Arrangement for a single Bell ringing from different rooms.** Sometimes it is desired to ring a single bell from different rooms. The arrangement for the purpose is shown in fig. 49 (*a*). The bell rings, whichever bell-push is pressed. To indicate the particular room of call, bulbs of different colours are attached for the purpose on a board. Thus in the diagram when *B*. 1 is pressed; besides the ringing of the bell, yellow bulb will also be lighted up. Similarly *B*. 2 and *B*. 3 will, when pressed light up blue and red lamps respectively, besides ringing the bell.

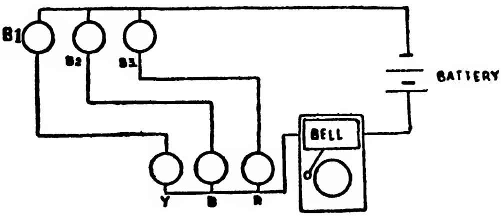


FIG. 49 (*a*)

It is also possible to ring different bells, with the same battery from one place. The arrangement for the purpose is shown in fig. 49 (*b*).

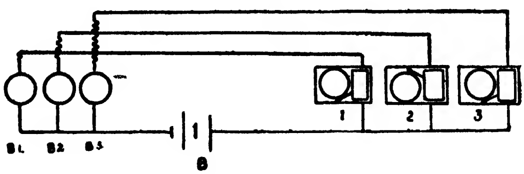


FIG. 49 (*b*)

335. Telegraphy. The figure shows the plan

of the most extensively used **open system** of telegraphy. By this arrangement, messages from one station can be transmitted simultaneously to all others, which are included in the same circuit. In this system, as in all other modern systems of telegraphy, only one wire called the *line wire* is used to connect several stations. The circuit is completed through the Earth.

Each of the stations *A* and *B* is provided with a battery as well as a Morse key and a receiver, which may be a galvanometer or any other device to detect electric currents. The negative pole of the battery and one end of the receiver are connected to the Earth; and the $+ve$ pole of the battery and the other end of the receiver, to the two screws of the Morse key, in such a manner that when the key is not pressed, the line wire is in contact

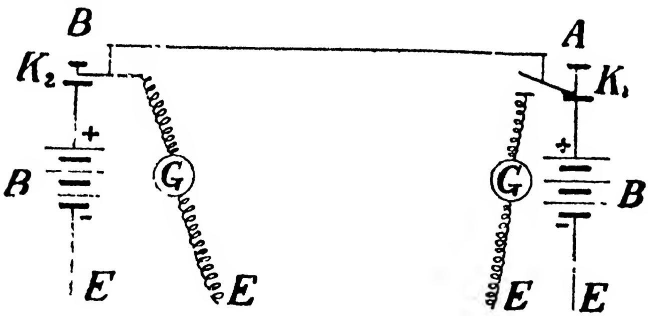


Fig. 50

with the receiver as at station *B* and when the key is pressed, contact of the line wire with the receiver is broken and it becomes connected to the positive pole of the battery as at station *A*. This is a very suitable arrangement, for each station when not sending out messages is in readiness to receive them from the other stations; and the signals of the sending station do not affect its own receiver. In the figure, *A* is shown as sending messages to *B*.

Nowadays the receiver used is the *sounder*, which consists of a long soft iron tube finely poised between

an electromagnet. When a signal is sent from the transmitting station, the iron tube is attracted towards the electromagnet at the receiving station and a click is produced by the tube coming in contact with the electromagnet. The receiver listens to these clicks and notices, whether the interval between two clicks is short or long. A short interval is called a dot and a long one, a dash. The Morse's code for interpreting dots and dashes into letters of alphabet is of the form given below:—

A —·— B—·· C—·—·

336. Relay. In working over long distances, or where there are a number of instruments included in one circuit, sometimes the current is not strong enough to affect the recording instrument directly. In such cases a device, known as the *relay*, is used. It consists of an electromagnet, round which the line current flows. This electromagnet when magnetized attracts a light armature, which makes contact for a local circuit, in which a cell and the receiving instrument are included. The object of this is, *that a current too weak to do the work may set a strong local current to do its work for it.*

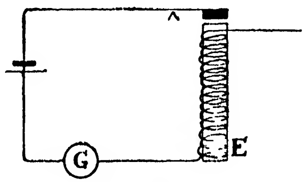


FIG. 50 (*a*)

337. The telephone. The telephone was first invented by Graham Bell in 1876. The instrument first devised by him is still used as a 'receiver' though the 'transmitter' differs, in various details from the original. In the earliest form of telephone, no battery is used. It depends for its transmitting action upon magneto-electricity and for its receiving action upon electromagnetism. They (*i.e.* both the receiver and the transmitter) consist of a very thin iron diaphragm, fixed round the edges of pieces of soft iron, attached to the ends of a permanent horse-shoe magnet fig. 51 (*i*).

The sound waves, falling on the diaphragm, set it

into vibrations. These vibrations change *the number of lines of force* in the coil, which is wound* round the soft iron pieces. The induced currents produced flow one way or the other in the coil, and are carried by the line wires to the other station, where they circulate round pieces of soft iron of a similar instrument, causing rapid variations in the pole-strength of the magnet. On this account the iron disc fixed near to its ends is set into vibrations, which correspond exactly with those of the first instrument and thus sound-waves are reproduced. The instrument, wherefrom speech is transmitted, is called the 'transmitter' and the one, where the sounds are reproduced, is called the 'receiver.'

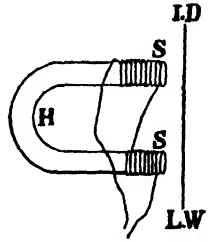


FIG. 51 (*i*)

338. Modern Telephone Nowadays in telephony the receiver used is similar to that used by Bell: but the transmitter, called the **microphone**, is different. The present day transmitter fig 51 (*ii*) consists of a thin diaphragm, which is in contact with pieces of carbon and these again are in contact with a block of carbon. The poles of the battery are connected through the primary of the inductorium and a switch to the carbon block and the iron diaphragm respectively, so that the circuit is completed through the loose contact of granulated carbon The ends of the secondary are joined to the receiver, which is similar to that used by Bell. The battery-switch *S* makes contact only when the receiver is taken out and held in hand. It is constructed on the principle that when a loose

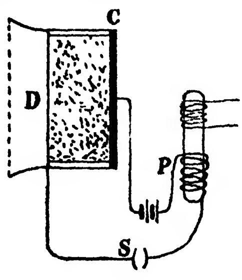


FIG. 51 (*ii*)

* Coils are wound in opposite directions on the soft iron pieces attached to the opposite poles of a horse-shoe magnet, so that induced currents may be in the same direction.

contact is included in a circuit; sound waves falling on that loose contact, cause variations in the resistance and therefore in the current. The variations in resistance produced in this manner are necessarily small; they produce appreciable effect, only when the total resistance of the circuit is comparatively small. But in telephony, long line wires are used and they have sufficient resistance; therefore small variations in resistance produce no effect in the current, when the total resistance is high. This difficulty is overcome by including an inductorium in the circuit near the transmitter Fig. 51 (*ii*). The variable current passes

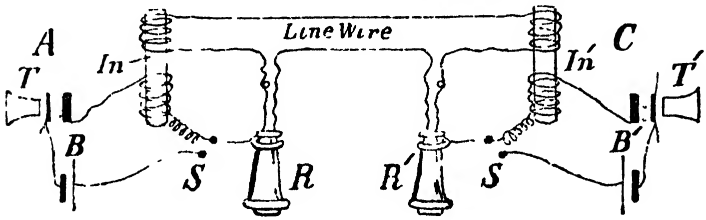


FIG. 51 (*iii*)

round the primary, while the line wires are joined to the secondary. The variations in *E. M. F.* induced in the secondary are sufficiently great and the currents sent through the line wires are strong enough to affect the receiver at the distant end. Figure 51 (*iii*) shows the arrangement of a modern transmitter and receiver.*

It may be noticed that in telegraphy, we use only one wire and make use of the earth as the *return*; while in telephony, the earth is not used as the *return*, but instead two wires are used. This is *not* due to the fact that currents in telephony are weak. The reason is, that in telegraphy we are concerned only with dots and dashes and the induction currents of the earth do not produce any appreciable effect; while in telephony, exact reproduction of sound is a necessity. If earth be used as the

*As is evident from the figure, the speaker hears his own sound as well in the receiver, but this makes no difference, for in ordinary conversation too, the speaker hears his own speech.

return, the Earth's induction currents produce distortion in the sound. To avoid this defect, two wires are used and the Earth is not used as the *return*.

339. The Dynamo. We have learnt in Chapter VIII, that if a closed circuit be moved in a magnetic field, in such a way as to vary the number of magnetic lines of force enclosed by it; then a current is induced, the direction of which depends upon the direction of the lines of force. From **Lenz's Law** it is evident, that if a coil of wire be held between the poles of a magnet with its plane perpendicular to the lines of force and then rotated about a vertical axis; the number of lines of force passing through the coil is reduced from maximum to zero, when the coil rotates from 0° to 90° and an induced *E.M.F.* is set up in one direction.

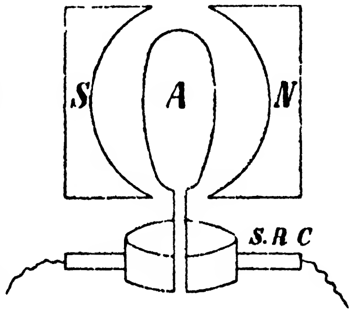


FIG. 52

On rotating it from 90° to 180°, *i.e.* from the plane of the field to a plane perpendicular to it; the induced *E.M.F.* is in the same direction as before, but diminishes from maximum to zero.

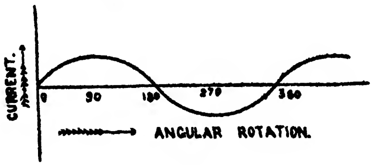


FIG. 52 (*i*)

When rotated from 180° to 270°, the *E.M.F.* is in the opposite direction, increases in value and becomes zero, when the coil has become vertical. At every half revolution, the current is reversed; such a current is called an *alternating current*. Fig. 52 (*i*) shows how the current in the circuit changes during one complete revolution of the coil. The number of complete cycles in each second, is called the *frequency* of the alternating current. The coil of wire, which is rotated in the magnetic field to give current, is called the *armature* and is wound over an iron core, which (*i*) serves as a rigid support for

the coil of wire and (*ii*) causes a great increase in the number of magnetic lines of force, cut by the coil of wire, when it rotates; for the magnetic lines of force prefer a path through soft iron, due to its great permeability. If the field be due to powerful magnets, and the armature consist of many turns, the apparatus called the *dynamo*, would be capable of lighting a small 6 volts lamp. The magnets, which produce the field are generally electromagnets and are known as *Field magnets.*

If the armature be rotated slowly, then the light would flicker; and it would appear continuous, if the number of revolutions be about 25 per second. The alternating current supplied for lighting and other purposes is generally of frequency 50.

The alternating current from the armature is conveyed by two brushes, touching the metal rings attached to the ends of the armature

To get *direct current*, the ends of the coil are joined to the two halves of a split tube or ring, as shown in fig. 52. Such a device is called a *commutator*. With such an arrangement, when the coil is rotating, one of the brushes will always be in contact with that half of the split ring, through which the current is coming from the armature and the other with that half, through which the current goes to the armature. The current thus obtained is graphically represented, as shown in fig. 52 (*ii*).

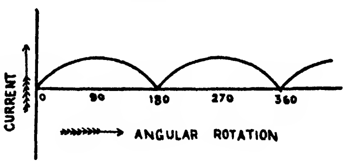


Fig. 52 (*ii*)

By rotating very rapidly an armature, having a number of coils wound round it, we can get a current of almost uniform strength.

The magnetic field, in which the armature rotates, is produced by powerful electromagnets. The current for the electromagnet is produced by the dynamo itself.

339 (*a*) **Transformers.** Whenever electric energy, popularly known as electric power, is to be sent from the generating

station to distant centres of consumption, the problem of dis-

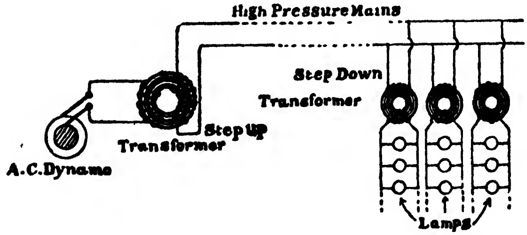


Fig. 53

tribution, with the minimum of wastage and expense is the one to be tackled with. Electric energy in *watts* is given by the product of *volts* and *amperes*. Thus if it is desired to transmit 1,00,000 watts from the Mandi Hydro-Electric supply station to Amritsar, a distance of about 75 miles; and if we are to send a current of 400 amps at a voltage of 250, we would in the first instance require very thick copper wires, the cost of which would be prohibitive; secondly there would be enormous wastage of energy resulting in the heating of transmission lines; and thirdly the voltage available at Amritsar will be much less than 250.

All these difficulties are successfully overcome by transmitting energy at a very high voltage (say 10000 in this particular case) and low current strength (of 10 amps.). High voltage does not require thick wires and thus ordinary wires are used. As the current is not very strong, the wastage of energy due to heating will also be negligible. Energy at such high voltage can neither be easily produced by dynamos nor easily utilized and is very dangerous. Therefore at the generating station, a device to convert a current of low voltage and high amperage into one of high voltage and low amperage, is used. Such a device is called a **step-up transformer** At the consumption station, a similar device is used to convert a current of high voltage and low current strength into one of low voltage and high current value. Such an arrangement is called a **step-down transformer.**

The alternating current transformers consist of a ring of soft iron having two coils, one of thick copper of few turns and the other of many turns of thin wire. Thus, if at the generating station the dynamo is giving a current of 500 volts and this is to be converted into a current of 10000

volts. The terminals of the dynamo are connected to the primary or thick copper coil of the transformer, the line wires are connected to the secondary or thin wire coil of the transformer. If the ratio of turns of the *secondary to the primary* is as 20 : 1; then the *E.M.F.* in the line wire (or secondary) will be $500 \times 20 = 10000$ volts. At the consumption station, if it is desired to lower the voltage to 250, the primary should consist of a coil of many turns and the secondary of thick copper wire of few turns. If the ratio of the turns of the primary to those of the secondary is as 1 · 40, the *E.M.F.* at the terminals of the secondary will be reduced to $10000/40 = 250$ volts, which is quite safe for consumption purposes. An arrangement of the installation is shown in fig. 53

How this transformation takes place, has already been indicated. The current in the primary magnetizes the soft iron ring and thus sets up a magnetic field within it. The lines of force due to this field, pass through the turns of the secondary coil and thus produce a momentary *E M F.* The *E.M.F.* at the terminals of the secondary bears the same ratio to the *E.M.F.* at the terminals of the primary, as the number of turns of the secondary bears to the number of turns of the primary. The energy obtained from the secondary can never be greater than that put into the primary.

To transform a direct current is a rather tedious affair. A direct transformer consists of a combination of a motor and a dynamo. It is not possible to get from dynamos a direct current, of voltage higher than 600; when higher voltages than this are required, use is made of rectifiers, which convert an alternating current of high *E.M.F.* into one of direct current.

340. The Electric Motor.

If two semi-circular iron bars be taken and a wire wound round both of them in the same direction, and if battery connections as shown in fig. 54 (*i*) be made; then we shall have two semi-circular electromagnets, having their similar poles near each other.

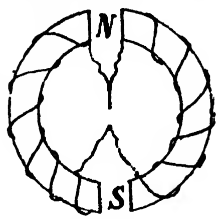


Fig. 54 (*i*)

These two semi-circular bars can be joined together so as to form a ring, which shall have a strong *S.* pole at one end and a strong *N.* pole at the other, as shown in

fig. 54 (*ii*). If such a ring be placed between the poles of a strong horse-shoe shaped magnet, then the *N*-seeking pole of the ring would be repelled by the *N*-seeking pole of the magnet and its *S*-seeking pole repelled by the *S*-seeking pole of the magnet; so that the ring would tend to move in the direction, shown by the arrows. Now if we could arrange to make the current always enter and leave at right angles to the direction of the lines of magnetic force, then the ring should go on moving continuously. This is the principle of an **electric motor**. The ring, with the coils of wire wound round it, is called the **armature**, as in a dynamo and the magnet is called the **field magnet**.

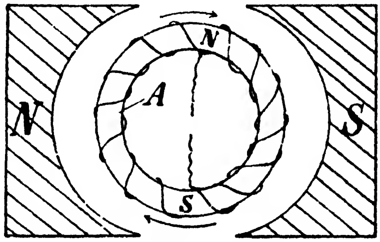


FIG 54 (*ii*)

The arrangement by which the current can be made always to enter and leave at right angles to the direction of the lines of magnetic force, due to the field magnet, is called a **commutator**. Its construction is as follows:—The battery terminals end in brass plates, as shown in fig. 54 (*iii*); and every turn of the coil wound round the ring, carries a small projecting wire, which when the armature is rotating, comes in contact with the brass plates, one after the other. Thus the current is always made to enter and leave at the two brass plates.

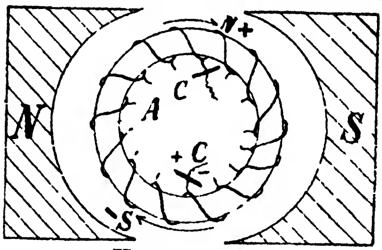


FIG. 54 (*iii*)

From what has been said, it is clear that there is no difference in the construction of a dynamo and an electric motor. In the latter, current is made to do

mechanical work, while in the former mechanical energy is converted into electric energy.

341. Wireless Telegraphy. It is the method of communication between distant places by electromagnetic waves, propagated through ether.

Production of waves. In 1888, Hertz the celebrated German Physicist showed, by arranging two knobs to form a spark gap across a big Leyden jar, that when a spark passes between the knobs, the positive and negative charges rush to and fro in opposite directions. The motion of the charges is similar to the motion of a pendulum bob. Just as the bob at its lowest point overshoots its mark due to its kinetic energy; similarly the opposite charges overshoot, and for a moment the knobs become oppositely charged. The process continues till the energy of the charge is dissipated away in the form of heat or in the form of radiations, known as electromagnetic waves. That waves are given out, when a spark passes, was predicted by Maxwell and experimentally demonstrated by Hertz.

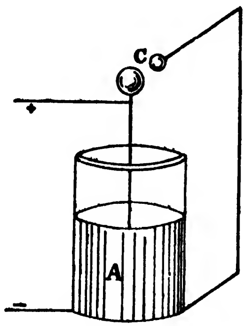


FIG. 55

Detection of electromagnetic waves. In Chapter VIII, it has been shown that when a conductor is moved so as to cut the lines of magnetic force, an *E M.F.* is induced in the conductor. The same result follows when the conductor is kept stationary and the magnetic lines of force move. All what is needed is, relative motion between the two. Thus when magnetic waves produced in the manner described above, strike against a vertically-held metal wire, an induced *E.M.F.* is set up in the wire. The *E.M.F.* so induced rapidly alternates, as the approaching waves undergo rapid changes of direction. The alternating *E.M.F.* sets up oscillating currents, which may be detected by suitable apparatus.

In wireless, arrangements are made to impress the modulations of the speaker's voice on ether at the transmitting station, while at the receiving station arrangements are made to receive these ether waves and convert them into sound.

342. Wireless transmitter. At the transmitting station, a long wire called the *aerial* is suspended from the top of a very high pole and the lower end is connected to an arrangement (like that of an induction-coil) to send an alternating current in the aerial wire. Every time the current goes up and down the aerial, electromagnetic waves are set up in the ether; they travel in all directions, though they become weaker as they go far off from the source. These waves are transmitted with the velocity of light. Signals are transmitted, by causing interruptions in the current of the aerial wire at the transmitting station. To get an alternating current of high frequency, several devices are resorted to, one of them being the induction-coil. One end of the secondary coil is earthed and the other end to the aerial wire; and a little sparking gap is provided between the two.

In order to get best results, the aerial wire has to

A=aerial
V.I.=variable inductance
V.C.=variable capacity in series
E=Earth
S.G.=spar gap
S.I.=secondary of induction-coil
P.I.=Primary of induction-coil
K=key for interruptions.

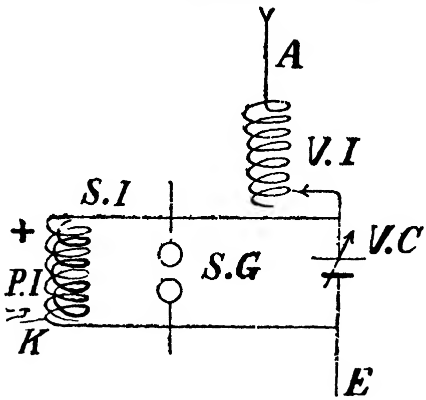


Fig. 56

be proportioned according to the frequency required,

i.e. for high frequency the aerial should be small and for low frequency the aerial should be long. In fact the aerial should be equal to half the wave-length.

It is difficult, if not impossible, to vary the length of the aerial every time; for this purpose a variable inductance and a capacity are arranged in the transmitting circuit. The effect of an **inductance** *in series* and a **capacity** *in parallel*, is to increase the effective length of the aerial; while the effect of a capacity *in series*, is to decrease its effective length. Thus an ordinary transmitting apparatus is of the form, shown in fig 54.

343. Wireless Receiver. When the electromagnetic waves sent by a transmitting station impinge against the aerial of a receiving station, oscillating currents are set up in it. To adjust the receiving aerial to different wave-lengths, a condenser and an inductance are included in it, as in a transmitting aerial. In order to listen to the signals, a telephone receiver is also essential.

A=Aerial
V.I.=Variable inductance
E=Earth
V.C.=Variable condenser
C=Condenser
T=Telephone
C.D.=Crystal detector.

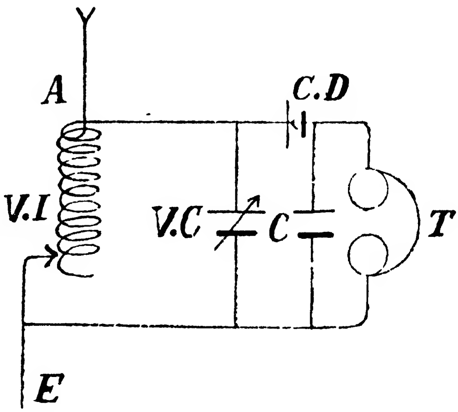


FIG. 57

A telephone does not respond to an alternating current of high frequency, due to its inertia; it requires a direct current for its operation. A steady current will not produce any sound in a telephone. It is a varying current, which produces sound in a telephone.

Therefore the oscillating alternate current in the receiving aerial is made uni-directional, by a device known as **crystal detector**, which consists of a crystal of carborundum. It allows current in one direction only and thus makes the sound audible in a telephone. This property of the crystal is called *rectification*. The receiving set of a simple crystal detector is shown in fig. 57.

Thermionic Valve. Thermionic valve, as first devised by Fleming in 1904, consists of a filament of tungsten and a cylinder of nickel, supported by a platinum wire, insulated from one another and enclosed in a vacuum bulb like an ordinary incandescent lamp. The tungsten filament called the *Cathode* or *Filament* is made to glow by passing a current from two or three accumulators. The nickel cylinder called the *Anode* or *Plate* is maintained, at a positive potential with respect to the filament. When the filament is heated by an electric current, electrons are shot out from it. Being negatively charged, they are attracted by the Anode. Thus a negative current may be supposed to flow from the filament to the Anode or a positive one from the Anode to the filament.

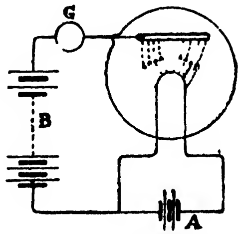


FIG. 58

If the Anode were negatively charged, no current would flow; because instead of attracting the electrons, it will repel them. Thus the valve allows current in one direction only and acts as a *rectifier*. In order to maintain the Anode at a constant positive potential, it is connected as shown in the diagram to the positive pole of a high tension battery; the negative pole of which is connected to the negative of the low tension battery, used to light the filament. When the filament is lighted, a current will be noticed to flow through the galvanometer, connected in the Anode circuit.

Fig. 59 shows how such a valve may be used as a rectifier. The electric waves falling on the aerial give rise to

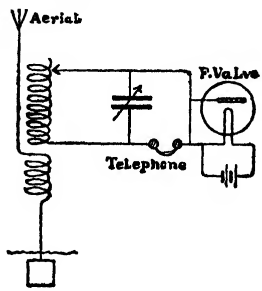


FIG. 59

alternating induced *E.M.F.* in the circuit and thus the anode becomes alternately charged positively and negatively. When it acquires negative charge, no current flows but when it acquires +charge, current flows and a click is heard in the telephone.

The Fleming valve described above has been modified by the introduction of a third electrode, between the filament and the Anode. This is generally made up of copper gauze or a spiral of thin wire and is called the *Grid.* Such a valve is called a Triode valve, see fig. 60.

Since the grid is nearer to the filament than the anode, a change of potential difference between it and the filament produces a greater change in the plate current. The grid thus provides a means of controlling the anode filament current by small potential changes. Thus the addition of the grid enables us to use the triode valve as an amplifier.

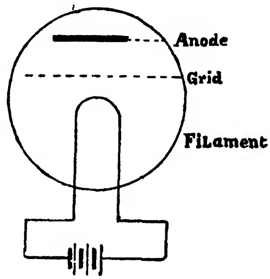


FIG. 60

The diagram illustrates how the first triode valve acts as an amplifier. The small *E.M F.s* induced in the aerial by the striking of waves are impressed on the grid. These cause big variations in the Anode current. This current passes through the primary of a transformer, the secondary of which forms part of a second circuit and so on. The amplification can be increased by using a number of valves. The several circuits are 'tuned' *i.e.* their times of vibrations are made the same, in order to get maximum effect. The phenomenon is then like resonance in sound.

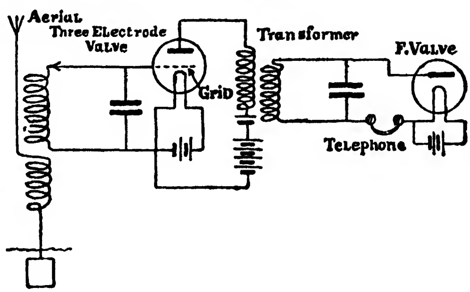


FIG. 61

344. Safety Fuses. To prevent a strong current

from running through any circuit, safety-fuses are used. These (safety fuses) usually consist of a short length of wire of an alloy with high specific resistance and low melting-point; and depend, for their action, upon the thermal effects of currents. The diameter of the wire is so selected that a current, stronger than that required in the circuit, would heat it up to its melting-point and so break the circuit. The diameter d, in millimetres, of a wire which fuses with any given current of C amperes, is given by the relation $d=\left\{\dfrac{C}{a}\right\}^{\frac{2}{3}}$, where a is a constant depending on the material.

For copper $a=80$
for tin $a=12.8$
and for lead $a=10.8$

345. Electroplating. The coating of an object with a coherent layer of any metal, is called electroplating. The metals usually employed for coating purposes are silver, gold, nickel and copper. The process of electroplating depends upon the principle of electrolysis. The object to be coated is thoroughly cleansed by washing in soda and acid respectively; and is then made the *kathode* of an electrolysis cell. For the metallic ions always travel with the current. The anode must consist of the metal with which the object is to be coated and the electrolyte should be the solution of a salt of the same metal. If the object to be coated is a non-conductor, its surface is rendered conducting by covering it with a thin layer of graphite.

In electro-silvering and gilding, the electrolytes used are: (*i*) double cyanide of silver and potassium and (*ii*) double cyanide of gold and potassium respectively.

EXAMPLES

1. Describe the construction and working of an ordinary electric bell.

2. What is the difference between a dynamo and an electric motor?

3. What is a safety fuse? What is its use and what conditions are necessary for the material of the fuse?

4. What do you understand by electroplating? How would you electroplate a ring with gold? Sketch the form of the apparatus.

5. Describe the open system of telegraphy and state the function of relays.

6. What is a telephone? Describe a modern transmitter.

CHAPTER X

X-RAYS & DISCHARGE OF ELECTRICITY THROUGH GASES

346. Spark Discharge. The difference of potential between the electrodes, required to start the discharge, is called the *Spark Potential.* It is independent of the material of the electrodes but is directly proportional to the distance between them and the pressure of the gas. This statement is known as **Paschen's law.**

In order to study the discharge at low pressures, we take a sufficiently long tube, as shown in fig. 62, with two metallic electrodes fused into the ends and a side tube for reducing the pressure. The latter is connected to a mercury air-pump and the electrodes, to the secondary terminals of an induction coil. No discharge takes place at the ordinary pressure, because the distance between the two electrodes is greater than the spark length for the coil.

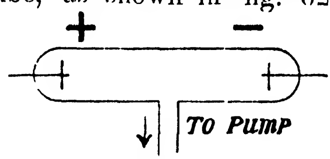


FIG. 62

Now work the pump and begin reducing pressure till a point is reached, when intermittent crackling flashes of rosy-coloured light begin to pass between the two electrodes. If the pressure be decreased still further, the crackling nature of the discharge disappears, its path widens and it takes place quietly. The colour of the discharge depends upon the nature of the gas contained in the tube At this stage, a difference between the two ends of the discharge becomes noticeable and a discontinuity near the cathode is seen. The glow loses its uniform character and assumes different characteristics at different places of

the tube, which becomes marked with **striæ**, consisting of layers of luminosity separated by dark spaces.

The discontinuity noticed near the cathode, called the **Faraday's dark space**, increases in size; and with decreasing pressure, it becomes covered with a luminous glow. If the pressure be reduced still further, then it is noticed that the striæ thicken and Faraday's dark space becomes less distinct. The luminous glow separates from the kathode, leaving a second dark space, known as **Crook's dark space**. The positive column, consisting of the striations, extends from the Faraday dark space up to the anode.

Further reduction of the pressure results in the growth of the Crook's dark space and the disappearance of the positive column. At this stage, the pressure is nearly 0·037 mm. The Crook's dark space is always bounded by a luminous glow; and when it extends to the glass walls of the tube, a bright phosphorescence is seen. The colour of the glow depends upon the nature of the glass used. So far, decrease in pressure is accompanied by decrease of resistance; but further reduction causes the resistance to increase enormously; the highest vacuum is thus a perfect non-conductor, and it becomes impossible to get a discharge.

347. Cathode rays. When the vacuum inside a discharge tube reaches such a stage, that Crook's dark space extends to the walls of the tube and produces phosphorescence; then the phenomena has been investigated by Sir William Crook, to be due to *something emitted with high velocity from the cathode.* These are called **Kathode rays,** which consist of a stream of very small particles, carrying a small *negative charge.* These particles are known as electrons. Their mass is nearly $\frac{1}{2000}$ of an hydrogen atom and their velocity is nearly $\frac{1}{10}$ th that of light. Their other properties are :—

(*i*) *They travel in straight lines.* This is demonstrated by placing an obstacle in the path of the rays (fig 63) The obstacle is observed to produce a shadow on the wall of the tube.

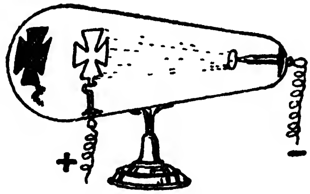


Fig. 63

(*ii*) *They possess momentum and thus produce mechanical pressure on the body, against which they impinge* This is shown by the help of the apparatus of fig. 64. The rays, when they impinge on the mica mill-wheel, set it into rotation.

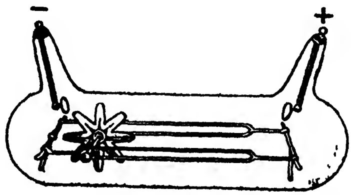


Fig. 64

(*iii*) *The rays produce heat when they fall on matter.* If the cathode be made concave and a piece of thin platinum sheet be placed at its focus, it is seen to become red hot.

(*iv*) *The cathode rays are deflected by a magnetic field, like an electric current.* The effect can be shown very easily, with the help of the tube of fig 63. The shadow of the cross is made to go up or down, by the application of a strong magnetic field, produced by powerful electromagnets.

348. Positive or Canal Rays. If the cathode be perforated, then rays are seen behind it. They carry a small amount of positive charge and are known as Positive or Canal rays.

349. X-Rays. If the kathode *be made concave* and the rays be focussed on a piece of platinum, by placing it at the centre of curvature of the electrode, with its plane at an angle of 45° to its axis, fig. 65; then radiations of very short wave-length, known as *X*-rays are given off from this platinum piece, called the **anti-kathode**. These rays, known also as **Rontgen rays**, after the name of their discoverer, differ from

the kathode rays.

These rays are not deflected by a magnetic or an electric field, and hence they *cannot* be a stream of charged particles. They are different from light, because they can pass through many substances, which are opaque to ordinary light. Substances like lead, which have high atomic weight, are opaque to X-rays; while those like aluminium, that have low atomic weight, are transparent to them. X-rays ionise the gas through which they pass and thus make it a conductor of electricity. Being waves of extremely short wave-length, they cannot affect our retina and hence are invisible. They readily affect a photographic plate and are capable of exciting powerful fluorescence in many substances. The latter property is most marked in barium platino-cyanide and its screens are used to detect X-rays. If a hand be interposed in the path of X-rays and then a screen, coated with the above compound, be placed between the hand and the eyes, a shadow of the bones becomes visible on the screen.

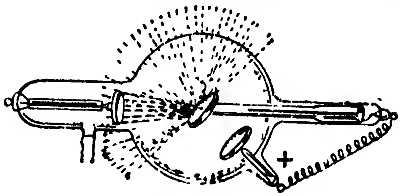


Fig. 65

X-rays are ether-waves of small wave-lengths. They are produced by the sudden stoppage of electrons by the anti-kathode. The higher the vacuum and the harder the anti-kathode, the greater is the penetrating power of X-rays. The wave-length of X-rays is roughly $\frac{1}{1000}$ of the wave-length of ordinary light. Their velocity is just the same as that of light.

X-rays are used by surgeons to locate fractures of bones or other extraneous matter in human body, by **Radiographs** fig. 66, which is an X-ray photograph of a child's hand, taken with Coolidge tube in the Government College, Lahore. They are now extensively used

to study the structure of crystals. X-rays may also be used to detect the contents of a closed box. Whenever

Radiograph of a child's hand.

FIG. 66

X-rays fall on a body, secondary rays of longer wave-lengths are given out by that body. This 'effect' is known as **Compton effect**, after the name of the discoverer.

350. The structure of the atom. The atom of a substance, according to the modern conception, is supposed to consist of a central nucleus of positive charge called the proton and a number of electrons, revolving round it in definite orbits like the planets, with the following differences.—(*i*) the ratio of the size of the electron to that of the atom is much smaller than that of any planet to the Sun; (*ii*) the planets are held in their courses by the force of gravitation, while the little electrons are kept to their circular paths by the force of electric attraction; and (*iii*) all electrons are identical, while the various planets differ in size.

EXAMINATION QUESTIONS XII

1. Give an account of the methods of producing (*i*) Cathode rays (*ii*) X-rays, and a general comparison of their properties. (*P.U.* 1931).

2. Draw a diagram to show how telegraph messages are sent from Lahore to Bombay; relay is to be shown in the connections.

3. Three wires, each having a resistance of 15 ohms, are connected in series and then in parallel to the poles of a battery of negligible resistance and of *E. M. F.* 5 volts. Compare the currents in the two cases.

4. A lead accumulator (*E. M. F.* $=2$ volts) furnishes a current of 10 amperes through an ammeter of 0·1 ohm resistance. Find the resistance of the accumulator.

5. A voltmeter (resistance 2000 ohms) is shunted with a wire of 1 ohm resistance, to act as an ammeter. What fraction of the total current will flow through the voltmeter coil?

6. An electric lamp of 25 watts requires $\frac{1}{8}$th. of an ampere. Will an accumulator, capable of giving 30 amperes on short circuit, light the lamp? Give reasons.

7. Describe the construction of an accumulator and state the actions, which take place.

8 Define specific resistance and give a method of finding the same by a Post-office box.

9. What is Local Action and Polarization? What methods are used to make a Volta cell work continuously?

10. Describe Ruhmkorff's induction coil Suppose you want to transform a current of high tension into one of low tension, what will you do?

11 Name and define the practical units of current, resistance, *E. M F.* and electrical energy

12. Describe how you will receive wireless signals with a crystal set. How will you arrange to tune the instrument?

13. Describe how to get X-rays What are their properties and how should an operator protect himself?

14. Describe either an electric motor or a telephone.

15. What are the chief differences between an Ammeter and a Voltmeter?

16. State Faraday's Laws of Electrolysis and Electromagnetic Induction.

17. Why does an electric lamp glow, while the *lead* wires remain cool?

18. Describe the construction and working of a tangent galvanometer. How will you make it sensitive? What advantages has a moving-coil galvanometer over the tangent form?

ANSWERS

Examples—(P. 9)

3. 24850 miles nearly.
4. 50·91 lbs. per c. foot.
5. 9952·38 gms.
6. ·5 lb. nearly.
7. 7079 litres; 250 c. feet; 7079 kilograms. 15625 lbs.

Examples—(P. P. 19—23)

8. 17·73 feet per second.
9. BC
10. $10\sqrt{2}$ in N.W. direction.
11. 44·7 miles at an angle tangent^{-1}=·5 with the north.
12. 70 feet per second.
13. 7 miles per hr. up the stream and 1 mile per hour down the stream.
15. 15 miles per hour due east.

Examples—(P. P. 29—33)

9. Retardation of $\frac{132}{225}$ ft. per second per second.
10. 38·4 feet.
11. $\frac{11}{30}$, $3\frac{17}{75}$, $40\frac{1}{3}$ ft. sec.^{2}
12. 337·5 feet.
13. velocity=86·4 ft; height=518·4 feet.
14. 540 miles per hr. per hr.
15. 16 feet; 256 feet.
16. 100 feet, 80 feet.
17. Total time=$3\frac{5}{8}$ secs. total distance=$210\frac{1}{4}$ ft.
18. 5 secs., 220 feet, $60\frac{1}{2}$
19. $32\sqrt{5}$ ft. per sec., $\sqrt{5}$ secs.
20. $\frac{11}{10}$ sec.
21. 200 feet.

Examples—(P. P. 46—47)

3. $562\frac{1}{2}$ Poundals.
4. 735750 units of momentum.
   73·575
5. 256 feet and 2000 feet.
6. 200 dynes.
7. 2·5 feet per second.
8. One minute.
9. $4·524\times10^7$
10. 224 : 675
11. 2 seconds, 64 feet from the roof, 320 units.
12. One revolution per second.

Examples—(P. P. 58—61)

8. It will be diminished.
9. $g=981\cdot3$
10. 16 feet per second.
11. 11200 poundals.
12. 5 : 16
13. $6\frac{9}{16}$ lbs. weight.
14. 25 feet.
15. 9.7
16. $179200\sqrt{3}$
17. $\theta=\sin^{-1}\frac{1}{48}$
18. 16 feet per second.
19. 28 feet per second.
20. $18^1/_3$

Examination Questions I—(P. P. 61—62)

6. $2^1/_2$ lbs. weight.
7. 4840 feet.
8. 32 feet per second.
9. $7^1/_2$ seconds.
10. As 1 : 327

Examples—(P. P. 67—68)

3. $^7/_8$ ton weight.
4. 24 feet and 12 feet per sec.
5. $9\cdot658$ feet per sec. per sec.
6. $2\cdot24$ lbs. wt. per ton.

Examples—(P. P. 74—76)

4. $4^1/_2$ feet from 4 lbs. end.
5. 5 feet.
6. $2^1/_2$ lbs. weight.
7. 40 lbs.
8. 50 lbs. weight.
9. 3 feet.

Examples—(P. P. 84—87)

10. $754\cdot3$
11. $6\cdot0\times10^6$ gms. wt. cms.
12. 1792 ft. lbs.
13. $346\cdot4$ metres. per sec.
14. 3320240 ft. lbs.
15. $163\cdot2$ H.P.
16. $\frac{1}{5}$ H. P.
17. $4065\frac{3}{5}$ lbs. wt.
18. $5\cdot12\times10^{10}$ ergs.
19. $6\cdot075\times10^8$ ergs.
20. 75 ft. lbs.

Examples—(P. P. 92—93)

4. $35\cdot50$ lbs. wt.

Examination Questions II—(P. P. 93—94)

5. 2685×10^{10} ergs.
9. 30° and 45° respectively.

Examples—(P. P. 108—109)

5. $1^2/_3$ and $1^1/_3$
6. $4\frac{2}{3}$ lbs. weight.
7. 2 lbs.
8. 16 feet.

Examples—(P. P. 118—119)

5. True weight=10·247 lbs. 7. 3*s*. 9*d*. and 2*s*. $4\frac{4}{5}d$.

Examples—(P. P. 139—140)

4. 22·8 feet.
5. 169125 lbs.
6. 14 tons and $3^1/_2$ tons.
7. 800 lbs.
9. 36·864 feet.
10. 5·63 lbs. wt per sq inch.

Examples—(P. P. 144—145)

5. 3218·75 c. yds.
6. $2\frac{7}{9}$

Examination Questions III—(P. P. 145—146)

1. 7·31
3. 2910, 3022461
6. $21\frac{13}{19}$
7. 2 gms.
8. ·675 . ·6
9. 90·96 gms. weight

Examples—(P. P. 154—156)

4. Specific gravity of solid=5 ; sp. gravity of liquid=2.
5. 200 c. c.
6. $\frac{6}{7}$
7. 2
8. 14 inches, $\frac{6}{7}$.
9. Sectional area= 04 sq cms and density=6 gms. per c.c.
10. 15 gms.
11. Alcohol is 0·309 of the mixture.
12. 88

Examples—(P. P. 163—164)

5. 71·35 cms
7. 27·2 inches.
8. 30 inches.
9. 75 cms.
10. 15 c. c.

Examples—(P. P. 170—171)

4. $42^1/_2$ feet. 30 inches.
5. 1070 cms. of water column.

Examination Questions IV—(P. 176)

3. 12·92 metres.
6. 3·04 metres.

HEAT

Examples—(P. 196)

4. 36·95°C., 89·32°C.
5. 20·76

Examples—(P. P. 223—226)

7. 0·124 inches.
8. 0·0001, 0·0003
9. 10·20
10. 0·000009
11. 64·1°C.
12. 3·6°C.
13. 20 for O and 300 for H on F.P.S. system, or $O=2{\cdot}75\times10^6$ and $H=4{\cdot}14\times10^7$
(In questions like these the equation $RV=PT$ refers to lb. or gm. molecule of the substance).
14. 452°C.
15. 234·7

Examination Questions V—(P. 226)

3. 26·6°C., —40°C., 50°F.

Examples—(P. P. 234—235)

3. 12 gms.
4. 79°C.
5. 0·091
6. 0 138
7. 77 9
8. 0·62

Examples—(P. P. 246—247)

3. 67·3°C.
4. Yes, 0·39°C.
5. ·916
6. 86·87
7 ·23

Examples—(P. P. 256—257)

3. 318 gms.
4. 29 9 gms.
5. 68·1°C.
6. 59·1 gms.

Examples—(P. 269)

3. 74·5 per cent.
4. 318·7 c.c.

Examination Questions VI—(P. 270)

6. ·11

Examples—(P. P. 287—288)

3. 18000 kilo-gms.
4. 82·5 gms.
5. 32·5°C.
6. ·2

Examples—(P. P. 296—297)

3. ·233°C.
4. $8·0 \times 10^5$ cms.
5. 132000 lbs. F units.
7. $2·058 \times 10^6$ cms.

Examination Questions VII—(P. 297)

1. 79 cms.
5. 800°C.
19. ·000187
22. 1 mm.

LIGHT

Examples—(P. 308)

1. 1920 Candle Power.
2. 1·414 feet.
3. 36 per cent.
4. 1·66 feet.
5. 210000 miles.
6. 264 feet.
7. Umbra circle 2″ diameter. Penumbra surrounds it and a ○ of 3″ radius.

Examples—(P. P. 328—329)

3. 33·33
4. 6 mms.
5. 30 cms.
8. 8 inches behind the mirror.
7. 171·9 cms.

Examples—(P. 349)

1. $\operatorname{Sin}^{-1}\frac{3}{4}$
2. 1·41
3. 6 inches.
4. 2 inches.
5. +20 inches; concave lens.
6. v=28 cms. beyond the second lens; image real, inverted, 3·2 cms. high.
7. $6\frac{2}{3}$ ft. from the candle, $4\frac{4}{9}$ ft.
8. 5 cms.
9. 12 cms.

Examples—(P. 371)

1. 6
2. $6^2/_3$ cms.
3. 100 and 302·68 cms.
4. −11·6 inches f. length.
5. 3·75 and 4
6. ·251 inch.

Examination Questions VIII—(P. 374)

1. $4^1/_2$ feet.
2. 2 inches and 4 inches.
3. 60 cms
5. 10 and $6^2/_3$

SOUND

Examples—(P. P. 390—391)

3. 3360 ft.
4. 4·7143 secs. slow.
5. 272·5 ft. per minute, distance=4360 ft.

Examples—(P. 395)

1. 45
2. 300
3. 23·26
4. 49 5
5. 400 lbs.

Examples—(P. 405)

1. 261
2. 256, 288, 320, etc.
3. $340\sqrt{1\cdot5}$ metres per sec.
4. 336·66 metres per sec.

Examples—(P. 413)

1. 1·1 ft.
2. 44·8 cms. approximately
3. 128 per sec.
4. 120 cms.
5. ·71 ft.

Examination Questions IX—(P.P. 413—414)

7. 336 ft.
8. 3·14 approximately.

STATICAL ELECTRICITY

Examples—(P. 423)

5. 1·33 dynes.

Examples—(P. 443)

4. $\frac{31}{64}\pi$
5. 4·4 approximately.
7. 3920π

Examples—(P. 461)

5. 4, $\frac{15}{8}$, 13·4 approximately.
6. $\frac{63}{80}$, $\frac{63}{640}$

Examples—(P. 465)

2. 1/45 dyne.

Examination Questions X—(P. P. 467—469)

6. 89·4 e.s. units.
11. Potentials 1 : 5
    Surface densities 1 : 5
12. 208·8 e.s. units.
13. $C=2$; $V=4$.
14. $V=40; Q_1=400; Q_2=600$.
15. 1 : 0·96

MAGNETISM

Examples—(P. 484)

7. $\frac{3}{7}$ cms. approximately.

Examples—(P. P. 497—498)

1. $M=50$
   $m=10$

14. 152
15. ·0081
16. 160 ergs.
17. 0·25 dynes and 50 units.
18. 13 : 28
19. 1·5 dynes.

Examination Questions XI—(P. 510)

3. 160 C.G.S. magnetic units.
4. ·55 very approximately.
5. 67°-40′

CURRENT ELECTRICITY

Examples—(P. P. 581—584)

8. 0·8 ohms.
9. 440 ohms, 110 watts.
10. 35 ohms.
11. 600 Volts.
12. $\frac{1}{90}$ of an ohm.
13. 15 amperes (v. approximately).
14. ·091 ampere, 2 pies approximately.
15. $200\dot{1}$ Volts.
16. 49·5, 2 annas.
17. 48·8 ohms, 1 hr. and 25 mts. very approximately.
18. 5% ; 2 amps. 4·2 watts per C.P.
19. 9 mts.

Examination Questions XII—(P.P. 615—616)

3. $\frac{1}{9}:1$
4. 0·1
5. $\frac{1}{2001}$
6. No

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(*The numbers refer to pages*)

THE END.